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An optimization model of economic order quantity with financial constraints and market tolerance in ud plastikq

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Abstract. An optimization model of Economic Order Quantity (EOQ) is one of the methods used to determine the inventory order quantity that can minimize the cost of storage and the cost of ordering supplies. The optimization model in this paper is an optimization model of Economic Order Quantity (EOQ) with financial constraints and market tolerance. Financial constraints include the limited capital available per period and the advanced payment. In the market tolerance factor, there are two periods of market tolerance and three levels of the backorder. Based on this model, numerical simulations can be formulated and carried out so that the best optimal solution was obtained when conditions are unlimited, complete backorder, no advanced payment and the market tolerance period is 0.2 with a total cost decrease of 28,92% and a total profit increase of 13,37% from the initial condition.

1. Introduction

Inventory management plays an important role in the progress of the trading industry. One inventory control that can be used is the Economic Order Quantity (EOQ) optimization model which is useful for determining the number of inventory orders that can minimize storage costs and inventory ordering costs. The Economic Order Quantity (EOQ) model was first introduced by [1] by developing an economic order quantity formula, but the assumptions contained in this model are still difficult to apply in daily life because there are still many factors that influence it.

In the Economic Order Quantity model it is not permissible for goods to be emptied (stockout), therefore [2] developed a basic Economic Order Quantity model with the existence of stockouts, which is one type of stockout case, namely the backorder model. Along with the development of many times, many researchers who study and develop the Economic Order Quantity (EOQ) model into a variety of new Economic Order Quantity (EOQ) models one of which according to [3] developed the Economic Order Quantity (EOQ) model with a partial backorder assuming that some a small purchase fee is paid in advance several times before the order is received. According to [4] discussing the need to reduce the number of orders, to overcome liquidity problems.

This paper discusses the Economic Order Quantity model with financial constraints and market tolerance that refers to [5]. The developed model is the Economic Order Quantity optimization model, the Economic Order Quantity optimization model with a partial backorder system, advance payments and budgetary limits.

2. Assumption and notations

In this EOQ model, there are several assumptions that need attention. The assumptions are as follows:
1. The level of demand is known and constant for each planning period.
2. The model developed is only for one type of item (single item) and there is no interaction with other items.
3. The level of additional inventory is unlimited.
4. Lack of permitted and partial backorder at known levels β, $0 \le \beta \le 1$; $\beta = 0$ represents a complete lost sale, $\beta = 0.85$ representing a partial backorder and $\beta = 1$ representing complete backordering.
5. Advance payments at a known level α, (with $0 \le \alpha \le 1$) of total purchase costs (per Inventory cycle). $\alpha = 1$ representing full prepayment and $\alpha = 0$ without prepayment.
6. Capital budget (per inventory cycle) is limited by B ($B \to \infty$ and $B = 600000$)
7. Limited-time period (T_t). $T_t = 0$ represents the absence of a market tolerance period and $T_t = 0.2$ represents a market tolerance period limit of 0.2.
8. Value $s - \omega \ge 0$.
9. Storage costs are a percentage of the purchase price of goods per unit.
10. Unlimited capital, no upfront payment, complete lost sale, and no period of market tolerance representing initial conditions.

In this journal several notations are used, including:

D : constant demand rate (units/ planning period).
h : unit holding cost (Rp/ unit/ planning period).
p : bought-out items purchasing price (Rp/ unit).
p_s : unit sales price (Rp/ unit).
K : ordering cost (Rp/ replenishment).
b : unit backorder cost (Rp/ unit).
s_g : penalty cost (Rp/ unit).
s : lost-sales cost, including lost profits (Rp/ unit) $\left(p = p_s - p + s_g\right)$.
B : available purchasing budget (limiting purchase per cycle)(Rp-).
β : backorder rate, $0 \le \beta \le 1$.
α : prepayment fraction, $0 \le \alpha \le 1$.
T : length of inventory cycle (a decision variable).
t_1 : part of inventory cycle with no shortages (a decision variable).
T_p : fixed advanced payment period.
T_t : fixed market tolerance period.
Q : order quantity for one order cycle (unit/ replenishment), $Q = D\left[t_1 + (T - t_1)\beta\right]$
ω : unit prepayment cost (Rp/ unit).
x^+ : max $\{0, x\}$.

3. Model formulation

This discussion examines the formulation of the Economic Order Quantity Optimization Model with Financial Constraints and Market Tolerance. This Economic Order Quantity Optimization Model is a combination of three main (but not exclusively) factors, namely advance payment, limited budget and market tolerance using backorder. So that numerical simulations can be performed in a case study. Relating to the assumption that there is a model of total average profit (per unit time) that uses two decision variables; there are, T (inventory cycle) and t_1 (time no stockout occurred) are behind the emergence of the total cost model. The inventory level can be illustrated in Figure 3.1 as follows.

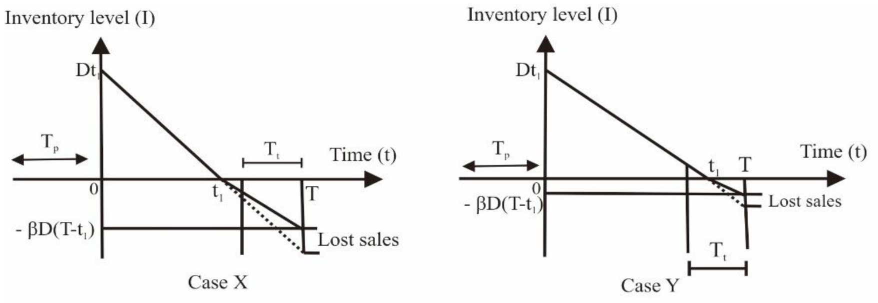

Figure 3.1 The Inventory level when $T_t < T - t_1$ (Case X)
and $T - t_1 < T_t$ (Case Y)[5]

Based on the development of previous studies on prepayments discussed by[6], [7] and partial *backorder* discussed by [8], [9], the total profit per unit of time can generally be stated in Equation (1) as follows:

$$TP(T,t_1) = \frac{1}{T}\left[p_s Q - (p+\omega)Q - K - \frac{hDt_1^2}{2} - s_g D(1-\beta)\left(T - t_1\right)\right] - \frac{b\beta D\left[(T-T_t-t_1)^+\right]^2}{2T} \tag{1}$$

Where the first term represents revenue, while all other sequences for purchase costs as well as costs caused by the time period of Tp advance payment, ordering cost, holding cost, penalty cost and finally backorder cost (as influenced by market tolerance). By substituting lost sales costs $s = p_s - p + s_g$ and $Q = D\left(T-(1-\beta)(T-t_1)\right)$ as a function of T dan t_1, to Equation (1) so $TP(T,t_1)$ can be rewritten in the form :

$$\begin{aligned} TP(T,t_1) &= (p_s - p - \omega)D - \left(\frac{K}{T} + \frac{hDt_1^2}{2T} + \frac{(s-\omega)D(1-\beta)(T-t_1)}{T} + \frac{b\beta D\left[(T-T_t-t_1)^+\right]^2}{2T} \right) \\ &= (p_s - p - \omega)D - TC(T,t_1) \end{aligned} \tag{2}$$

Viewed in Equation (2), the total profit per unit time is obtained from the reduction in the total marginal gross profit reduced $TC(T,t_1)$ which represents all inventory-related costs. Since $argmaxTP(T,t_1) = argminTC(T,\mathrm{t}_1)$ then it can minimize $TC(T,t_1)$, can be analyzed as follows:

$$TC(T,t_1) = \begin{cases} TC_X(T,t_1),\ if\ 0 \le t_1 \le T - T_t \\ TC_Y(T,t_1),\ if\ T - T_t \le t_1 \le T \end{cases} \tag{3}$$

from Equation (3), can be obtained

$$TC_{\mathrm{X}} = \frac{K}{T} + \frac{hDt_1^2}{2T} + \frac{(s-\omega)(1-\beta)D(T-t_1)}{T} + \frac{b\beta D(T-T_t-t_1)^2}{2T}$$
$$TC_{\mathrm{Y}} = \frac{K}{T} + \frac{hDt_1^2}{2T} + \frac{(s-\omega)(1-\beta)D(T-t_1)}{T}$$

3.1. Optimal policy determination

The problem for optimization $TC(T,t_1)$ is related to the value of the decision variable T and t_1 are determined to minimize costs in the inventory system under the capital constraints can be written as follows:

$$P_0 \begin{cases} \min TC(T,t_1) \\ s.t. \\ pD[t_1+\beta(T-t_1)] \le B \end{cases}$$

As already mentioned the cost function $TC(T,t_1)$, is not, in general, convex in T and t_1. So, in order to proceed with the optimization, the two branches $TC(T,t_1)$ will be optimized separately and consequently, the following problems will be solved:

$$P_1 \begin{cases} \min TC_{\mathrm{X}}(T,t_1) \\ s.t. \\ 0 \le t_1 \le T-T_t \\ pD[t_1+\beta(T-t_1)] \le B \end{cases} \qquad P_2 \begin{cases} \min TC_{\mathrm{Y}}(T,t_1) \\ s.t. \\ T-T_t \le t_1 \le T \\ pD[t_1+\beta(T-t_1)] \le B \end{cases}$$

The optimal solution for P_1 and P_2 leads to the optimal solution from P_0. The results obtained from the two problem solutions are compiled in order to find a globally optimal solution, the optimal solution from P_0 with the optimal number of goods orders is $Q^* = D\left[t_1^* + \beta\left(T^* - t_1^*\right)\right]$. The following will be discussed the optimal solution from P_0.

Theorem 3.1 *Presents the optimal solution for* $\Omega_0 > 0$ and $\Omega_0 < 0$

1. *If* $\Omega_0 > 0$, the cost function $TC(T,t_1)$ is differentiable and convex in T *and* t_1. Hence, the optimal solution to the problem $\min TC(T,t_1)$ $B \to \infty$ *is:*

   1.1. If $T_t < \dfrac{B}{pD\beta}$, *then* $t_{1,\mathrm{u}}^* = t_{1,\mathrm{X}}^* = \dfrac{b\beta(T_{\mathrm{X}}^* - T_t) + (s-\omega)(1-\beta)}{h+b\beta}$,

   $$T_{\mathrm{u}}^* = T_{\mathrm{X}}^* = \left(\frac{\Omega_0}{b\beta hD} + \frac{2K}{hD} + \frac{2(s-\omega)(1-\beta)T_t}{h} + T_t^2\right)^{\frac{1}{2}}$$, with

   $$TC(T_{\mathrm{u}}^*, t_{1,\mathrm{u}}^*) = \frac{Dh\left((s-\omega)(1-\beta) + b\beta(T_{\mathrm{u}}^* - T_t)\right)}{h+b\beta}.$$

1.2. If $T_t \geq \dfrac{B}{pD\beta}$, then $t_{1,\mathrm{u}}^* = 0,\ T_{\mathrm{u}}^* = \dfrac{B}{pD\beta}$, with

$$TC\left(T_{\mathrm{u}}^*, t_{1,\mathrm{u}}^*\right) = \frac{K\beta pD}{B} + (s-\omega)(1-\beta)D\ .$$

2. *If* $\Omega_0 < 0$, then $t_{1,\mathrm{u}}^* = T_u^* = \left(\dfrac{2K}{hD}\right)^{\frac{1}{2}}$, with $TC(T_{\mathrm{u}}^*, t_{1,\mathrm{u}}^*) = \sqrt{2KhD}$

Theorem 3.2 *The optimal solution to the problem* P_0 *subject to* $pD\left[t_1 + \beta(T - t_1)\right] = B$ *is*

1. *If* $T_t < \dfrac{B}{pD\beta}$:

1.1. If $\Omega_2 > B^2\left(h + \dfrac{b}{\beta}\right) - \beta b(B + Dp(1-\beta)T_t)^2$, then $t_1^* = 0, T^* = \dfrac{B}{\beta pD}$, with

$$TC\left(T^*, t_1^*\right) = \frac{b(B - Dp\beta T_t)^2 + 2Dp(Kp\beta + B(s-\omega)(1-\beta))}{2Bp}.$$

1.2. If $h\beta^2(B + p(1-\beta)DT_t)^2 \leq \Omega_2 \leq B^2\left(h + \dfrac{b}{\beta}\right) - \beta b(B + Dp(1-\beta)DT_t)^2$ then

$$t_1^* = \frac{B}{pD(1-\beta)} - \frac{\beta T^*}{(1-\beta)}, T^* = \left(\frac{\Omega_1}{D^2p^2\beta(h\beta + b)}\right)^{\frac{1}{2}} \text{ with}$$

$$TC\left(T^*, t_1^*\right) = \frac{Dp\beta(h\beta + b)T^* - B\beta(h+b) + Dp(1-\beta)((s-\omega)(1-\beta) - \beta bT_t)}{p(1-\beta)^2}$$

1.3. If $B^2\beta^2 h \leq \Omega_2 \leq h\beta^2(B + p(1-\beta)DT_t)^2$, then

$$t_1^* = \frac{B}{pD(1-\beta)} - \frac{\beta T^*}{(1-\beta)}, T^* = \left(\frac{\Omega_2}{D^2p^2\beta^2 h}\right)^{\frac{1}{2}} \text{, with}$$

$$TC\left(T^*, t_1^*\right) = \frac{h\beta(Dp\beta T^* - B) + Dp(s-\omega)(1-\beta)^2}{p(1-\beta)^2}.$$

1.4. If $\Omega_2 < B^2\beta^2 h$, then $t_1^* = T^* = \dfrac{B}{pD}$ with $TC\left(T^*, t_1^*\right) = \dfrac{Bh}{2p} + \dfrac{DKp}{B}$.

2. *If* $T_t \geq \dfrac{B}{pD\beta}$

2.1. If $B^2\beta^2 h \leq \Omega_2 \leq B^2 h$, *then* $t_1^* = \dfrac{B}{pD(1-\beta)} - \dfrac{\beta T^*}{(1-\beta)}$, $\quad T^* = \left(\dfrac{\Omega_2}{D^2p^2\beta^2 h}\right)^{\frac{1}{2}}$ with

$$TC\left(T^*, t_1^*\right) = \frac{h\beta(Dp\beta T^* - B) + Dp(s-\omega)(1-\beta)^2}{p(1-\beta)^2}.$$

2.2. If $\Omega_2 > B^2 h$ then $t_1^* = 0$ and $T^* = \dfrac{B}{\beta pD}$ with $TC\left(T^*, t_1^*\right) = \dfrac{K\beta pD}{B} + (s-\omega)(1-\beta)D$

2.3. If $\Omega_2 < B^2\beta^2 h$ *then* $t_1^* = T^* = \dfrac{B}{pD}$ *with* $TC\left(T^*, t_1^*\right) = \dfrac{Bh}{2p} + \dfrac{DKp}{B}$.

Remark 3.1 *The optimal solution for complete backorders and lost sale when* $\beta = 0$ *and* $\beta = 1$ *in a case* P_0 *subject to* $pD[t_1 + \beta(T - t_1)] = B$ *is,*

1. When complete back-ordering is assumed then the optimal solution is

   1.1. If $T_t < \dfrac{B}{pD}$, then $t_1^* = \dfrac{b\left(T^* - T_t\right)}{h+b}, T^* = \left(\dfrac{2K}{D}\left(\dfrac{h+b}{bh}\right) + T_t^2\right)^{\frac{1}{2}}$ provided that $pDT^* \le B$.

   If $pDT^* > B$, then $t_1^* = \dfrac{b\left(T^* - T_t\right)}{h+b}$ and $T^* = \dfrac{B}{pD}$.

   1.2. If $T_t \ge \dfrac{B}{pD}$, then $t_1^* = 0, T^* = \dfrac{B}{pD}$.

2. When complete lost sales are assumed then the optimal solution is:

   2.1. If $2Kh - (s-\omega)^2 D > 0$ then $T \to \infty$ which implies that $Q = 0$ and should allow all demands to be lost sale like [9].

   2.2. If $2Kh - (s-\omega)^2 D < 0$ then $t_1^* = T^* = \left(\dfrac{2K}{hD}\right)^{\frac{1}{2}}$, provided that $pDT^* \le B$. *If* $pDT^* > B$, then $t_1^* = T^* = \dfrac{B}{pD}$

4. Numerical simulation

The EOQ model was simulated at UD PlastikQ. Basic data is obtained $p_s = \text{Rp. } 1000/$ unit, $p = \text{Rp. } 650/$ unit, $h = \text{Rp. } 65/$ unit/ 1 month, $D = 1063$ unit/ 1 month, $s = \text{Rp. } 635/$ unit, $b = \text{Rp. } 100/$ unit, $K = \text{Rp. } 100000/$ 1 month. Assuming three different types of customer reactions to stockout, as explained by the level of back-ordering (β = 0, β = 0,85 and β = 1), the combined effect of the three factors studied here on profit and costs related to inventory will be sought. in the following arrangements: advanced payment costs, $(\omega = 0 \text{ and } \omega = 1)$, capital $(B \to \infty \text{ and } B = 600000)$, and market tolerance periods $(T_t = 0 \text{ and } T_t = 0.2)$.

Based on the theorem that exists for the problem when $B \to \infty$ given 12 different conditions will be solved using Theorem 3.1 by finding first Ω_0. For case $\beta = 0$ because $\Omega_0 < 0$ so used Theorem 3.1 (2), for the case $\beta = 0{,}85$ and $\beta = 1$ because $\Omega_0 > 0$ and $T_t < \dfrac{B}{pD\beta}$ so used Theorem 3.1 (1.1). Another case when $B = 600000$ given 12 different conditions will be solved using Theorem 3.2 by finding first $\dfrac{B}{pD\beta}$, while for special cases $\beta = 0$ and $\beta = 1$ will be solved using Remark 3.1. For case, $\beta = 0$ because $2Kh - (s-\omega)^2 D < 0$ so used Remark 3.1 (2.2), for the case $\beta = 0{,}85$ because $T_t < \dfrac{B}{pD\beta}$ dan $\Omega_2 < B^2\beta^2 h$ so used Theorem 3.2 (1.4) and for the case $\beta = 1$ because $T_t < \dfrac{B}{pD}$

so used Remark 3.1 (1.1). Comparison of optimal solutions for p_0 when $B \to \infty$ and $B = 600000$ shown in Table 3.1.

Table 3.1 The optimal solution for p_0 when $B \to \infty$ and $B = 600000$

		$B \to \infty$			$B = 600000$		
		$\beta = 0$	$\beta = 0,85$	$\beta = 1$	$\beta = 0$	$\beta = 0,85$	$\beta = 1$
$\omega = 0$	$T_t = 0$	$t_1^* = 1,7013$ $T^* = 1,7013$ $Q^* = 1808,5268$ $TC(T^*,t_1^*) = 117554,242$ $TP(T^*,t_1^*) = 254495,757$	$t_1^* = 1,6899$ $T^* = 1,8617$ $Q^* = 1951,6135$ $TC(T^*,t_1^*) = 116768,405$ $TP(T^*,t_1^*) = 255281,594$	$t_1^* = 1,3244$ $T^* = 2,1854$ $Q^* = 2323,0947$ $TC(T^*,t_1^*) = 91515,8539$ $TP(T^*,t_1^*) = 280534146$	$t_1^* = 0,8683$ $T^* = 0,8683$ $Q^* = 923,0769$ $TC(T^*,t_1^*) = 145158,333$ $TP(T^*,t_1^*) = 226891,667$	$t_1^* = 0,8683$ $T^* = 0,8683$ $Q^* = 923,0769$ $TC(T^*,t_1^*) = 145158,333$ $TP(T^*,t_1^*) = 226891,667$	$t_1^* = 0,5262$ $T^* = 0,8683$ $Q^* = 923,0769$ $TC(T^*,t_1^*) = 133340,152$ $TP(T^*,t_1^*) = 238709,848$
	$T_t = 0,2$	$t_1^* = 1,7013$ $T^* = 1,7013$ $Q^* = 1808,5268$ $TC(T^*,t_1^*) = 117554,242$ $TP(T^*,t_1^*) = 254495,757$	$t_1^* = 1,6679$ $T^* = 2,0228$ $Q^* = 2093,748$ $TC(T^*,t_1^*) = 115248,783$ $TP(T^*,t_1^*) = 256801,216$	$t_1^* = 1,2088$ $T^* = 2,1945$ $Q^* = 2332,8026$ $TC(T^*,t_1^*) = 83523,132$ $TP(T^*,t_1^*) = 288526,867$	$t_1^* = 0,8683$ $T^* = 0,8683$ $Q^* = 923,0769$ $TC(T^*,t_1^*) = 145158,333$ $TP(T^*,t_1^*) = 226891,667$	$t_1^* = 0,8683$ $T^* = 0,8683$ $Q^* = 923,0769$ $TC(T^*,t_1^*) = 145158,333$ $TP(T^*,t_1^*) = 226891,667$	$t_1^* = 0,4050$ $T^* = 0,8683$ $Q^* = 923,0769$ $TC(T^*,t_1^*) = 125929,416$ $TP(T^*,t_1^*) = 246120,532$
$\omega = 1$	$T_t = 0$	$t_1^* = 1,7013$ $T^* = 1,7013$ $Q^* = 1808,5268$ $TC(T^*,t_1^*) = 117554,242$ $TP(T^*,t_1^*) = 253432,757$	$t_1^* = 1,6897$ $T^* = 1,863$ $Q^* = 1962,833$ $TC(T^*,t_1^*) = 116753,633$ $TP(T^*,t_1^*) = 254233,366$	$t_1^* = 1,3244$ $T^* = 2,1854$ $Q^* = 2323,0947$ $TC(T^*,t_1^*) = 91515,8539$ $TP(T^*,t_1^*) = 279471,1461$	$t_1^* = 0,8683$ $T^* = 0,8683$ $Q^* = 923,0769$ $TC(T^*,t_1^*) = 145158,333$ $TP(T^*,t_1^*) = 225828,667$	$t_1^* = 0,8683$ $T^* = 0,8683$ $Q^* = 923,0769$ $TC(T^*,t_1^*) = 145158,333$ $TP(T^*,t_1^*) = 225828,667$	$t_1^* = 0,5262$ $T^* = 0,8683$ $Q^* = 923,0769$ $TC(T^*,t_1^*) = 13340,152$ $TP(T^*,t_1^*) = 237646,848$
	$T_t = 0,2$	$t_1^* = 1,7013$ $T^* = 1,7013$ $Q^* = 1808,5268$ $TC(T^*,t_1^*) = 117554,242$ $TP(T^*,t_1^*) = 253432,757$	$t_1^* = 1,6675$ $T^* = 2,0239$ $Q^* = 2094,6316$ $TC(T^*,t_1^*) = 115220,757$ $TP(T^*,t_1^*) = 255766,242$	$t_1^* = 1,2088$ $T^* = 2,1945$ $Q^* = 2332,8026$ $TC(T^*,t_1^*) = 83523,1329$ $TP(T^*,t_1^*) = 287463,8671$	$t_1^* = 0,8683$ $T^* = 0,8683$ $Q^* = 923,0769$ $TC(T^*,t_1^*) = 145158,333$ $TP(T^*,t_1^*) = 225828,667$	$t_1^* = 0,8683$ $T^* = 0,8683$ $Q^* = 923,0769$ $TC(T^*,t_1^*) = 145158,333$ $TP(T^*,t_1^*) = 225828,667$	$t_1^* = 0,4050$ $T^* = 0,8683$ $Q^* = 923,0769$ $TC(T^*,t_1^*) = 125929,468$ $TP(T^*,t_1^*) = 245067,532$

5. Conclusion

The selling and buying activities implemented by UD PlastikQ meet the assumptions of the Economic Order Quantity model with financial constraints and market tolerance. Therefore, the calculation to find out the optimal solution with the basic data taken from UD PlastikQ will remain constant but the level of the backorder and the three factors studied changes in value. When simulating using this model the best optimal solution is obtained $\left(B \to \infty, \beta = 1, \omega = 0, T_t = 0,2\right)$ when conditions with a total cost decreased by 28,92% and total profits increased by13,37% from the initial conditions. While the worst optimal solution when the condition $\left(B = 600000, \beta = 0, \omega = 1, T_t = 0\right)$ with total costs increased by 23,48% and total profits decreased by 11.26% of the initial conditions.

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