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### Study of stress-strain behavior of a compressed elastic strip

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Abstract The paper considers the plane strain of a compressed elastic strip made from incompressible material. The deviation of the cross-section contour from the rectangle, as well as the intensity of the force applied to the upper and the lower edge of the section are known up to small parameters. The authors have carried out a study of the problem solution analyticity with respect to small near zero parameters. With the help of the perturbation method, they have found a solution for a particular case of the function describing the deviations up to the second order of smallness. It has been shown that if an experimental research is carried out and the applied force meets the condition described, the average stress and displacement values will differ from those typical for homogeneous state only by a second order infinitesimal.

#### 1. Introduction

The researchers studying the properties of elastic bodies often consider the displacement and deformation to be small. Given these assumptions, such an approach provides the results suitable for various purposes [1-3]. Strip and plate distortion by various types of external force for different bearing types has been considered in [4-6].

It has been shown in [7] that when due regard is given to boundary surface deformation, the problems associated with elastic bodies stability can be solved by the methods of mathematical elasticity theory. The example of elastic strip compression with p force shows that the results obtained through the methods applied in the studies of materials strength are the limiting results for the critical loads determined by the mathematical elasticity theory. This approach was used in [8]. Here, the rotation components were considered not only under boundary conditions, but also in equilibrium equations.

The properties of materials, the factors of production associated with technology, as well as the external factors are stochastic. Analytical methods of solving stochastic boundary problems for struc-

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turally heterogeneous materials are well developed for linear elastic media [9, 10]. The development of analytical methods for solving stochastic problems is associated with certain obstacles, the most important of which are physical and statistical nonlinearity of the defining equations. One of the methods frequently used for analytical solution of stochastic boundary problems is perturbation method. The use of the small parameter method in the study of stress-strain behavior of elastoplastic bodies is described in the monograph by D.D. Ivlev and L.V. Yershov [12]. However, this approach is associated with computation difficulties, and for this reason the solutions of specific stochastic problems are normally restricted to the first approximation. This is justified for weakly inhomogeneous media [11-15].

The use of the small parameter method for the study of creep processes in various stochastically inhomogeneous media has been shown in [16, 17]. In [16] stochasticity was introduced into the defining relation of the creep, in compliance with the nonlinear theory of viscous flow. The authors applied the small parameter expansion method, where allowance is made for both, linear and quadratic terms.

In [17] an analytical solution to a nonlinear boundary problem of steady-state creep for nonhomogenous endless strip is provided. The problem can be solved approximately by perturbation method in relation to stress tensor components. The solution was used for statistical analysis allowing us to determine the basic edge effect singularities.

The ways of solving stochastic boundary problems with a higher order of expansion terms for multidimensional problems, as well as the problems of solution convergence, are to be further explored.

In [21] the author describes the conditions under which (for several spaces of solutions and given data [18-20]) the solution will be analytical in small parameters close to zero. The main condition for that is the continuous dependence of the solution on small parameters.

#### 2. Problem description

The present paper looks at the stress-strain behavior of a compressed elastic strip made from incompressible material.



Figure 1. Cross-Section of the Strip.

It is assumed that the functions characterizing the upper and the lower edge of the section in undeformed state, as well as the intensity of the force applied, are accurate up to small parameters. They are represented by random variables.

$$f^{(i)}(x) = (-1)^{i}(h + \varepsilon_{1}h^{(i)}(x)); \quad P_{y} = -p_{y} + \varepsilon_{2}q(x) \qquad (i=1,2)$$
(1)

Given [8], let us consider the following problem as a mathematical model describing the state of a strip made from incompressible material.

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$$\frac{\partial(\sigma_x - \omega\tau)}{\partial x} + \frac{\partial(\tau - \omega\sigma_y)}{\partial y} = 0; \quad \frac{\partial(\tau + \omega\sigma_x)}{\partial x} + \frac{\partial(\sigma_y + \omega\tau)}{\partial y} = 0$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0; \quad \omega = \frac{1}{2} \left( \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} \right)$$

$$\sigma_x - \sigma_y = 4G \frac{\partial u}{\partial x}; \quad \tau = G \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)$$

$$\tau_n \Big|_{y=g_i} = 0; \quad \sigma_n \Big|_{y=g_i} = P_y \quad (3)$$

$$\sigma_{x}\Big|_{x=\pm\ell} = -p_{x}; \quad v\Big|_{x=\pm\ell} = \varepsilon_{y}^{0}y \tag{4}$$

Functions  $g_1(x)$  and  $g_2(x)$  describe the upper and the lower edge of the deformed strip.

If we ignore the imperfections, i.e.  $h^{(i)}(x) = 0$ , q(x) = 0, then the problem (2)–(4) can be solved as [4, 8]:

$$\sigma_x = \sigma_x^0 = -p_x; \quad \sigma_y = \sigma_y^0 = -p_y; \quad \tau = \tau^0 = 0$$

$$v = v^0 = \varepsilon_y^0 y; \quad u = u^0 = \varepsilon_x^0 x; \quad \omega = \omega^0 = 0$$
(5)

where constants  $\varepsilon_y^0$ ,  $\varepsilon_x^0$  are derived from (5) in accordance with the following relation:

$$\varepsilon_y^0 = \frac{p_x - p_y}{4G}, \qquad \varepsilon_x^0 = -\varepsilon_y^0 \tag{6}$$

The solution to this problem should be obtained by the perturbation method and expressed as a power series, while (5) will be used as a zero approximation:

$$u = \sum_{k,l=0}^{\infty} \varepsilon_{1}^{k} \varepsilon_{2}^{l} u^{kl}(x, y); \qquad v = \sum_{k,l=0}^{\infty} \varepsilon_{1}^{k} \varepsilon_{2}^{l} v^{kl}(x, y)$$

$$\sigma_{x} = \sum_{k,l=0}^{\infty} \varepsilon_{1}^{k} \varepsilon_{2}^{l} \sigma_{x}^{kl}(x, y); \qquad \sigma_{y} = \sum_{k,l=0}^{\infty} \varepsilon_{1}^{k} \varepsilon_{2}^{l} \sigma_{y}^{kl}(x, y); \qquad (7)$$

$$\tau = \sum_{k,l=0}^{\infty} \varepsilon_{1}^{k} \varepsilon_{2}^{l} \tau^{kl}(x, y)$$

In compliance with [21], it is important to study the continuous dependence of the solution on the small parameters close to zero, so that the series (7) could converge.

## **3.** Continuous dependence of the solution describing the stress-strain behavior of the strip on the initial data

In accordance with the results obtained in [7, 8], [18] describes various mathematical models of the strain boundary conditions. The analysis shows that while studying continuous dependence of the solution on the initial data (functions, parameters of the mathematical model), the statistical boundary conditions should be formulated on the boundary of a real body in the strained state. In [21] special cases of differential operators used in the solutions of solids' quasistatic deformation problems are described. The conditions for continuous dependence of the solution on the initial data have been obtained for several Banach spaces with certain norms, which are normally used for solving the problems

of continuum mechanics (Hölder spaces, Hilbert space,  $C^2([a,b], \mathbf{R}^m)$ ,  $C^4([a,b], \mathbf{R}^m)$  etc.), where Fréchet derivative of mapping is an isomorphism [18-20].

As it follows from [21], in order to study the problem of continuous dependence, we need to have a look at an auxiliary homogeneous problem for the functions  $\zeta_i$ , which was formulated on the basis of (1), (4), (5)  $(\sigma_x = \sigma_x^{(0)} + \zeta_1; \sigma_y = \sigma_y^{(0)} + \zeta_2; \dots v = v^{(0)} + \zeta_5)$ . Let us change the variable in compliance with [7]  $y_1 = y + (1 + \varepsilon_v^0)h$ :

$$\begin{aligned} \frac{\partial \zeta_{1}}{\partial x} + \frac{\partial \zeta_{3}}{\partial y_{1}} + \frac{p_{y}}{2} \frac{\partial}{\partial y_{1}} \left( \frac{\partial \zeta_{4}}{\partial y_{1}} - \frac{\partial \zeta_{5}}{\partial x} \right) &= 0; \\ \frac{\partial \zeta_{3}}{\partial x} + \frac{\partial \zeta_{2}}{\partial y_{1}} - \frac{p_{x}}{2} \frac{\partial}{\partial x} \left( \frac{\partial \zeta_{5}}{\partial x} - \frac{\partial \zeta_{4}}{\partial y_{1}} \right) &= 0 \\ \frac{\partial \zeta_{4}}{\partial x} + \frac{\partial \zeta_{5}}{\partial y_{1}} &= 0; \\ \zeta_{1} - \zeta_{2} &= 4G \frac{\partial \zeta_{4}}{\partial x}; \\ \zeta_{3} &= G \left( \frac{\partial \zeta_{4}}{\partial y_{1}} + \frac{\partial \zeta_{5}}{\partial x} \right) \\ \left| \zeta_{3} - p_{x} \frac{\partial \zeta_{5}(x, 0)}{\partial x} \right|_{y_{1}=0} &= 0; \quad \left| \zeta_{3} - p_{x} \frac{\partial \zeta_{5}(x, y)}{\partial x} \right|_{y_{1}=2(1+\varepsilon_{y}^{0})h} &= 0; \\ \zeta_{2}(x, y) \Big|_{y_{1}=0} &= \zeta_{2}(x, y) \Big|_{y_{1}=2(1+\varepsilon_{y}^{0})h} &= 0; \end{aligned}$$
(9)

In compliance with [12], we will assume that

ζ

$$\zeta_4 = \frac{\partial \Phi}{\partial y_1}; \quad \zeta_5 = -\frac{\partial \Phi}{\partial x} \tag{11}$$

Given the expressions (11), we can create an equation for finding the function  $\Phi(x, y_1)$ 

$$\left(1 - \frac{p_y}{2G}\right)\frac{\partial^4 \Phi}{\partial y_1^4} + \left(2 - \frac{p_y}{2G} - \frac{p_x}{2G}\right)\frac{\partial^4 \Phi}{\partial x^2 \partial y_1^2} + \left(1 - \frac{p_x}{2G}\right)\frac{\partial^4 \Phi}{\partial x^4} = 0$$
(12)

The equation (12) will be solved as:

$$\Phi(x, y_1) = \phi(y_1) \cos \lambda x \tag{13}$$

The conditions (10), where  $\lambda = n \frac{\pi}{\ell}$ , will be met. Let us use (13) in (12), and we will get an equation for determining the function  $\varphi(y_1)$ 

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$$(1 - \gamma_{1})\frac{d^{4}\phi}{dy_{1}^{4}} - (2 - \gamma_{1} - \gamma)\lambda^{2}\frac{d^{2}\phi}{dy_{1}^{2}} + (1 - \gamma)\lambda^{4}\phi = 0$$

$$\gamma = \frac{p_{x}}{2G}; \qquad \gamma_{1} = \frac{p_{y}}{2G}$$
(14)

The equation (14) can be easily integrated. After the function  $\Phi(x, y_1)$  has been determined, let us find  $\zeta_1$  and  $\zeta_2$ , while the relations (8) allow us to find the other functions. Now we will need the following expressions:

$$\begin{split} \zeta_{5} &= \lambda (C_{1} \mathrm{sh} \lambda y_{1} + C_{2} \mathrm{ch} \lambda y_{1} + C_{3} \mathrm{sh} \lambda \gamma_{2} y_{1} + C_{4} \mathrm{ch} \lambda \gamma_{2} y_{1}) \mathrm{sin} \, \lambda x \\ \zeta_{3} &= -G \lambda^{2} \Big[ 2 (C_{1} \mathrm{sh} \lambda y_{1} + C_{2} \mathrm{ch} \lambda y_{1}) + \Big( \gamma_{2}^{2} + 1 \Big) (C_{3} \mathrm{sh} \lambda \gamma_{2} y_{1} + C_{4} \mathrm{ch} \lambda \gamma_{2} y_{1}) \Big] \mathrm{sin} \lambda x \\ \zeta_{2} &= \Bigg[ -G \lambda^{2} \Big( 2 C_{1} \mathrm{ch} \lambda y_{1} + 2 C_{2} \mathrm{sh} \lambda y_{1} + \frac{\gamma_{2}^{2} + 1}{\gamma_{2}} C_{3} \mathrm{ch} \lambda \gamma_{2} y_{1} + \frac{\gamma_{2}^{2} + 1}{\gamma_{2}} C_{4} \mathrm{sh} \lambda \gamma_{2} y_{1} \Big) + \\ &+ \frac{p}{2} \frac{\lambda^{2} (\gamma_{2}^{2} - 1)}{\gamma_{2}} (C_{3} \mathrm{ch} \lambda \gamma_{2} y_{1} + C_{4} \mathrm{sh} \lambda \gamma_{2} y_{1}) \Bigg] \mathrm{cos} \lambda x \\ \gamma_{2}^{2} &= \frac{1 - \gamma}{1 + \gamma_{1}} = \frac{2 G - p_{x}}{2 G - p_{y}} \end{split}$$

The simultaneous equations (8) have a nontrivial solution, when:

$$2\frac{\operatorname{ch}(\beta(2-\gamma_{1}+\gamma))\operatorname{ch}\left(\beta\sqrt{\frac{\gamma-1}{\gamma_{1}-1}}(2-\gamma_{1}+\gamma)\right)-1}{\operatorname{sh}(\beta(2-\gamma_{1}+\gamma))\operatorname{sh}\left(\beta\sqrt{\frac{\gamma-1}{\gamma_{1}-1}}(2-\gamma_{1}+\gamma)\right)} = (15)$$

$$=\frac{4+\gamma_{1}+\gamma-\gamma(\gamma_{1}-1)}{2+\gamma_{1}+\gamma+\gamma(\gamma_{1}-1)}\sqrt{\frac{\gamma-1}{\gamma_{1}-1}}\frac{1}{2-\gamma} + \frac{2+\gamma_{1}+\gamma+\gamma(\gamma_{1}-1)}{4+\gamma_{1}+\gamma-\gamma(\gamma_{1}-1)}\sqrt{\frac{\gamma_{1}-1}{\gamma-1}}(2-\gamma)$$

where  $\gamma = \frac{p_x}{2G}$ ;  $\gamma_1 = \frac{p_y}{2G}$ ,  $\lambda = n \frac{\pi}{\ell}$ ;  $\beta = \lambda h$ .

If  $\gamma = 0$ , then we can get the condition mentioned in [8] up to the designations.

In Figure 2a we can see a curve corresponding to (15) in case  $\frac{h}{\ell} = 0.05$ , where n = 1. Here the value of  $\gamma$  is virtually the same as in [7]. For  $\frac{h}{\ell} = 0.5$  the line corresponding to (15) is shown in Figure 2b (n = 1). The results are the same as those obtained in [8].



Figure 2. The boundaries of the series convergence region (7).

Therefore, with parameters  $\gamma_1$  and  $\gamma'$  determining the location of the point within the region limited by the curve (15), the solution to the problem (2)-(4) will be analytical around the point  $\varepsilon = 0$ , and the series (7) will be convergent. It must be noted that if the point is beyond the region determined by the function curve (15), the solution (5) cannot be used as an approximate solution to the problem (2)-(4), even if the parameter  $\varepsilon$  is indefinitely small. In this case, if  $\varepsilon = 0$ , we need to find a different solution to the problem (2)-(4), i.e. the one that does not refer to homogeneous stress-strain state.

#### 4. Perturbation method application

Normally the conduction of research is fairly complicated. Thus, we will consider a typical case where  $\frac{p}{G} \ll 1$ . In compliance with (9), the strain will also be small.

$$\varepsilon_v^0 \ll 1; \quad \varepsilon_x^0 \ll 1.$$

If we use (7), (5) in (2)-(4), we will get the following problems for the first approximation:

$$\frac{\partial \sigma_x^{10}}{\partial x} + \frac{\partial \tau^{10}}{\partial y_1} + \frac{p_y}{2} \frac{\partial}{\partial y_1} \left( \frac{\partial u^{10}}{\partial y_1} - \frac{\partial v^{10}}{\partial x} \right) = 0;$$

$$\frac{\partial \tau^{10}}{\partial x} + \frac{\partial \sigma_y^{10}}{\partial y_1} - \frac{p_x}{2} \frac{\partial}{\partial x} \left( \frac{\partial v^{10}}{\partial x} - \frac{\partial u^{10}}{\partial y_1} \right) = 0$$

$$\frac{\partial u^{10}}{\partial x} + \frac{\partial v^{10}}{\partial y_1} = 0;$$

$$\sigma_x^{10} - \sigma_y^{10} = 4G \frac{\partial u^{10}}{\partial x}; \quad \tau^{10} = G \left( \frac{\partial u^{10}}{\partial y_1} + \frac{\partial v^{10}}{\partial x} \right)$$
(16)

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where  $y_1 = 0; 2h$ 

$$\sigma_{y}^{10}(x, y_{1}) = 0;$$
  

$$\tau^{10} + p_{x} \frac{\partial v^{10}}{\partial x} = -p_{x} \frac{\partial f^{(1)}}{\partial x}$$
(17)

$$\sigma_x^{10}(\pm \ell, y_1) = v^{10}(\pm \ell, y_1) = 0$$
(18)

As the equations (16) and (8) are in agreement up to the designations, we should find a general solution to the problem (16)-(18) in the form similar to (11). As a result, we will get:

$$\Phi(x, y) = (D_1 shay_1 + D_2 chay_1 + D_3 sha\gamma_2 y_1 + D_4 cha\gamma_2 y_1) \cos ax$$
(19)

As it follows from (19), the boundary conditions will be fulfilled if  $a = n \frac{\pi}{l}$ .

It can be derived from (17) that

$$\begin{split} \sigma_{x}^{10} &= -a^{2}G[2D_{1}\operatorname{chay}_{1} + 2D_{2}\operatorname{shay}_{1} + \left(\frac{\gamma_{1} + \gamma - 2}{\gamma_{1} - 1}\sqrt{\frac{\gamma_{1} - 1}{\gamma_{1} - 1}} - 4\sqrt{\frac{\gamma - 1}{\gamma_{1} - 1}}\right) \times \\ &\times (D_{3}\operatorname{cha}\sqrt{\frac{\gamma - 1}{\gamma_{1} - 1}}y_{1} + D_{4}\operatorname{sha}\sqrt{\frac{\gamma - 1}{\gamma_{1} - 1}}y_{1}) + \frac{p_{x}}{2G}\frac{\gamma - \gamma_{1}}{\gamma_{1} - 1}\sqrt{\frac{\gamma_{1} - 1}{\gamma_{-1}}} \times \\ &\times (D_{3}\operatorname{cha}\sqrt{\frac{\gamma - 1}{\gamma_{1} - 1}}y_{1} + D_{4}\operatorname{sha}\sqrt{\frac{\gamma - 1}{\gamma_{1} - 1}}y_{1})) \sin ax \\ \sigma_{y}^{10} &= a^{2}G(2D_{1}\operatorname{chay}_{1} + 2D_{2}\operatorname{shay}_{1} + \frac{\gamma_{1} + \gamma - 2}{\gamma_{1} - 1}\sqrt{\frac{\gamma_{1} - 1}{\gamma_{-1}}}(D_{3}\operatorname{cha}\sqrt{\frac{\gamma - 1}{\gamma_{1} - 1}}y_{1} + D_{4}\operatorname{sha}\sqrt{\frac{\gamma - 1}{\gamma_{1} - 1}}y_{1})) \sin ax \\ \tau^{10} &= a^{2}G[2D_{1}\operatorname{chay}_{1} + 2D_{2}\operatorname{chay}_{1} + \left(1 + \frac{\gamma - 1}{\gamma_{1} - 1}\right)(D_{3}\operatorname{sha}\sqrt{\frac{\gamma - 1}{\gamma_{1} - 1}}y_{1} + D_{4}\operatorname{cha}\sqrt{\frac{\gamma - 1}{\gamma_{1} - 1}}y_{1})] \cos ax \\ u^{10} &= a[D_{1}\operatorname{chay}_{1} + D_{2}\operatorname{shay}_{1} + \sqrt{\frac{\gamma - 1}{\gamma_{1} - 1}}(D_{3}\operatorname{cha}\sqrt{\frac{\gamma - 1}{\gamma_{1} - 1}}y_{1} + D_{4}\operatorname{sha}\sqrt{\frac{\gamma - 1}{\gamma_{1} - 1}}y_{1})] \cos ax \\ v^{10} &= a(D_{1}\operatorname{shay}_{1} + D_{2}\operatorname{chay}_{1} + D_{3}\operatorname{sha}\sqrt{\frac{\gamma - 1}{\gamma_{1} - 1}}y_{1} + D_{4}\operatorname{cha}\sqrt{\frac{\gamma - 1}{\gamma_{1} - 1}}y_{1}) \sin ax \\ v^{10} &= a(D_{1}\operatorname{shay}_{1} + D_{2}\operatorname{chay}_{1} + D_{3}\operatorname{sha}\sqrt{\frac{\gamma - 1}{\gamma_{1} - 1}}y_{1} + D_{4}\operatorname{cha}\sqrt{\frac{\gamma - 1}{\gamma_{1} - 1}}y_{1}) \sin ax \\ (20) \\ \omega^{10} &= -a\left(1 - \frac{\gamma - 1}{\gamma_{1} - 1}\right)(D_{3}\operatorname{sha}\sqrt{\frac{\gamma - 1}{\gamma_{1} - 1}}y_{1} + D_{4}\operatorname{cha}\sqrt{\frac{\gamma - 1}{\gamma_{1} - 1}}y_{1}) \cos ax \\ \end{array}$$

If  $h^{(i)}(x) = \cos \lambda x$ , we will get a system of algebraic equations for arbitrary constants  $D_j$ :

$$\sum_{j=1}^{4} \alpha_{ij} D_j = d_i \qquad (i = 1, ..., 4), \tag{21}$$

where

$$\alpha_{11} = \operatorname{ch}ah; \ \alpha_{12} = \operatorname{sh}ah; \ \alpha_{13} = -\alpha_{33} = h\operatorname{ch}ah; \ \alpha_{14} = \alpha_{34} = h\operatorname{sh}ah; \ \alpha_{21} = -\alpha_{41} = (1+\gamma)\operatorname{sh}ah;$$
  
$$\alpha_{22} = (1+\gamma)\operatorname{ch}ah; \ \alpha_{23} = \alpha_{43} = \frac{1}{a}\operatorname{ch}ah + h(1+\gamma)\operatorname{sh}ah;$$
  
$$\alpha_{24} = \alpha_{44} = \frac{1}{a}\operatorname{sh}ah + h(1+\gamma)\operatorname{ch}ah; \ d_1 = d_3 = 0; \ d_2 = d_4 = -p_x$$

In case of an approximation (01), the form of the defining equations and boundary conditions for the side edges of the section is analogous, while in case  $y_1 = 0$ ; 2h, the boundary conditions will be the following:

$$\sigma_{y}^{01}(x, y) = q(x);$$
  

$$\tau^{01} + p_{x} \frac{\partial v^{01}}{\partial x} = 0$$
(22)

If  $q(x) = \cos \lambda x$ , the system of algebraic equations for arbitrary constants  $D'_j$  will be as follows:

$$\sum_{j=1}^{4} \alpha_{ij} D'_j = d'_i \qquad (i = 1, ..., 4), \tag{23}$$

where  $d_1 = d_3 = 1$ ;  $d_2 = d_4 = 0$ .

In second order approximation problems the rheological relationships, incompressibility condition and expressions for the angles of rotation are the same as in the original problem. The equilibrium equations will be as follows:

$$\frac{\partial \sigma_x^{20}}{\partial x} + \frac{\partial \tau^{20}}{\partial y_1} + \frac{p_y}{2} \frac{\partial}{\partial y_1} \left( \frac{\partial u^{20}}{\partial y_1} - \frac{\partial v^{20}}{\partial x} \right) = \frac{\partial}{\partial y_1} (\omega^{10} \tau^{10}) + \frac{\partial}{\partial x} (\omega^{10} \sigma_y^{10});$$

$$\frac{\partial \tau^{20}}{\partial x} + \frac{\partial \sigma_y^{20}}{\partial y_1} - \frac{p_x}{2} \frac{\partial}{\partial x} \left( \frac{\partial v^{20}}{\partial x} - \frac{\partial u^{20}}{\partial y_1} \right) = -\frac{\partial}{\partial x} (\omega^{10} \tau^{10}) - \frac{\partial}{\partial y_1} (\omega^{10} \sigma_x^{10})$$
(24)

$$\frac{\partial \sigma_x^{11}}{\partial x} + \frac{\partial \tau^{11}}{\partial y_1} + \frac{p_y}{2} \frac{\partial}{\partial y_1} \left( \frac{\partial u^{11}}{\partial y_1} - \frac{\partial v^{11}}{\partial x} \right) = \frac{\partial}{\partial y_1} (\omega^{10} \tau^{01} + \omega^{01} \tau^{10}) + \frac{\partial}{\partial x} (\omega^{10} \sigma_y^{01} + \omega^{01} \sigma_y^{10});$$
$$\frac{\partial \tau^{11}}{\partial x} + \frac{\partial \sigma_y^{11}}{\partial y_1} - \frac{p_x}{2} \frac{\partial}{\partial x} \left( \frac{\partial v^{11}}{\partial x} - \frac{\partial u^{11}}{\partial y_1} \right) = -\frac{\partial}{\partial x} (\omega^{10} \tau^{01} + \omega^{01} \tau^{10}) - \frac{\partial}{\partial y_1} (\omega^{10} \sigma_y^{01} + \omega^{01} \sigma_y^{10});$$

The boundary conditions at the side edges are homogeneous, while at the other two edges ( $y_1 = 0$ ; 2h) are as follows:

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$$\sigma_{y}^{20} = -\frac{p_{y}}{2} \left(\frac{dg^{10}}{dx}\right)^{2} + \tau^{10} \frac{dg^{10}}{dx} - \frac{\partial \sigma_{y}^{10}}{\partial y_{1}} g^{10};$$

$$\tau^{20} + p_{x} \frac{dg^{20}}{dx} = \sigma_{x}^{10} \frac{dg^{10}}{dx} - \frac{\partial \tau^{10}}{\partial y_{1}} g^{10}$$
(25)

$$\sigma_{y}^{11} = -p_{y} \frac{dg^{10}}{dx} \frac{dg^{01}}{dx} + \tau^{10} \frac{dg^{01}}{dx} + \tau^{01} \frac{dg^{10}}{dx} - \frac{\partial \sigma_{y}^{10}}{\partial y_{1}} g^{01} - \frac{\partial \sigma_{y}^{01}}{\partial y_{1}} g^{10};$$

$$\tau^{11} + p_{x} \frac{dg^{11}}{dx} = \sigma_{x}^{10} \frac{dg^{01}}{dx} + \sigma_{x}^{01} \frac{dg^{10}}{dx} - \frac{\partial \tau^{10}}{\partial y_{1}} g^{01} - \frac{\partial \tau^{01}}{\partial y_{1}} g^{10}$$
(26)

where

$$g^{10} = h^{(1)} + v^{10}; \quad g^{01} = v^{01}; \quad g^{20} = h^{(1)} \frac{\partial v^{10}}{\partial y_1} + v^{20} - (h^{(1)} + v^{10})u^{10};$$
$$g^{02} = v^{02} - \frac{\partial v^{10}}{\partial x}u^{01}; \qquad g^{11} = v^{11} + h^{(1)} \frac{\partial v^{01}}{\partial y_1} - \frac{dg^{10}}{\partial x}u^{01} - \frac{\partial v^{01}}{\partial x}u^{10}$$

For the components (02) the equilibrium equations and boundary conditions are analogous to (25), (26).

Here, according to [12], given

$$u^{ij} = \frac{\partial \Phi^{(ij)}}{\partial y_1}; \quad v^{ij} = -\frac{\partial \Phi^{(ij)}}{\partial x}$$

we will get the equations for determining the function  $\Phi(x, y_1)$ 

$$\frac{\partial^4 \Phi^{(ij)}}{\partial y_1^4} + 2 \frac{\partial^4 \Phi^{(ij)}}{\partial x^2 \partial y_1^2} + \frac{\partial^4 \Phi^{(ij)}}{\partial x^4} = \Psi^{(ij)}(x, y_1),$$

where the function  $\Psi^{(ij)}(x, y_1)$  depends on the first order approximation components and is known.

Functions (20)-(23) helped us to find the expressions for the second-order approximation stress and displacement components.

$$\sigma_x^{ij} = \sigma_x^{ij}(2a, h, \gamma, \gamma_1, y) \sin 2ax, \quad \sigma_y^{ij} = \sigma_y^{ij}(2a, h, \gamma, \gamma_1, y) \sin 2ax,$$
$$\tau^{ij} = \tau^{ij}(2a, h, \gamma, \gamma_1, y) \cos 2ax$$
$$u^{ij} = u^{ij}(2a, h, \gamma, \gamma_1, y) \cos 2\lambda x, \quad v^{ij} = v^{ij}(2a, h, \gamma, \gamma_1, y) \sin 2ax$$

The functions are not given here because of their length. Hence, the functions

$$\sigma_{x} = \sigma_{x}^{0} + \varepsilon_{1}\sigma_{x}^{10} + \varepsilon_{2}\sigma_{x}^{01} + \varepsilon_{1}^{2}\sigma_{x}^{20} + \varepsilon_{2}^{2}\sigma_{x}^{02} + \varepsilon_{1}\varepsilon_{2}\sigma_{x}^{11}, \dots,$$
  
$$v = v^{0} + \varepsilon_{1}v^{10} + \varepsilon_{2}v^{01} + \varepsilon_{1}^{2}v^{20} + \varepsilon_{2}^{2}v^{02} + \varepsilon_{1}\varepsilon_{2}v^{11}$$

describe the stress-strain behavior of a strip up to the second order of smallness.

As random variables  $\varepsilon_1$  and  $\varepsilon_2$  are independent,

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#### 5. Conclusion

We have found the analyticity condition for the solution in small parameters for a pipe with crosssection close to a rectangle and external force close to a constant. If, according to the given data, the point is located within the region limited by the curve (15), the proximity of the section shape is retained under strain. If this condition is not met, the stress-strain state will no longer be homogeneous and corresponding to (5). In such a case we need either to find a different solution to the nonlinear problem or to alter the mathematical model.

We have found a solution describing stress-strain behavior of a strip up to the second order of smallness, which can be applied to a particular case of the section shape deviation from a rectangle and external force deviation from its constant value. It can be deduced from (27) that, in case of an experimental research carried out, the average stress and displacement values will differ from those typical for homogeneous state (5) by a second order infinitesimal  $\varepsilon_1$  and  $\varepsilon_2$ , on condition that the external influence parameters fall within the region limited by (15).

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