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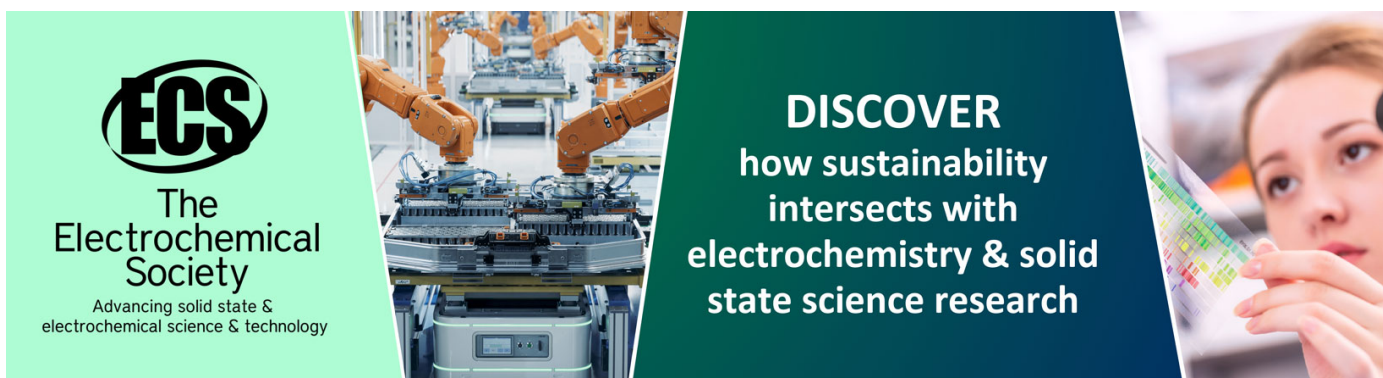
Mathematical modelling of the process of internal technological stress fields development in cylindrical solids being layer-by-layer made of ageing viscoelastic materials

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Mathematical modelling of the process of internal technological stress fields development in cylindrical solids being layer-by-layer made of ageing viscoelastic materials

D A Parshin^{1,2}

¹ Laboratory of Modelling in Solid Mechanics, Ishlinsky Institute for Problems in Mechanics of the Russian Academy of Sciences (IPMech RAS), Vernadskogo Ave. 101 Bldg 1, Moscow 119526, Russia

² Department of Applied Mathematics, Bauman Moscow State Technical University (BMSTU), 2nd Baumanskaya Str. 5 Bldg 1, Moscow 105005, Russia

E-mail: parshin@ipmnet.ru

Abstract. With the help of current approaches in mechanics of additively manufactured deformable solids, a non-classical mathematical model is developed to predict the stress-strain state of layers being gradually formed on a rotating substrate in an arbitrary number of continuous stages of ageing viscoelastic material application. On the basis of numerous calculations performed, the qualitative and quantitative features of the internal technological stresses distributions in the finished cylindrical product for different variants of its release from the mechanical loading and the kinematic bonds acting during the manufacturing process are analyzed in detail. Previously unknown mechanical effects are found. The appropriate recommendations of a practical nature are given.

1. Introduction

Additive manufacturing processes accompanied with increase of solids in size by means of adding new material layers to their surface are characterized by the fact that the solids do not exist in their final configurations before the start of deforming, versus classical solid mechanics, as all additively manufactured solids keep being formed in course of the deformation. External mechanical loads arise due to specific features of an additive process. One can observe, inter alia, that bulk forces often act as mechanical loads in different additive processes. In particular, those include the forces of inertia of the motion of the being formed solid as a rigid whole. Thus, the centrifugal forces of inertia can not be ignored in the analysis of centrifugal material application technologies used for manufacturing or strengthening various axisymmetric products, as well as for applying layers of coatings onto such products. The presented research is devoted to the modeling of such technologies from the point of view of mechanics of deformable solids. It is clear that the loads acting on any additive manufactured solid will cause appearance and further development of stress fields throughout this solid. The corresponding technological stresses can, on the one hand, cause significant geometry distortions of the formed product or even its destruction (perhaps, already in the manufacturing process!) and, on the other hand, compensate the influence of various negative influences on the product during its further



exploitation. So, knowledge of the correct stress distributions in the mentioned products is essential for investigation of problems on assessing strength, stiffness, stability, bearing capacity and durability of these products in different conditions of their loading and contact interaction [1–7]. A detailed understanding of the mechanisms of these distributions formation in a particular technological process can be, evidently, achieved only by mathematical modelling the additive manufacture processes in question from the standpoint of mechanics. Moreover, merely mathematical modeling provides an opportunity not only to predict these distributions, but also to effectively control them by suitable varying certain technological parameters in the course the process.

The present work is devoted to the study of mechanical problems associated with the gradual centrifugal application on the inner surface of the axisymmetric cylindrical substrate of a layer of ageing viscoelastic material uniform in thickness. The angular velocity ω of rotation of the substrate around its axis in the simulated additive process can change arbitrarily over the time t . It is however considered that the derivative of the given function $\omega(t)$ has not too high values, that is

$$\left| \frac{d\omega(t)}{dt} \right| \ll \omega^2(t). \quad (1)$$

This corresponds to a fairly slow variation in the speed of rotation of the substrate during the application of the material onto it. Specified condition (1) on the angular velocity rate imposed in the considered problem allows to neglect tangential forces of inertia of rotation of the formed layer of material together with a substrate in comparison with the corresponding centrifugal forces. It is also assumed that the instantaneous interaction of the applied additional material with the surface of the already formed layer is carried out in such a way that the potentially possible dynamic effects of this interaction are not significant. Therefore, it is possible to neglect the forces of inertia of the deformation of the formed layer in comparison with the centrifugal forces of inertia of its rotation together with the substrate and to consider the corresponding problem of mechanics in a quasi-static formulation.

In the considered process of multilayer applying the additional material onto a rotating substrate, the thickness of each additional elementary layer formed on the surface of a growing axisymmetric solid in one revolution around its axis is considered to be very small compared to the radius of this layer. This, firstly, means that the rate of inflow of additional material to the formed solid in the circumferential direction is much higher than the rate of its inflow in the radial direction, which makes it possible to simulate the process of centrifugal application of the material by axisymmetric additive process of replenishment of the formed solid with a new material simultaneously over the whole inner side surface. Secondly, this assumption makes it possible to simulate this process as a continuous one, in which an infinitely thin layer of additional material is attached to the formed solid each infinitesimal period of time. Thus, the variable inner radius $a(t)$ of the formed layer will be a continuous function of time t . The outer radius a_0 of the formed layer throughout the process coincides with the radius of the working surface of the substrate used. Neglecting the flexibility of the latter in comparison with the flexibility of the layer formed on it, we consider the outer radius of the layer unchanged in time, i.e. $a_0 = \text{const}$.

It should be noted that the assumption of constancy of the formed layer outer radius is not essential in the model proposed in this paper. On the one hand, this assumption is adopted to simplify the analysis of all the relations obtained. On the other hand, the absence, in the used model, of the deformation response of the substrate to the pressure exerted on it by the applied material seeking to expand under the action of centrifugal forces allows us to give a refined demonstration of the mechanical effects that occur when the layer-by-layer formation of the solid with the simultaneous action of the field of bulk forces on it (in our problem, this is the forces of inertia of the rotational motion) regardless of the influence of deformation processes occurring in that part of the solid in question which existed before the organization of the influx of additional material to it. This must be considered as a very significant special case of deforming an additively manufactured solid, because

of the fact that if a layer of additional material is attached to the surface of any solid that is already in the process of deformation under the influence of any external impacts, then the joined layer will be inevitably also involved in the deformation process.

The objectives of the present study are:

- to build a mechanical model of the above described additive process of manufacturing a layer on a cylindrical substrate by centrifugal method, under the assumption of complex rheological properties of the material used;
- to predict the evolution of the stress-strain state of the being formed layer under the action of centrifugal forces of inertia of its rotational motion together with the substrate;
- to identify quantitative and qualitative characteristics of this evolution and to explain the mechanical effects observed in the simulated process;
- to formulate practical recommendations that should be taken into account in the design and organization of additive processes of centrifugal material application.

2. Material and methods

As it is known (see, e.g., [8–11]), various engineering materials (for instance, polymers, concretes) exhibit pronounced rheological properties. The deformation response of such materials to loads applied to them has an deformation aftereffect. In addition, the mechanical characteristics of such materials, as a rule, change significantly with the material age, regardless of actual stresses. The last effect, called ageing, is manifested in a decrease in the material compliance with time, that is, in the weakening of its deformation response, both instantaneous and developing over time.

This paper uses a model of the mechanical behavior of an ageing viscoelastic material described in the framework of the linear hereditary theory of viscoelasticity of ageing isotropic media [9]. The relationship between the stress tensor $\mathbf{T}$ and the small strain tensor $\mathbf{E}$ for such a medium has the following form:

$$Q_{\tau_0} \mathbf{T} = 2\mathbf{E} + \phi \mathbf{1} I_1[\mathbf{E}] \quad (2)$$

where $\mathbf{1}$ is the rank 2 unit tensor, I_1 is the linear invariant of a tensor of the rank 2,

$$\phi = (1 - 2\nu)^{-1} - 1$$

is the material constant related with the Poisson's ratio ν which corresponds both to instantaneous elastic strain and to creep strain developing with time. Relationship (2) contains the following Volterra integral operator over time:

$$Q_{\tau_0} \varphi(t) = \frac{\varphi(t)}{G(t)} - \int_{\tau_0}^t \frac{\varphi(\tau)}{G(\tau)} K(t, \tau) d\tau \quad (3)$$

where $G(t)$ is the elastic shear modulus of the material at the age t , and $K(t, \tau)$ is the creep kernel. The latter can be expressed through various characteristics of the material which describe its behavior in different elementary stress state. For example, using the characteristics for pure shear we have

$$K(t, \tau) = G(\tau) \frac{\partial \Delta_{\text{shear}}(t, \tau)}{\partial \tau}$$

where the function $\Delta_{\text{shear}}(t, \tau)$ is usually referred to as the function of specific strain in pure shear. It is given by the expression

$$\Delta_{\text{shear}}(t, \tau) = \frac{1}{G(\tau)} + c_{\text{shear}}(t, \tau) \quad (4)$$

and describes the development over time t of the shear strain caused by the stress state of the pure

shear applied to a specimen at a time moment τ and remaining not changed further, divided by the acting shear stress magnitude. The function $c_{\text{shear}}(t, \tau)$ is called the creep measure in pure shear. For this function it is valid the identity

$$c_{\text{shear}}(\tau, \tau) \equiv 0.$$

Note that the time t is counted in (3) from the instant of origination of the material as a solid structure.

The time parameter τ_0 of operator (3) has a sense of the time of occurrence of the non-trivial stress state in the neighbourhood of the given point of the solid.

The structure and the characteristic features of a creep curve described with expression (4) is shown schematically in figure 1. One can find typical experimental creep curves representing the development over time of specific strain of specimens made of materials with the above mentioned deformation properties, for different ages at which the shearing load was applied to them, in figure 2.

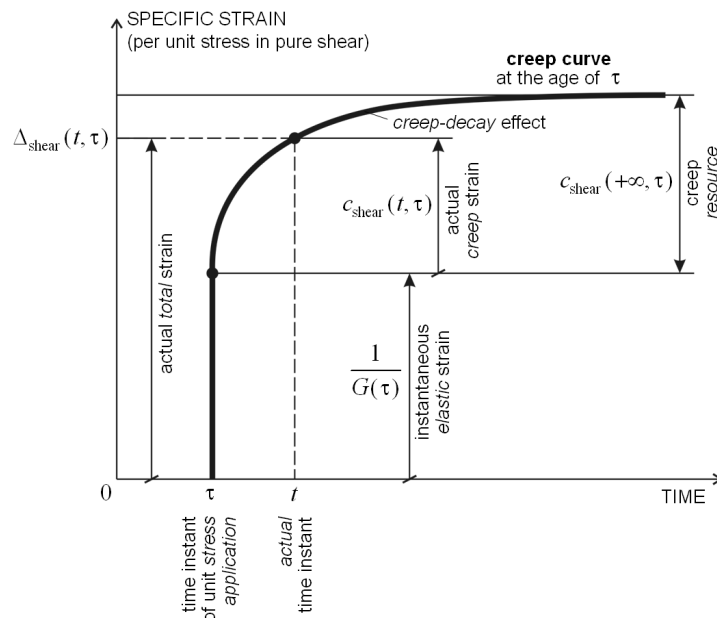


Figure 1. Structure and characteristic features of considered creep curves.

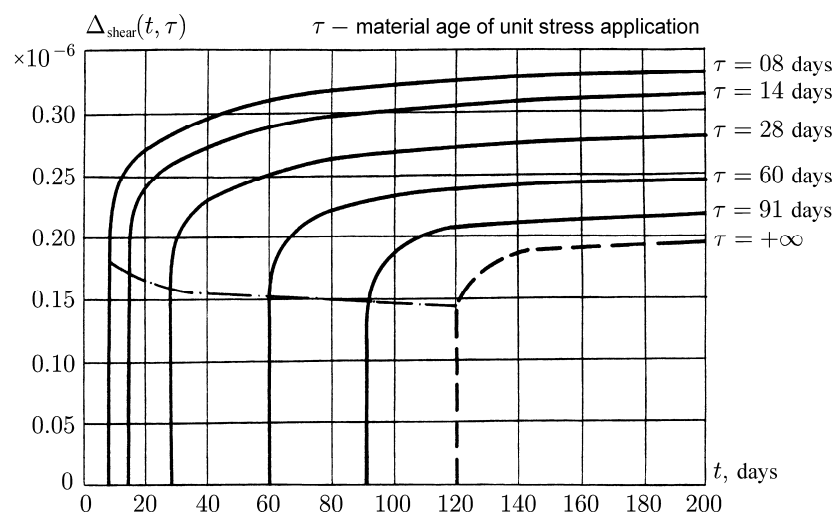


Figure 2. Typical experimental creep curves of the used material for different material ages.

The mechanical analysis of additive processes has to take into account external loads actual in the simulated technological process, including those acting on the additional material, simultaneously with mechanical peculiarities of gradual attaching the material to the solid surface. Such account cannot be correctly carried out within the framework of classical solid mechanics, even if the traditional equations and boundary conditions are formulated for a time variable spatial domain. This is due to absence of any unstressed configuration for the entire additive-manufactured solid.

Thus, the problems on mechanical modeling for additive-manufactured solids constitute a special class of problems in solid mechanics and possess a number of non-classical features. Approaches to statement and solution of such problems are being successfully developed nowadays by Russian scientific school in mechanics of additive processes founded by Professor A.V. Manzhirov (see, e.g., works [12–18]). According to these approaches, statements of initial boundary value mechanical problems at all stages of continuous application of the additional material, in the pauses between these stages and after the final termination of the additive process are non-classical in solid mechanics. They are formulated in the present study in terms of the rates of change of the stress-strain state characteristics of the being formed solid in the variable in time region occupied by this solid at the current time instant. The set initial and boundary conditions take into account the kinematic and force specifics of the interaction of the additionally attached material with the already formed part of the solid over the entire instantaneous surface of its growth.

A significant mathematical difficulty in solving the initial boundary value problems stated is the dependence of the moment of the beginning of stresses development in the material on the point of the formed solid due to the non-simultaneous inclusion of various material elements in the process of deformation. The mathematical apparatus developed in the framework of the established by Professor A.V. Manzhirov Russian scientific school allows to overcome the announced difficulty. According to this apparatus, the procedure of solving the stated non-classical problems can be reduced to solving a series of boundary problems having a mathematical form of classical problems of solid mechanics, followed by the treatment of certain integral equations in time for each point of the finally formed solid. In the presented work, the solutions of the corresponding classical problems are constructed analytically in a closed form containing quadratures depending in a complex way on different parameters of the simulated additive process and on rheological properties of the material used for manufacturing. Calculation of these quadratures and further inversion of integral equations are carried out by numerical methods.

Let us discuss the basic mechanical features of the in the present study considered additive process.

Since in the simulated processes the deformable solid does not exist in its final composition before the application of loads but is replenished with new material elements already in the course of deformation caused by their action, the moment τ_0 of appearance of stresses at the points of such a solid will change from point to point, so will not be a constant.

Thus, the lower limit of integration in (3) in the simulated processes should be set by some function $\tau_0 = \tau_0(\mathbf{r})$ of the radius-vector $\mathbf{r}$ of the solid points. The specific type of this function will be determined by the program of solid formation implemented in the concrete additive process. In our case, due to the axial symmetry of the additive process, it is obvious that $\tau_0(\mathbf{r}) = \tau_0(\rho)$ where ρ is the distance from the given point $\mathbf{r}$ of the material layer being manufactured on the substrate to the axis of symmetry being simultaneously its rotation axis. So, the function $\tau_0(\rho)$ for $a_{\text{fin}} < \rho < a_0$, where a_{fin} is the final (achieved by the end of the manufacturing process) value of the layer inner radius, should be determined from the condition $a(\tau_0(\rho)) \equiv \rho$ which means that non-zero stresses in the neighbourhood of the considered point of the formed material layer arise directly at the time of inclusion of this point in its composition. If the simulated additive process starts at a time instant $t = t_0 > 0$ than we have $\tau_0(\rho) > t_0$ for all $a(t) < \rho < a_0$, and $\tau_0(\rho) \rightarrow t_0$ as $\rho \rightarrow a_0 - 0$.

Elements of additional material attached to the solid in question in the process of its gradual formation may by virtue of certain mechanical, physical or chemical features of this process acquire at the time of joining some non-trivial stress state. Taking into account the initial stresses in the elements of any solid formed in the additive process should be part and parcel of the formulation of correct boundary conditions on that part of its surface to which the new material flows. This part of the boundary surface of an additively formed solid is usually called the surface of its growth. In this paper we consider the case when by applying additional material onto the surface of the growth of the produced layer, i.e. its inner surface $\rho = a(t)$, this surface does not experience the action of any external loads. In other words, the initial stresses at the points of the formed layer, set at these points at the time when the growth surface passes through them, are consistent with the zero load on this surface:

$$\mathbf{e}_\rho \cdot \mathbf{T}|_{t=\tau_0(\rho)} = \mathbf{0}$$

where $\mathbf{e}_\rho = \mathbf{e}_\rho(\mathbf{r})$ is the unit vector that specifies the radial direction in the cross-sectional plane of the cylindrical layer being produced. One can explicitly define the vector field $\mathbf{e}_\rho$ as

$$\mathbf{e}_\rho = \frac{\partial \mathbf{r}}{\partial \rho} / \left| \frac{\partial \mathbf{r}}{\partial \rho} \right|.$$

The problem of deformation of the produced cylindrical layer is solved in the presented study in the approximation of plane strain state. The case of small strain is considered. In view of the latter circumstance, it does not make sense to take into account the deformation component in the time-dependence of the formed layer internal radius a which is decreasing in time due to the influx of additional material to the layer. So this dependence can be considered as given by virtue of the implemented program of material application. Thus, the function $a(t)$ is a known function, strictly decreasing at those time intervals in which the application of the material is carried out, and constant at those time intervals, in which the inflow of additional material to the formed layer is temporarily or permanently terminated, that is, in pauses between the individual stages of continuous application of the material and after the final completion of the layer additive formation.

In the context of the problem under study in this paper, it is necessary to pay attention to the following circumstances. If we were talking about the additive formation of a purely elastic solid, the rate of change of its stress-strain state at each point would be determined only by the instantaneous characteristics of the processes of its loading and replenishment with new material, and after the termination of the formation, fixation or removal of loads and kinematic bonds, the state of the solid would no longer change. The situation is different in the case of using the material which is viscoelastic and ageing in their mechanical properties. Here, the nature of the development of the stress-strain state of the whole formed solid at each time instant depends on the full history of the formation and loading of the solid, that is, on the duration of staying of the various material elements in its composition, on the age of their entry into this composition, and on the laws of change of the loads applied to the solid in all previous moments of time. Along with this, in the pauses between the stages of continuous inflow of new material to the solid, as well as after the completion of the solid manufacturing, its stress-strain state will continue to change over time, striving for some final characteristics.

3. Results and discussion

Additive processes in which solids are formed from materials exhibiting pronounced rheological properties (strain aftereffect, ageing) are very difficult to model. This is explained by the continuous interaction of the deformation reaction of the formed solid to the loads acting on it, on the one hand, with the arising redistribution of stresses throughout the solid due to its replenishment with new material elements, on the other hand. Despite this, the study of such additive processes is very

important for a variety of technological applications. The present research deals with materials of right the mentioned kind.

On the basis of current approaches in mechanics of additive processes, the non-classical initial boundary value problem describing the simulated process is stated. It has the form:

$$\begin{aligned} \nabla \cdot \mathbf{S} + \mathbf{e}_\rho f(\rho, t) &= \mathbf{0} \quad \text{as } a(t) < \rho < a_0, \quad t > t_0; \\ \mathbf{S} &= 2\mathbf{D} + \phi \mathbf{1} I_1[\mathbf{D}], \quad \mathbf{D} = \frac{1}{2}(\nabla \mathbf{v}^{\text{transp}} + \nabla \mathbf{v}); \\ \mathbf{e}_\rho \cdot \mathbf{S} &= \mathbf{e}_\rho g(t) \quad \text{as } \rho = a(t); \quad \mathbf{v} = \mathbf{0} \quad \text{as } \rho = a_0; \\ \mathbf{T} &= \mathbf{e}_\varphi \mathbf{e}_\varphi \sigma_{\varphi,0}(\rho) \quad \text{as } t = \tau_0(\rho). \end{aligned} \quad (5)$$

Problem (5) is formulated in terms of the following variable stress-strain state characteristics:

$$\mathbf{S} = \frac{\partial Q_{\tau_0(\rho)} \mathbf{T}}{\partial t}$$

is the so called operator stress rate tensor, $\mathbf{D}$ is the classical strain rate tensor, $\mathbf{v}$ is the vector field of the solid deformation displacements velocities. The known scalar functions $f(\rho, t)$ and $g(t)$ are given by the following expressions:

$$\begin{aligned} f(\rho, t) &= 2\mu\rho \left[\frac{\omega(\tau_0(\rho))^2}{2} \frac{\partial c_{\text{shear}}(t, \tau_0(\rho))}{\partial t} + \frac{\omega(t)}{G(t)} \frac{d\omega(t)}{dt} + \int_{\tau_0(\rho)}^t \omega(\tau) \frac{d\omega(\tau)}{d\tau} \frac{\partial c_{\text{shear}}(t, \tau)}{\partial t} d\tau \right], \\ g(t) &= -\frac{1}{G(t)} \frac{da(t)}{dt} \left[\frac{\sigma_{\varphi,0}(a(t))}{a(t)} - \mu\omega(t)^2 a(t) \right]. \end{aligned}$$

Here μ is the used material mass density and $\sigma_{\varphi,0}(\rho)$ is the circumferential component of the initial stress tensor $\mathbf{T}|_{t=\tau_0(\rho)}$. We assume that this component is the only non-zero component of this tensor and arbitrarily depends on the radius of the attached elementary material layer. As one can see, the functions $f(\rho, t)$ and $g(t)$ are identified by:

- the entire history of the material attachment,
- the entire substrate rotation history,
- the material mass density,
- the creeping and aging properties of the material, and
- the material elastic shear modulus.

The closed-form analytical solution of problem (5) is constructed. This solution is quit cumbersome and therefore is omitted here. After the time evolution of the operator stress rate tensor $\mathbf{S}(\rho, t)$ is calculated by means of this solution, we can reconstruct the evolution of the operator stress tensor $\tilde{\mathbf{T}}(\rho, t) = Q_{\tau_0(\rho)} \mathbf{T}(\rho, t)$ via integrating procedure:

$$\tilde{\mathbf{T}}(\rho, t) = \frac{\mathbf{T}(\rho, \tau_0(\rho))}{G(\tau_0(\rho))} + \int_{\tau_0(\rho)}^t \mathbf{S}(\rho, \tau) d\tau.$$

The evolution of the true stress tensor $\mathbf{T}(\rho, t)$ at each point of the solid is to be further determined by solving the following integral equation:

$$\frac{\mathbf{T}(\rho, t)}{G(t)} - \int_{\tau_0(\rho)}^t \frac{\mathbf{T}(\rho, \tau)}{G(\tau)} K(t, \tau) d\tau = \tilde{\mathbf{T}}(\rho, t).$$

The numerical procedure of solving this equation is implemented in the present study with the help of trapezoid quadrature formula.

Numerous numerical calculations were performed using the developed mathematical model. In the calculations, a variant of centrifugal application of the material is considered, in which the features of its entry into the composition of the formed solid do not lead to the appearance in the latter of initial stresses other than zero near the surface of its growth.

We take in the computations the following approximating expressions for the creep measure and the elastic shear modulus [9]:

$$c_{\text{shear}}(t, \tau) = C(\tau)[1 - e^{-\gamma(t-\tau)}], \quad G(t) = G_{\infty} - \Delta G e^{-\alpha t}$$

where γ , α , G_{∞} and ΔG are the positive constants fitting experimental data, and the function

$$C(\tau) = c_{\text{shear}}(+\infty, \tau),$$

called the function of ageing, or the creep resource, is taken in the form [9]:

$$C(\tau) = C_{\infty} + \Delta C e^{-\beta \tau}$$

where β , C_{∞} and ΔC are the experimentally chosen positive constants. We use the following values for the material constants [9]:

$$\alpha/\gamma = 2, \quad \beta/\gamma = 31/60, \quad \Delta G/G_{\infty} = 0.5, \quad G_{\infty}\Delta C = 4, \quad G_{\infty}C_{\infty} = 0.5522, \quad \nu = 0.2.$$

We conduct all the analysis of mechanical behaviour of the being additively formed material layer in terms of the dimensionless stress values $\bar{T}$ evolving over the dimensionless time variable $\bar{t}$. These quantities are introduced as

$$\bar{T} = T/G_{\infty}, \quad \bar{t} = \gamma t.$$

The stress distributions in a material layer additively formed on a substrate are compared with the corresponding classical distributions in a similar layer coupled with the same substrate and rotating with it around its axis but completely made in advance without any residual stresses before the start of rotation. The latter distributions are found from the solution of the corresponding classical solid mechanics problem which does not take into account the process of solid formation and assumes the application of all loads to it already in its final composition. The principal qualitative and quantitative differences of the above distributions are demonstrated: they differ in the character of monotony, positions and values of extreme values, have different intervals of sign-constancy. It is extremely important to point out that the revealed differences are due not to the rheological manifestations in the deformation response of the material used, but to the mechanical features of the additive process itself of layer-by-layer formation of a solid at the same time with the loads acting on it. It can be shown (see, for example, [12]) that in the absence of rheological manifestations (in the case of using a purely elastic material) the demonstrated differences would be even more pronounced, whereas the creep process in the solid during its gradual formation affords slurring over these differences.

The evolution of the technological stress state of the formed material layer during its manufacture is analyzed. It is shown that the intensity of shear stresses, which is an important stress characteristic of the product in terms of assessing its strength, reaches a maximum in the immediate vicinity of the substrate. This fact can not be found on the basis of the solution of the classical mechanical problem on deformation under the action of centrifugal forces of a material layer coupled with the rotating substrate, not taking into account the mechanical features of the manufacturing process of this layer. Indeed, from such solution it would follow that the maximum of the intensity of shear stresses should be achieved at the free surface of the layer, the most distant from the substrate, which directly contradicts the results obtained taking into account the specifics of the additive technological process.

In the presented work it is demonstrated that the technological stresses in the material layer arising in the process of its manufacture depend in a decisive way on the speed and nature of the whole process. Thus, the presence of pauses in the process of applying the material leads to a qualitative and very significant quantitative change in the pattern of the technological stress state of the finished layer compared to that obtained in a continuous process.

The very specificity of the simulated technological processes dictates the appearance of residual technological stresses in the resulting products. The final distributions of residual stresses, which are eventually settled in the finally formed material layer, are found in the present study in two various situations: after the rotation stops at the completion of the layer formation, as well as after the rotation stops and the finished layer is detached from the substrate (if the latter is provided by the simulated technological process). It could be expected that the stresses arising under the action of centrifugal forces in the layer during its fabrication, together with the stop of rotation and therefore the disappearance of these forces should essentially reduce their values. However, as the calculations show, this does not happen: for example, the maximum values of compressive circumferential stress and the intensity of shear stresses may even increase significantly after the termination of the rotational motion. The reduction of all residual stresses is observed only after the finished layer is detached from the substrate, but the corresponding stresses can still not be neglected compared to those that have acted during the centrifugal application of the material.

If the finished layer remains bonded to the substrate after the termination of the centrifugal application of the material, then, despite the absence of inertial effects on the non-rotating material layer, the normal contact stresses at the interface of this layer and the substrate will remain compressive indefinitely. This circumstance, obviously, prevents spontaneous detachment of the finished layer from the substrate, which is especially important in cases where it is supposed to be further used as an entity together with the substrate on which it was manufactured. If the substrate was used in the simulated process only as a tooling and is disconnected from the finished layer after the material application process, then the corresponding residual radial stresses will be compressive throughout the thickness of the layer, and the residual circumferential stresses will be stretching in the finished layer part that was adjacent to the substrate and they will be compressing in another part.

4. Conclusion

In the present study, the additive technological processes of layer-by-layer manufacturing ageing viscoelastic pieces on the inner surface of a rotating cylindrical substrate are investigated. The material behavior is described in the study in the framework of the linear hereditary theory of viscoelasticity of ageing media. The following fundamental scientific and practical results are obtained:

- 1) A non-classical mechanical model of the processes in question is proposed. The model is based on considerations and approaches of the current theory of additively formed deformable solids. The corresponding initial boundary value problem for description of the deformation process of the being formed material pieces under the simultaneous action of centrifugal forces is stated in terms of these pieces stress-strain state rate characteristics.
- 2) The numerical-analytical solution of the problem is obtained. On the basis of this solution numerous calculations are performed. The evolution of the stress-strain state of a cylindrical material layer during and after its manufacturing is described.
- 3) It is demonstrated that the distributions of technological stresses in the produced material layer depend in a decisive way on the realized manufacturing regime and essentially differ from the classical stress distributions in a similar rotating material layer that has not undergone any deforming impacts during the process of its manufacturing.
- 4) The mentioned difference can be explained by basic mechanical features of the additive process itself and causes the inevitable occurrence of residual stresses in the finished product after stopping the substrate rotation and, if it is necessary, after detaching the produced piece from the substrate. The distributions of these residual stresses are found and analyzed in detail.

- 5) The practically significant mechanical effects related with additive manufacture processes with the use of viscoelastic materials are detected.

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