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Graphs with table constraints on reachability

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Abstract. On directed graphs defined a new kind of reachability restriction – table constraints on reachability. Each edge of the graph assigned a certain element of the monoid. Some element of the monoid selected and fixed. This element does not equal to the unit of the monoid. This element is called forbidden. Each way on the graph is associated with characteristic – a vector, whose length is equal to the number of way edges. First element of the characteristic is equal to the element of the monoid which corresponded to the first edge of the way. Each successive element of the characteristic is equal to the previous element of the characteristics plus the value of an element of the monoid, corresponding to the next edge of the way. A way on the graph is considered as valid if the way characteristic does not contain a forbidden element. The construction of the scan-graph, which is built on the original graph, is described. The transition to the scan-graph allows you to solve the problems of the reachability on such graphs, shortest ways and random walks on the graphs with table constraints on reachability.

1. The first section in your paper

Graphs with different reachability constraints (or graphs with non-standard reachability) we considered in [1-6]. The most complete and detailed results and methods of working with such objects described in [1]. Reachability constraints generally mean that there are additional constraints on the sequence of way edges.

The first constraints we studied [1] was a mixed reachability. In this case, have selected subset of edges, along which the way cannot pass in a row. That is, edges from this subset as elements of a subset of edges are selected on the graph, along which the path cannot pass in a row. The edges from such a subset in the on the sequence of way edges must be mixed (separate) by other edges of the graph. The transition from considering the entire set of ways on a graph to considering a subset of ways on a graph (constraints on reachability) makes classical problems about shortest ways, random walks on the vertices of the graph and flows in networks is non-trivial. Classical algorithms for solving them become inapplicable because they do not provide for restrictions on the set of admissible ways.

For all the introduced and studied types of constraints on reachability, we proposed a general approach to solving classical problems on graphs with constraints on reachability, consisting in constructing an appropriate scan-graph – an auxiliary graph without constraints on reachability. The original problem on the graph with reachability constraints transfer to the scan-graph, where it is treated as a problem without constraints on reachability, and the result is "returned" to the original graph with constraints on reachability. Thus, if for the entered restrictions on reachability it is possible



to describe the design of the scan-graph corresponding to this restriction, then the offered scheme allows solving classical problems on graphs with constraints on reachability.

In this article, we introduce a new rather General type of constraints on reachability, which we called tabular. The design of the scan-graph described for it. The problem of reachability, shortest ways and random walks on the vertices of graphs with different constraints on reachability solved.

2. Definitions and Examples

Under the directed graph, following [11], we will understand the triple $G(X;U;f)$, $X (\neq \emptyset)$ is the set of vertices of the graph, U is the set of edges of the graph ($X \cap U = \emptyset$), $f:U \rightarrow X \times X$ – mapping from the set of edges of the graph to the Cartesian product $X \times X$.

This definition of a graph is the most general, since it admits the presence on the graph of a parallel (multiple) edges. In classical problems on graphs with no constraints on reachability, it is possible to "get rid" of multiple edges, and in the case of constraints on reachability, the multiplicity of edges can be present in essence, especially when solving specific application problems.

Definition 1. Under the way μ of length n on the graph we will mean the mapping $\mu:[1;n]_{\mathbb{N}} \rightarrow U$, in which each subsequent edge $\mu(i+1)$ begins at the vertex at which the previous edge $\mu(i)$ ends. ($[1;n]_{\mathbb{N}} = \{1;2;\dots;n\}$).

If $f(u)=(x;y)$, then the vertex x is called the beginning vertex of the edge u , and the vertex y is called end vertex of the edge u .

The beginning vertex of the edge $\mu(1)$ is called the beginning of the way μ , and the ending vertex of the edge $\mu(n)$ is called the end of the way μ .

Definition 2. Let M be a finite monoid ([7]) with a binary operation $*$ and a unit (neutral element) e . We assume that on the edges of the graph $(X;U;f)$ are given a mapping $r:U \rightarrow M$. Consider the way μ of length n on the graph. Define the characteristic of the way μ - r $r_{\mu}(i)$, ($1 \leq i \leq n$) inductively:

$$r_{\mu}(1)=r(\mu(1)), r_{\mu}(i+1)=r_{\mu}(i)*r(\mu(i+1)) \tag{1}$$

We can define what does it means table constraints on reachability.

Let us assume that the monoid M contains an element z , which we will call forbidden. Let's consider a graph with the mapping $r: U \rightarrow M$ given on it.

Definition 3. The way μ of length n will be called valid under a table constraints on reachability, defined by a monoid M with a forbidden element z and a mapping r , if the characteristic of the way does not take a value equal to z , i.e. $r_{\mu}(i) \neq z, \forall i \in [1;n]_{\mathbb{N}}$, otherwise, the way μ is invalid.

Definition 4. The graph on which we considered only valid ways in the sense the definition 3, we will call a graph with table constraints on reachability generated by the monoid M with forbidden element z and the mapping r .

Such graph will be denoted by $G(X;U;f;M;r)$ and called a graph with a table constraints on reachability.

Example 1. Let the monoid M be given by the following Kelly table (table 1).

Table 1. Kelly table

	e	a	b	z
e	e	a	b	z
a	a	a	b	z
b	b	a	z	z
z	z	z	z	z

Consider the graph shown in figure 1.

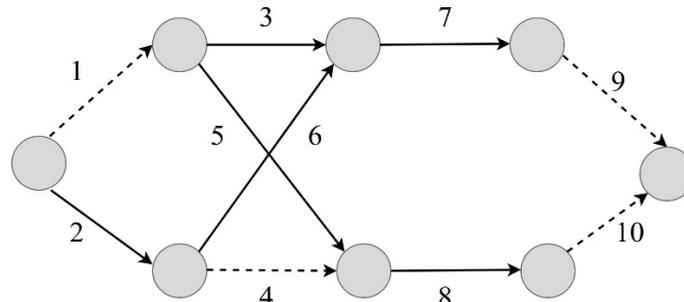


Figure 1. The graph with table constraints on reachability.

The mapping r on the edges indicated by the dotted line is equal to a , and on the edges indicated by the solid line it is equal to b .

It is clear that on this graph, there is only one valid way leading from the left vertex to the right vertex satisfy to a given table constraints on reachability.

This way is specified by the rule $\mu(1)=2, \mu(2)=4, \mu(3)=8, \mu(4)=10$.

Indeed, we will write out with the help of table 1 the characteristic r_μ of this path:

$$r_\mu(1)=b; r_\mu(2)=a; r_\mu(3)=b; r_\mu(4)=a.$$

Let's check that the way specified by the rule $\beta(1)=1, \beta(2)=3, \beta(3)=7, \beta(4)=9$, is not a valid way. Find using table 1 characteristic r_β of this way:

$$r_\beta(1)=a; r_\beta(2)=b; r_\beta(3)=z; r_\beta(4)=z.$$

3. Plotting a scan-graph for a graph with table constraints on reachability

Under the directed graph, following Each vertex x of the original graph on the scan-graph corresponds to a set of vertices in an amount equal to $|M|-1$, denoted by $x_g \in M \setminus \{z\}$. Each edge u on the scan-graph corresponds to a set of edges, which are constructed by the following rule: if $f(u)=(x,y)$ and $r(u)=g$, then the set of edges consists of edges connecting the following pairs of the vertices on scan-graph:

$$(x_t; y_s); t \in M \setminus \{z\}; s = t * g. \tag{2}$$

Example 2. Let the monoid Consider the graph shown in figure 2. The constraints are specified in table 1. The designations are similar to example 1.

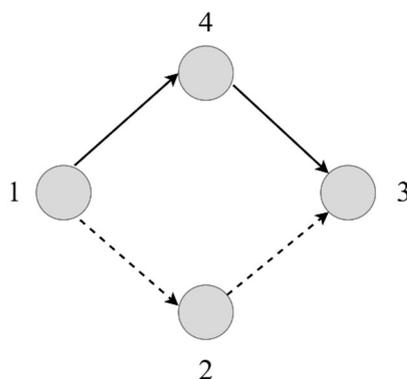


Figure 2. A graph with table constraints on reachability.

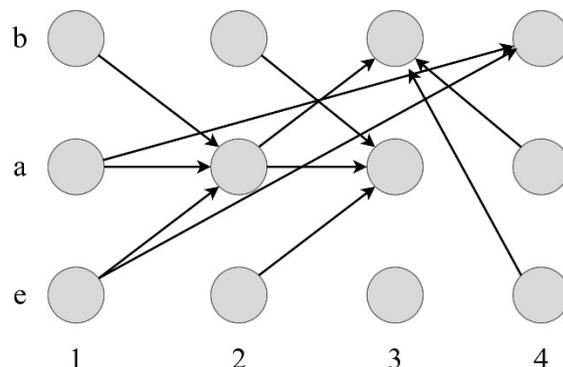


Figure 3. A scan of the graph shown in figure 2.

Figure 3 shows the scan-graph for the graph on figure 2.

Theorem 1. A vertex y is reachable from a vertex x on a graph $G(X,U,f,M,r)$ with table constraints on reachability generated by a monoid M with a forbidden element z , if and only if there is at least one way from vertex x_e to a set of vertices $\{y_g, g \in M \setminus \{z\}\}$ on a scan-graph for the graph $G(X,U,f,M,r)$.

Obviously, the ways on the original graph correspond to the ways on the scan-graph leading from "layer e ". In this case, the set of indices of the vertices along which the way passes coincides with the characteristic of the way on original graph. Absence on the scan-graph vertices from "layer z " guarantees the validity of the way. By "layer g " on the scan-graph is meant the set of all vertices having the same index $g \in M \setminus \{z\}$.

4. The shortest ways

Under the directed graph, Let the edges of the graph have a length, i.e. the mapping $\rho: U \rightarrow (0; \infty)$ is given. Under the length of the way we assume the sum of the lengths of the edges along which the way passes. Consider the problem of finding the shortest paths on a graph with table constraints on reachability.

It is clear that classical algorithms (see [8]) do not work in this case, since they find the shortest way in the set of all ways of the graph, and in the case of constraints on reachability we consider the problem on a subset of valid ways.

Let's build a scan-graph of the original graph. Let us transfer to the edges of scan-graph the lengths of the edges of the original graph which generated its.

The next theorem holds:

Theorem 2. The shortest way on a graph $G(X,U,f,M,r)$ with a table constraints on reachability generated by a monoid M with a forbidden element z leading from vertex x to vertex y can be restored (by matching edges of scan-graph to the edges of original graph that gave rise to them) by the shortest way on the scan-graph leading from vertex x_e to the set of vertices $\{y_g, g \in M \setminus \{z\}\}$ on a scan-graph for the graph $G(X,U,f,M,r)$.

5. Reducibility of some known constraints on the reachability to table constraints on reachability

Let the edges of the graph have a length, i.e. the mapping $\rho: U \rightarrow (0; \infty)$ is given. Table constraints on reachability seem to the author quite wide class restrictions on reachability. In particular, the mixed reachability that we talked about in the introduction can be interpreted as table constraints on reachability. It is generated by a monoid, the specified in table 1, when to each edge from a set of those edges, on which it is impossible to pass in a row, correspond the element of a monoid b ($r(u)=b$), and to neutral edges correspond the element of a monoid a ($r(u)=a$).

Now we show that the barrier reachability of height k (see [1]) can also be treated as table constraints on reachability. In the case of barrier reachability there are three types of edges on the graph: neutral, boosting and barrier. The reachability constraints are as follows – passing through the barrier edge is possible if the way has passed before through at least k boosting edges, passing through the next barrier edge required the passing through of at least k new boosting edges.

Barrier reachability is well models cycles in computer programs (see figure 4).

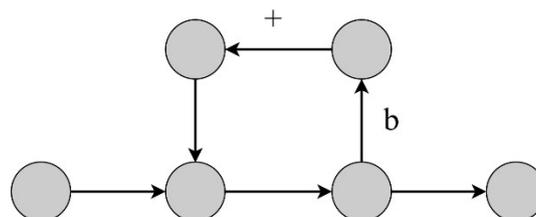


Figure 4. Program cycle.

On figure 4 the sign "+" denotes the boosting edge, the sign "b" – the barrier edge, other edges – neutral. The height of the barrier k determines the number of "scrolls" of the cycle.

It is obvious that the monoid M defining this kind of constraints on reachability is given by table 2 (In table 2 filled in only the cells needed to determine barrier reachability). The mapping $r:U \rightarrow M$ is defined by the equality (3):

$$r(u) = \begin{cases} e, & \text{if } u \in U_N; \\ a_1, & \text{if } u \in U_+; \\ b, & \text{if } u \in U_B. \end{cases} \tag{3}$$

U_N – denotes the set of neutral edges, U_+ – denotes the set of boosting edges, U_B – the set of barrier edges.

Table 2. Kelly table for barrier reachability

	e	a ₁	a ₂	a ₃	...	a _{k-1}	a _k	b	z
e	e	a ₁	a ₂	a ₃	...	a _{k-1}	a _k	b	z
a ₁	a ₁	a ₂							z
a ₂	a ₂	a ₃							z
a ₃	a ₃	a ₄							z
...
a _{k-1}	a _{k-1}	a _k							z
a _k	a _k	a _k							e
b	b								
z	z								

6. Reducibility of some known constraints on the reachability to table constraints on reachability

Let the transition probabilities be given on the edges of the graph. In this case the process of random walk of the particle on the vertices of the graph is a Markov process. In its motion, the particle randomly plots a way beginning at the starting vertex. Each next arc is selected according to the given probabilities.

We assume that on the graph is given table constraints on reachability. In this case the choice of the next edge is also determined by an additional condition on valid way. In this situation the random walk process is not Markovian.

A similar situation occurred for the previously studied reachability constraints ([1-5]). In [1] we proposed a General scheme that allows us to consider the corresponding random process on the scan-graph instead of the original random process.

The random process on the scan-graph is already a Markov process. Since we have described the scan-graph design for table constraints on reachability, the General scheme applies in this case as well.

We give an example showing the difference between a random walk process without constraints on reachability and a random walk process with table constraints on reachability.

Example 3. Consider the graph shown in figure 5. The notations for edges are similar to the graph in figure 2, the monoid M is given by table 1. The numbers near the edges are the transition probabilities. We assume the vertex "1" as the starting vertex. Table 3 corresponds to the process in the graph figure 5 without constraints on reachability, table 4 corresponds to the process in the graph figure 5 with table constraints on reachability. The columns of the tables correspond to the vertices of the graph, and the rows correspond to the time counts.

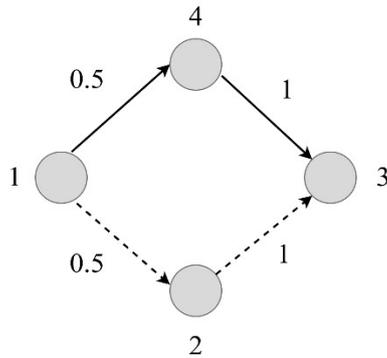


Figure 5. Graph with table constraints on reachability.

Table 3. Random walks without constraints

	1	2	3	4
0	1	0	0	0
1	0	0.5	0	0.5
2	0	0	1	0
3	0	0	1	0

Table 4. Random walks with constraints

	1	2	3	4
0	1	0	0	0
1	0	0.5	0	0.5
2	0	0	0.5	0.5
3	0	0	0.5	0.5

Comments

- In addition to the restrictions on the passage of the way along the edges of the graph, we considered graphs on which restrictions were imposed on the sequence of vertices of the way. Table constraints on reachability can be entered at once on edges and vertices of the graph. The mapping r in this case should be defined as $r:UUX \rightarrow M$. The way in this case should be understood as an alternating sequence of vertices and edges connecting them. The characteristic of the way is defined the similar as was done in definition 1 by equality (1).
- Graphs with reachability constraints are described in detail in the review article by L Yu Zhilyakova [13] and in it's with O P Kuznetsov monography [10].

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