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Fluxon Dynamics in Boundary Driven Josephson Transmission Line

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Abstract. Analysis of the different stationary states of a boundary driven Frenkel-Kontorova Model and the associated sine-Gordon equation shows the existence a new regime that can be represented by a kink (fluxon) motion back and forth in the restricted geometry. This dynamics is produced by driving the system at one end at a given frequency. As a consequence our result suggests the conception of a device which could switch to the conducting regime not according to intensity range but rather to given (quantized) frequency range. Moreover our findings indicate also a frequency converting scenario by choosing appropriately the system length and injected fluxon number.

A short Josephson junction has a simple interpretation in terms of a mechanical pendulum (see e.g. [1]), and a chain of Josephson junctions connected in parallel (or a long Josephson junction transmission line) is effectively described by a Frenkel-Kontorova model (or its continuous analogue the sine-Gordon model) [2, 3]. At the same time these models have a simple experimental counterpart, namely the chain of linearly coupled pendula [3] which offers an interesting opportunity to easily visualize all the main nonlinear characteristics of the sine-Gordon system. On the other hand, this simple laboratory tool allows to observe novel effects [4, 5, 6, 7] which may then apply in completely different physical situations.

As a matter of fact, a recent experimental discovery of supratransmission effect in the pendula chain [8] has led to the study of similar phenomena in optical Bragg gratings [9], Josephson Junction transmission line [10], waveguide arrays [11, 12]. By this approach, many similar phenomena observed in the same systems [13, 14, 15, 16, 17] have been identified as effects of *nonlinear bistability*. Moreover it allowed us to predict the existence of bistable magnetization profiles in magnetic films [18] and to suggest ultrasensitive detectors (or digital amplifiers) in optical waveguides [19] quantum Hall bilayers [20] and Josephson junction parallel arrays [21].

We report here the discovery of a stationary state which can be qualitatively understood as the motion back and forth of a kink-like structure (some analogy of the fluxon in long Josephson junctions) in the pendula chain. The new stationary regime appears to be completely different from the two cases considered earlier e.g. in [18, 19, 20, 21, 22]. Such a dynamics creates a new frequency in the system and furnishes a tool to divide the input frequency by odd fractions (we shall illustrate chain end oscillations with frequency $\Omega/3$ where Ω is the driver frequency).

Let us consider a one dimensional array of N short Josephson junctions coupled through

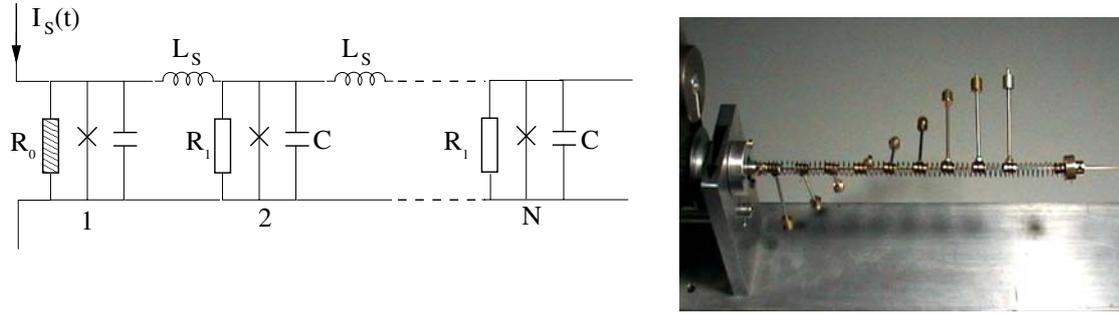


Figure 1. Scheme of a parallel array of Josephson junctions (left) and its mechanical counterpart: the linearly coupled pendula chain (right).

super-conducting wires as represented by figure 1. It obeys the following model [3]

$$\ddot{u}_1 + \gamma_0 \dot{u}_1 - \lambda_J^2 [u_2 - u_1] + \sin u_1 = I_s(t), \quad (1)$$

$$\ddot{u}_N + \gamma \dot{u}_N - \lambda_J^2 [u_{N-1} - u_N] + \sin u_N = 0,$$

$$\ddot{u}_n + \gamma \dot{u}_n - \lambda_J^2 [u_{n+1} + u_{n-1} - 2u_n] + \sin u_n = 0, \quad (2)$$

where $n = 2, \dots, N$ number underdamped Josephson junctions, while the first junction is considered to be overdamped with a large damping parameter γ_0 . The injected current $I_s(t)$ is normalized to the Josephson critical current I_c in the single junction. The time is normalized to the inverse plasma frequency $\omega_p = 1/\sqrt{L_J C}$, C stands for the junction capacitance and $L_J = \hbar/(2eI_c)$ is the Josephson inductance. The parameter λ_J is defined by $\lambda_J^2 = L_J/L_S$ where L_S is the inductance representing by the superconducting wires connecting the junctions. $\gamma_0 = \sqrt{\hbar/(2eI_c R_0^2 C)}$ is a damping parameter for overdamped first junction (R_0 is its resistance) and $\gamma = \sqrt{\hbar/(2eI_c R_1^2 C)}$ represents damping constant of other junctions with resistance R_1 . For our numerical simulations we choose $\lambda_J = 2$ and $\gamma = 0.02$.

The first junction being overdamped ($\gamma_0 \gg \gamma$), one may neglect in (1) all left-hand side terms except $\gamma_0 \dot{u}_1$. Consequently with a sinusoidal injected current $I_s(t) = b \sin(\Omega t)$ this means that we control the function $u_1 \sim \cos(\Omega t)$. Thus the dynamics of parallel array of Josephson junctions is effectively described by the Frenkel-Kontorova model [2] with applied boundary conditions:

$$\ddot{u}_n + \delta \dot{u}_n - \sigma^2 (u_{n+1} + u_{n-1} - 2u_n) + \omega_0^2 \sin u_n = 0, \quad u_0(t) = b \cos(\Omega t), \quad u_{N+1} = u_N \quad (3)$$

which is the model of linearly coupled pendula where the variable u_n is the angular deviation of the n^{th} pendulum, ω_0 is the eigenfrequency of a single pendulum and σ is proportional to the linear torsion constant of the spring.

Our experiments on the pendula chain [23] correspond to $\omega_0 = 15.1$ Hz, $\sigma = 32.4$ Hz. The damping coefficient δ is phenomenological, it has been evaluated approximately $\delta = 0.02\omega_0$. The experiments consist in driving the short chain pictured in Fig. 1 with a frequency in the forbidden band gap ($\Omega < \omega_0$), which actually does not excite linear modes. Without external perturbation the system locks to a periodic solution with low output amplitude $u_N(t)$. Depending on the value of an external kick one makes the system bifurcate to different stationary state. Introducing dimensionless time variable $t \rightarrow t/\omega_0$ one sees that the effective parameters of the pendula chain are very close to those of parallel array of Josephson junctions (1), (2) for which we now perform numerical simulations.

To develop an analytical description of the process, let us consider the continuous approximation of eq.(3) by substitutions $t \rightarrow \omega_0 t$, $n = \omega_0 x/\sigma$. Neglecting dissipation we obtain

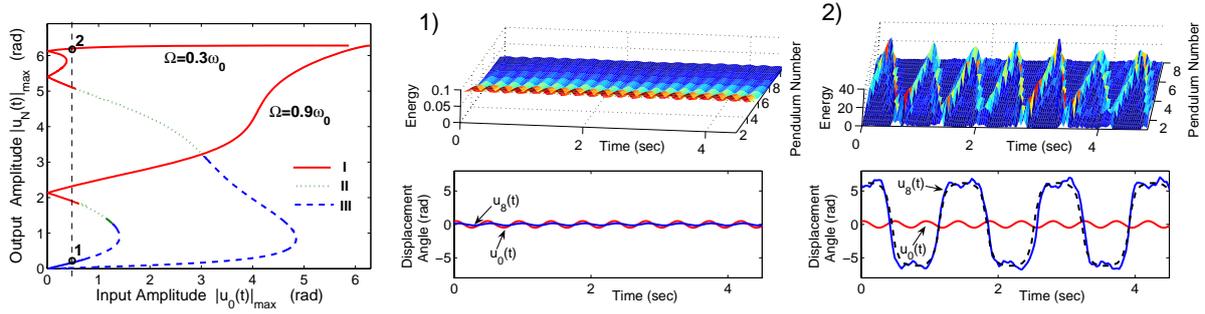


Figure 2. Left: Analytic input-output amplitude dependences for different oscillation frequencies of stationary states given by formulas (6). The two lines correspond to frequency $0.9\omega_0$ and $0.3\omega_0$ as indicated in the graph. The intersections of the vertical line with the curves correspond to the different regimes for the single driving amplitude $|u_0(t)|_{\max} = 0.5$ rad. All intersections except point 2 represent the situations when the whole chain oscillates with the driving amplitude $0.9\omega_0$ but with different output amplitudes. The point 3 corresponds to the driving frequency $0.3\omega_0$ and describes kink motion forth and back. As the experiments and numerical simulations show (and this is a main finding of the paper), the latter regime can also be reached with a driving frequency $0.9\omega_0$, three times larger than the one actually used for the analytic solution used in the plot. Two other plots describe numerical simulations on the model (1), (2 with a damping constant $\gamma = 0.02$ and 8 junctions. The time evolution of junctions energy and input-output oscillations are displayed corresponding to the points 1) and 2). The driving amplitude is $|u_0(t)|_{\max} = 0.5$ rad and its frequency $\Omega = 0.9$ for both cases. This results in the same output frequency oscillations Ω in graph 1) but $\Omega/3$ output oscillations in graph 2). Dashed lines display analytical curves obtained from (6).

the sine-Gordon equation

$$x \in [0, L] : u_{tt} - u_{xx} + \sin u = 0, \quad (4)$$

where $L = N\sigma/\omega_0$. The mixed Dirichlet and Neumann Boundary Conditions $u(0, t) = b \sin(\Omega t)$ (driven boundary), $u_x(L, t) = 0$ (free end boundary) allows to seek the following periodic stationary solutions [22]

$$u(x, t) = 4 \arctan \left[\sqrt{\left| \frac{rs}{b} \right|} \mathcal{X}(x) \mathcal{T}(t) \right], \quad (5)$$

where one has three choices (cn, sn and dn are the standard Jacobi elliptic functions)

$$\begin{aligned} \text{(I)} \quad & \mathcal{X} = \text{cn}(\beta(x - L), \mu), \quad \mathcal{T} = \text{cn}(\omega t, \nu), \\ \text{(II)} \quad & \mathcal{X} = \text{dn}(\beta(x - L), \mu), \quad \mathcal{T} = \text{sn}(\omega t, \nu), \\ \text{(III)} \quad & \mathcal{X} = \text{dn}(\beta(x - L) + \mathcal{K}(\mu), \mu), \quad \mathcal{T} = \text{sn}(\omega t, \nu). \end{aligned} \quad (6)$$

Here $\mathcal{K}(\mu)$ stands for a complete elliptic integral of the first kind of modulus μ . These families of solutions are parametrized by the two free constants ω and $\nu \in [0, 1]$, then for solutions of type (I) the remaining parameters are given by $b = \omega^4 \nu^2 (1 - \nu^2)$, $s = \omega^2 \nu^2$, $2r = 1 - \omega^2 + 2\omega^2 \nu^2 + \sqrt{(1 - \omega^2)^2 + 4\omega^2 \nu^2}$, $\beta^2 = (b + r^2)/r$, $\mu^2 = r^2/(b + r^2)$. While in both cases (II) and (III) they read $b = \omega^4 \nu^2$, $s = -\omega^2 \nu^2$, $2r = 1 - \omega^2(1 + \nu^2) + \sqrt{[1 - \omega^2(1 + \nu^2)]^2 - 4\omega^4 \nu^2}$, $\beta^2 = r$, $\mu^2 = 1 - (b/r^2)$. Note that r should be real valued and positive which may restrict the allowed values of ω .

Since the experiments show that the frequency $\Omega/3$ can also be excited (if one puts initially large energy in the chain), we assume that the period of the time dependent part $\mathcal{T}(t)$ of

the stationary solutions (6) coincide with an odd integer fractions of the driving frequency Ω . Recalling that the period of $\mathcal{T}(t)$ is $4\mathcal{K}(\nu)/\omega$ we require thus $\omega = 2\Omega\mathcal{K}(\nu)/(m\pi)$, where m is an odd integer. For a given value of the parameter $\nu \in [0, 1]$, the above relation fixes the second parameter ω in terms of the driving frequency Ω . Therefore fixing Ω (driver frequency) and varying ν one can plot the output amplitude $u(N, t)$ in terms of the input $u(0, t)$ from the analytic expressions (6). We display this dependence for $\Omega = 0.9$ (in units of ω_0) as a the full line in fig.2 where different colors report to different solutions. We also plot (dashed line) the output amplitude for a driving frequency $\Omega = 0.3$. Therefore, to the given driver amplitude $\max_t |u(0, t)| = 0.5$ may correspond stable synchronized states (intersections of vertical line with the curve corresponding to $\Omega = 0.9$ on the left graph of fig. 1, which has been analyzed previously [21]) and one more stable state with frequency 0.3 indicated by point 2.

It is then a simple matter to check that the stationary state related to point 2 of the left graph in fig. 2 corresponds effectively to our numerical simulations, and hence to the experiments of fig. 1 (see also [23] for the movie file describing the experiment). It is done in fig. 2 where the last plot shows the result of a numerical simulation (full line) compared with the analytic solution (dashed line) related to point 2 of left graph in fig. 2. We have also plotted the time evolution of the total energy of each pendulum (middle graph in Fig. 2) in that stationary state and compare its behavior with one of the early examined stationary state which is fully synchronized with driver (right graph in Fig. 2). Thus we have actually demonstrated the possibility of conceiving a *frequency divider* with which the driving frequency can be divided by factor 3.

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