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Interpolation of experimental values for working parameters of a construction machine in the data space of an arbitrary dimensionality

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Abstract. A technique has been developed for interpolating the experimentally obtained scattered data using radial basis functions, which can be used to study the constraints of kinematic, power, energy, economic and other parameters in construction, road and hoisting-and-transport machines, as well as their working patterns. The technique consisting of two computational steps arranged as two separate algorithms allows the design and machine working process parameters to be optimised. The technique can be used for the range of machine parameters with an arbitrarily large dimensionality. For the first time, detailed block diagrams of algorithms are presented for interpolating scattered data using radial basis functions. At the first stage, the weighting factor determination is performed for the radial basis function method in each experimental point of scattered data. The second stage involves the actual interpolation or extrapolation of the function value at a given point with arbitrary coordinates. The developed methodology allows rigid interpolation of scattered data, such as the field experiment values of various working parameters for the construction, road, hoisting-and-transport machines and their design parameters. The data space dimensionality for the experimental points is unlimited and can be arbitrarily large.

Keywords: interpolation, radial basis functions, algorithm, scattered data

1. Introduction

The development of improved control methods for construction, road and hoisting-and-transport machines is an urgent research task [1]. The use of mathematical models in the field of control for construction, road and hoisting-and-transport machines may produce a significant technical and economic effect by optimising the loading of working equipment, parameters of the working process, the machine cycle, etc. [2].

The study of the constraints and properties of construction objects, as well as the kinematic, power [3], energy [4], economic and other parameters of machines and the patterns of their work processes, is impossible without the use of modern numerical methods of modelling and optimisation [5]. For example, these methods solve the problem of minimising the fuel and energy consumption, the optimal choice of equipment working characteristics, etc. [6].

An important role in solving the problem of modelling and optimising the working processes of machines, as well as the development of algorithms and control programs for construction, road and hoisting-and-transport machines, is played by the interpolation of the machine working parameters obtained as a result of field experiments [1].

When conducting field experimental studies on real physical objects [1], experimentally measured values are frequently presented as scattered data (i.e., unevenly distributed points) [7, 8]. This is due to



the difficulty or impossibility of setting strictly defined values for all controlled parameters of construction, road and hoisting-and-transport machines, particularly in the dynamic mode [1].

A problem arises connected with interpolating the scattered (chaotic, random) data at intermediate points or on a regular grid [9]. In order to solve this problem, the global polynomial interpolation [10] can be used. This method is based on a single arbitrary shaped polynomial [11]. Global polynomial interpolation is a non-rigid interpolation method, i.e. the polynomial surface, as a rule, does not pass through all given points [12]. In addition, difficulties arise in determining an optimal analytical expression of the polynomial. With a large number of experimental points, the size of the polynomial, which provides acceptable accuracy, increases. This may be considered as a disadvantage of the method.

Another widely-used approach is based on the triangulation of given data points [13] with a subsequent verification that the interpolated point enters inside each of the obtained triangles [14] and finalisation by linear interpolation inside one of the triangles [15]. Delaunay triangulation is typically used in this case [16, 17].

Although linear interpolation along the planes of triangles is rigid, it demonstrates relatively low accuracy with respect to experimental data, which can be described by smooth nonlinear functions. This is particularly true for cases with a relatively small number of experimental points, i.e. when triangles are large.

In order to overcome the above-listed disadvantages, more precise, rigid methods for interpolating scattered points [18] were developed including: the Kriging method [19, 20] and the radial basis function method (RBF) [21]. The latter is used for constructing explicitly defined surfaces [22] and solving other problems [23].

The RBF method is quite simple and straightforward. However, available publications provide no algorithms for the implementation of the RBF method in a space of arbitrarily large dimensionality. The description of such an algorithm is the aim of this paper.

2. Formulation of the problem

A set of n_{rt} experimentally obtained scattered points is given in a space of d_{im} dimensionality. The coordinates of the points are denoted as x_i , where $i=1,2,\dots,d_{im}$. A single point with the $j=1,2,\dots,n_{rt}$ number in this space has the $[x_{1,j}, x_{2,j}, \dots, x_{i,j}, \dots, x_{dim,j}]$ coordinates. The function value at the j point is designated as F_j .

In addition, a *Point* point with $[x_{1,p}, x_{2,p}, \dots, x_{i,p}, \dots, x_{dim,p}]$ coordinates is given, the value of which must be interpolated. The value of the interpolated function, indicated as F_p , is to be determined at the *Point* point.

3. Interpolation algorithm using the radial basis function method

When solving the problem, the experimental points from the initial data set were presented in the form of a *Matrix_{input}* matrix with each row, except for the last element, presenting a coordinate vector of a separate experimental point. The last column of this matrix contains the values of the function at the corresponding experimental points:

$$Matrix_{input} = \begin{bmatrix} x_{1,1} & x_{2,1} & \dots & x_{i,1} & \dots & x_{dim,1} & F_1 \\ x_{1,2} & x_{2,2} & \dots & x_{i,2} & \dots & x_{dim,2} & F_2 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ x_{1,j} & x_{2,j} & \dots & x_{i,j} & \dots & x_{dim,j} & F_j \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ x_{1,nrt} & x_{2,nrt} & \dots & x_{i,nrt} & \dots & x_{dim,nrt} & F_{nrt} \end{bmatrix}.$$

The first step of the method, performed only once, involves the calculation of weighting factors (w) for each experimental point. To this end, in a space of the d_{im} dimensionality, the r Cartesian distance between all the experimental points is determined in pairs using nested cycles. The full expression of

the Cartesian distance depends on the space dimensionality. The calculation of the r distance between two points with i and j indices is based on summing the squares of the differences for the individual coordinates of the points in space using the recurrence formula in the inner loop.

$$r(i, j) = r(i, j) + (matrix_{input}(i, j_1) - matrix_{input}(j, j_1))^2,$$

where the i and j indices denote the numbers of two experimental points. The $j_1 \in [1; d_{im}]$ index denotes the serial numbers of the individual space coordinates (the inner loop of the recurrence formula is performed according to j_1). After completion of the recurrence formula inner loop, the square root is extracted:

$$r(i, j) = \sqrt{r(i, j)}.$$

Hereinafter, the presence of the same variables in the left and right parts of the formulas is determined by the syntax features in the algorithm software implementation in most common programming languages. The target (calculated) variable is sometimes used as one of the operands in the long right side of the expression. This provides for the total number of variables in the program to be reduced. In this case, the previous value of the target variable on the right side is taken. This is the value taken by the variable until the moment of the calculation described by the current formula.

Using the values of r pairwise distances, a RBF_{korn} square matrix of the $(n_{rt} \times n_{rt})$ size is compiled. Matrix elements are calculated by the formula of the polyharmonic spline most commonly used in the RBF method [24]:

$$RBF_{korn}(i, j) = (r(i, j))^2 \cdot \ln(r(i, j)). \quad (1)$$

The weighting factor of each experimental point is formed as a result of solving a system of linear algebraic equations (SLAE) with the RBF_{korn} coefficient matrix. The *Right* column of SLAE free terms (right parts) is the last right column of the $Matrix_{input}$ source data matrix:

$$Right = Matrix_{input}(1:n_{rt}, d_{im}).$$

In a matrix form, the SLAE of the considered problem has the form:

$$RBF_{korn} \cdot w = Right,$$

where RBF_{korn} is the SLAE matrix, w is the column of unknown weighting factors, *Right* is the column of free terms.

As a result of solving SLAE by known methods [22, 23], a w column of weighting factors for experimental points of n_{rt} size is formed. In this work, in order to solve SLAE, the *linsolve* function of the MATLAB system programming language was applied.

The second stage of the methodology includes the actual interpolation of the F_p multidimensional function value at the *Point* point with the given $[x_{1,p}, x_{2,p}, \dots, x_{i,p}, \dots, x_{dim,p}]$ coordinates.

The function value at the *Point* interpolated point is calculated as the sum with the number of terms equal to the number of experimental points. Each term of this sum corresponds to a specific experimental point with the i index and represents the product of the $w(i)$ weighting factor and the $RBF_{tek}(i)$ current RBF value for this experimental point relative to the interpolated *Point* point:

$$F_p = \sum_{i=1}^{n_{rt}} (w(i) \cdot RBF_{tek}(i)). \quad (2)$$

The $RBF_{tek}(i)$ current value of the experimental point RBF is determined by the dependence similar to the dependence (1):

$$RBF_{tek} = (r(i))^2 \cdot \ln(r(i)), \quad (3)$$

with the only difference that the r distance in this case is the Cartesian distance from the *Point* interpolated point to the experimental point with the i index. The RBF value is calculated using the recurrence formula

$$r(i) = r(i) + (matrix_{input}(i, j) - Point(j))^2, \quad (4)$$

where the j index denotes the serial numbers of the individual coordinates in the multidimensional space of d_{im} dimensionality.

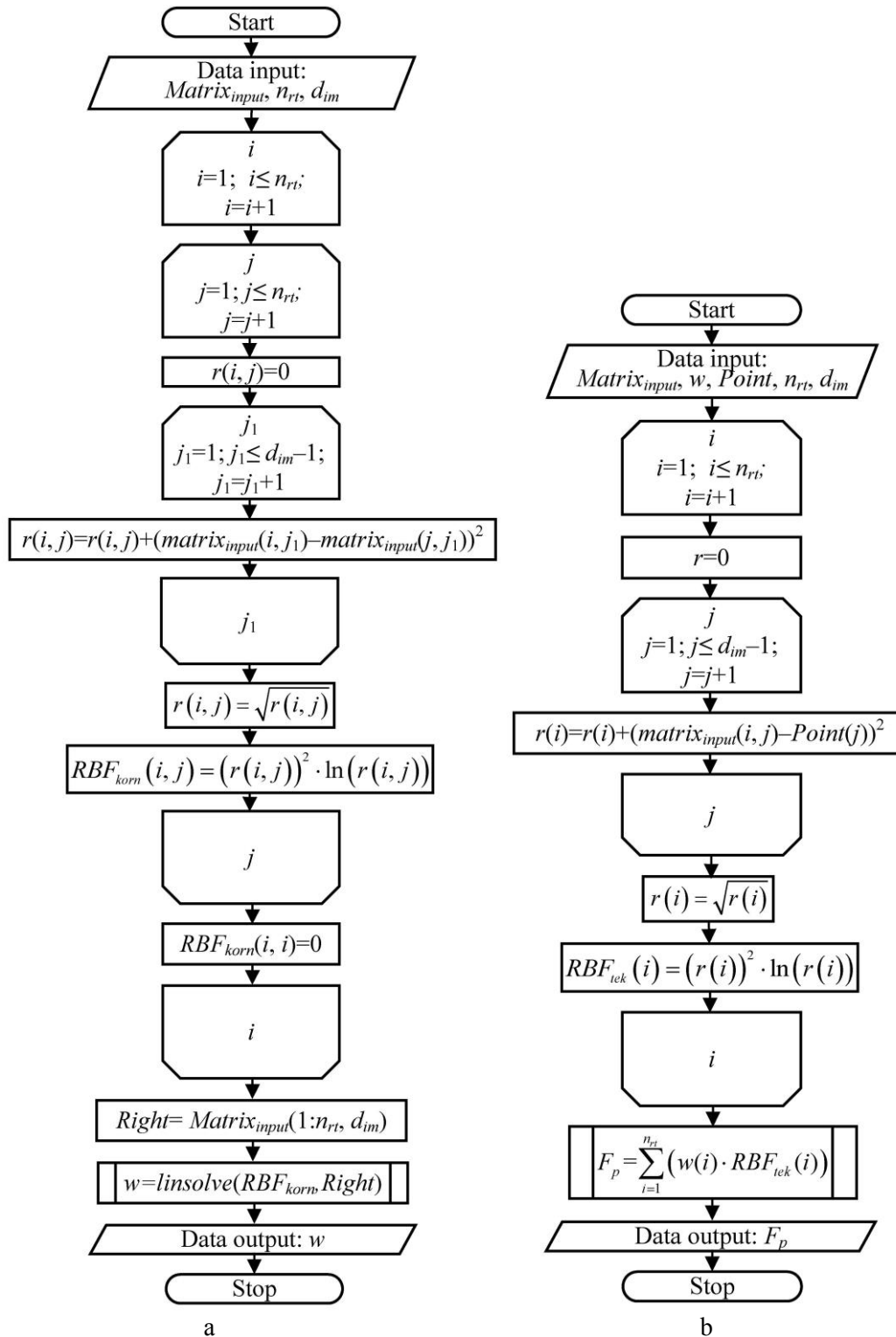


Figure 1. Block diagrams of algorithms for (a) calculation of weighting factors for experimental points and (b) interpolation by the method of radial basis functions in a space of arbitrary dimensionality.

When the w weights are known, the RBF interpolation is quite simple and basically consists in the following sequence of steps:

1. The n_{rt} of distances (r) are determined from the current interpolated *Point* point to all points from the set of experimental points according to formula (4).
2. For each point ($i=1, 2, \dots, n_{rt}$) from the set of experimental points, the $RBF_{tek}(i)$ RBF value is determined relative to the interpolated *Point* point according to the formula (3).
3. The value of the required F_p function is calculated at the interpolated point by the formula (2).

The detailed block diagrams of algorithms for determining weights and interpolation using the RBF method for a single point are shown in figure 1.

4. Experimental results

Let us consider the solution for the problem of interpolating scattered data by the RBF method using the interpolation example of the z force acting on the hydraulic cylinder rod for lifting the excavator boom with an empty bucket. In this case, the z force appears as a continuous function of two independent arguments: the rotation angles of the moving parts in the excavator working equipment - the x boom angle relative to the spindle and the y dipper angle relative to the boom:

$$z = f(x, y).$$

A set of n obtained experimentally scattered points is given with $z_j(x_i, y_i)$, $i=1, 2, \dots, n_{rt}$ coordinates (see figure 2). Due to the impossibility of setting absolutely exact values of the angles during the full-scale experiment, the indicated angle coordinates at the experimental points take random values within the working ranges of the $[x_{\min}; x_{\max}]$ boom and $[y_{\min}; y_{\max}]$ deeper angles each:

$$x_{\min} \leq x_i \leq x_{\max}; \quad y_{\min} \leq y_i \leq y_{\max}.$$

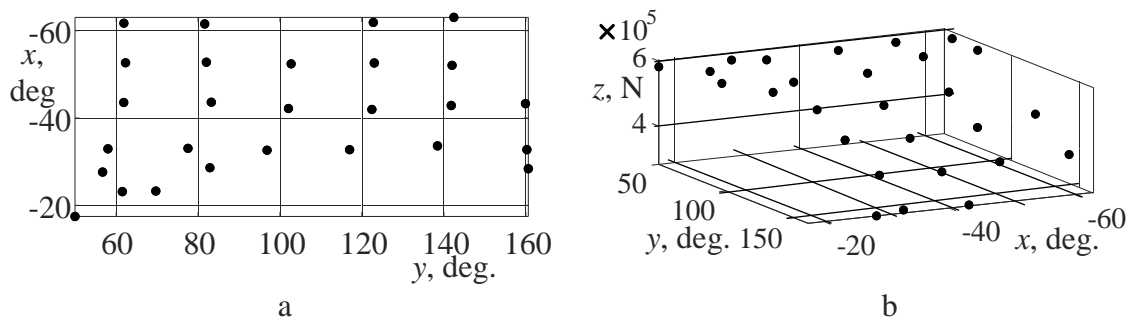


Figure 2. The experimentally measured z force on the hydraulic cylinder rod for lifting the excavator boom at different x boom and the y deeper angles (scattered data example): a - plot of the experimental point set in terms of two factors (top view); b - three-dimensional plot of the force function at the experimental points.

As an example, the function values were interpolated in the z_{jk} , ($j=1, 2, \dots, m$; $k=1, 2, \dots, n$) set of points with the given values (coordinates) of two angles (x_j, y_k). The angles in the (x_j, y_k) set of points for interpolation were given on a uniform grid in the range of $x_j = [-15^\circ; -65^\circ]$, $y_k = [40^\circ; 180^\circ]$, in increments of 2° . In the general case, these values do not exactly coincide with the coordinates of any of the (x_i, y_i) , $i=1, 2, \dots, n_{rt}$ scattered experimental points.

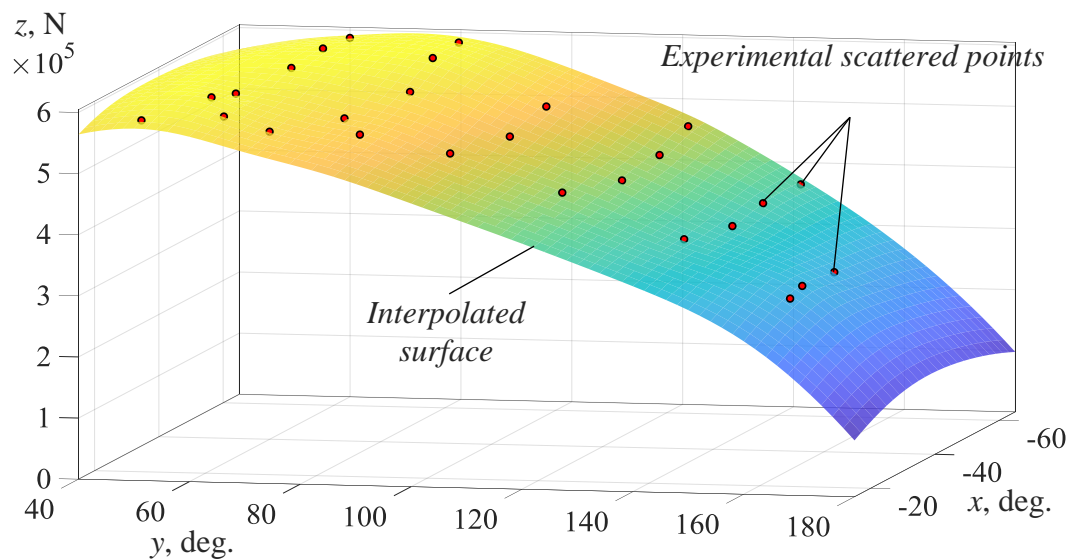


Figure 3. The surface of the z force values on the hydraulic cylinder rod obtained by interpolation of experimental scattered points using the radial basis function method (example).

The above argument ranges for interpolation include all experimental scattered points. The graphic results of interpolation by the RBF method are presented in figure3.

An analysis of the results obtained as an example demonstrated that the interpolated surface passes exactly through all the experimental scattered points and provides both interpolation and extrapolation of data, while being simultaneously rigid and smoothed.

5. Conclusions

As a result of the research, a technique was developed for interpolating scattered data by the RBF method in the argument space of arbitrary dimensionality. For the first time, block diagrams for two algorithms included in this technique and presenting its two separate stages are provided. The first stage involves the determination of the weighting factors for each experimentally obtained point of scattered data. In this case, the procedure for solving SLAE is used. The second stage involves actual interpolation. For each experimentally obtained point, the RBF value is determined relative to the current interpolated point. Further, the function value is determined at the interpolated point as the sum of the products for the RBF values of the experimental points and the corresponding weighting factors.

The developed methodology allows rigid interpolation of scattered data obtained as a result of field experiments, such as the values of various working parameters of construction, road and hoisting-and-transport and other machines. Among other things, interpolation of hydraulic fluid pressures in hydraulic elements of machines, forces and moments created by drives of machines and their working equipment is possible.

The developed technique can be used for interpolating scattered experimental data in a space of arbitrary dimensionality, which is quite typical of works describing complex working processes of construction, road, hoisting-and-transport and other technological machines.

For the first time, detailed block diagrams of algorithms for interpolating scattered data in a space of arbitrarily large dimensionality are presented. The feasibility of the developed algorithms was confirmed by a computational experiment.

The developed algorithms can be used by researchers involved in modelling and optimising the parameters of construction, road, hoisting and transport and other technological machines, as well as the parameters of their work processes and control systems.

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