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On a property of curves given by the determining equations

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Abstract. The motion of a mechanical system consisting of a carrier and a load is considered. The carrier, located all the time in a horizontal plane, can move translationally along a rectilinear trajectory. The carrier has a rectilinear channel through which the load can move. The load is considered further on as a material point. The load can move in the channel according to a predetermined motion law. The channel axis is located in a vertical plane passing through the trajectory of the carrier. The Coulomb dry friction model is applied for simulation the forces of resistance to the motion of the carrier from the side of the underlying plane. In the conditions of the carrier motion along horizontal plane without detachment, the carrier motion differential equations are a system of three linear second-order differential equations. The influence of the system parameters on the motion of the carrier from the rest state is studied in the point of view of determining functions. An important property of the determining expressions is proved: the existence of a single intersection point of the curves given by the determining equations, which corresponds to the zero angle of setting of the channel along which the load moves.

1. Introduction

The vibration-driven systems whose motion is carried out by periodic motion of internal masses are a new type of movable dynamical systems. These systems application in disaster rescue, pipeline inspection and cardiovascular surgery is preferable in comparison with the usual robots (with legs or with wheels) because of their simplicity, controllability and miniaturization potential.

Such systems have been studied from many points of view. The load motion laws, providing the required carrier motion, have been found in [1–5]. The inverse problem of finding the carrier motion for the *initial conditions* $x(t_0) = 0$ and $\dot{x}(t_0) = 0$ at $t_0 = 0$ for a preassigned law of load motion $\ell \cdot \sin(\omega t)$, where $\ell = Const$, $\omega = Const$, when the carrier moves along the *horizontal* plane, has been posed and solved in [6–8]. The problem of definition of load motion influence on carrier dynamics has been considered in [9] for the other load motion law.

An important property of the determining equations $I_1(\varphi, \beta) = 0$, $I_2(\varphi, \beta) = 0$ and $I_3(\varphi, \beta) = 0$ is proved in this article: the existence of a single intersection point corresponding to the setting angle equal to zero.

2. Carrier motion differential equations

The motion of a mechanical system (figure 1) consisting of a carrier and a load is considered [6–8, 10–14]. The carrier, located all the time on a horizontal plane, moves translationally along a rectilinear trajectory. The carrier has a rectilinear channel through which the load can move. The channel axis is located in a vertical plane passing through the trajectory of the carrier.

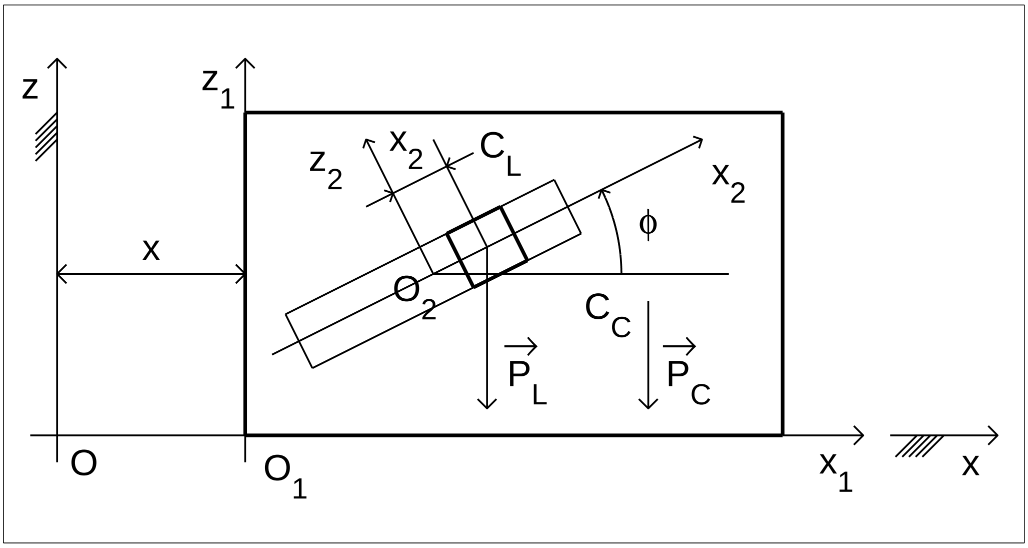

Figure 1. The carrier with the mobile load

Here (in figure 1)

$Oxyz$ is the fixed coordinates;

$O_1x_1y_1z_1$ is the moving carried-fixed coordinates;

$O_2x_2y_2z_2$ is the moving channel-fixed coordinates;

C_{C} is the carrier center of mass;

C_{L} is the load center of mass (further on the load is considered as material point);

$\vec{P}_{\mathrm{C}}$ is the carrier weight;

$\vec{P}_{\mathrm{L}}$ is the load weight.

Let the law of motion of the load in the channel be given in the form $x_2(t) = \ell \cdot \sin(\omega t)$, where $\ell = const$, $\omega = const$, and the forces of resistance of the medium to the motion of the carrier are modeled by forces of the Coulomb friction type, then the carrier motion differential equations (CMDE), according to [6, 7], are

$$\ddot{x} = \beta \cdot (\cos\varphi + f \cdot \sin\varphi) \cdot \sin(\omega t) - \gamma \qquad \text{for} \qquad \dot{x} > 0\,; \tag{1}$$

$$\ddot{x} = \beta \cdot (\cos\varphi - f \cdot \sin\varphi) \cdot \sin(\omega t) + \gamma \qquad \text{for} \qquad \dot{x} < 0\,; \tag{2}$$

$$\ddot{x} = 0 \qquad \text{for} \qquad \dot{x} = 0\,, \tag{3}$$

where

x is the carrier coordinate; φ is channel setting angle;

$\beta = \ell \cdot \omega^2 \cdot \dfrac{m}{m+M}$; $\gamma = g \cdot f$;

M is mass of the carrier; m is mass of load;

g is the acceleration of gravity; f is coefficient of sliding friction in motion (equal to the coefficient of sliding friction at rest) for a pair of materials “carrier–underlying horizontal plane”.

Let the channel setting angle be

$$-\frac{\pi}{2} < \varphi < \frac{\pi}{2}\,. \tag{4}$$

It is assumed that the inequality [10]

$$\beta \cdot |\sin\varphi| < g \tag{5}$$

is satisfied. Inequality (5) guarantees the motion of the carrier without detachment from the horizontal plane. If $\beta > g$, then the restriction on the channel setting angle

$$-\arcsin\left(\frac{g}{\beta}\right) < \varphi < \arcsin\left(\frac{g}{\beta}\right) \tag{6}$$

follows from (5) instead of (4).

3. Conditions for bilateral motions of a carrier from a state of rest

A necessary and sufficient condition for CMDE (1) to take place in the real dynamics of the carrier is the inequality

$$\beta \cdot (\cos\varphi + f \cdot \sin\varphi) > \gamma$$

or

$$\beta > \gamma_+(\varphi)\ , \tag{7}$$

where

$$\gamma_+(\varphi) = \frac{\gamma}{\cos\varphi + f \cdot \sin\varphi}\,,$$

and for the realization of CMDE (2) in real dynamics there is an inequality

$$\beta \cdot (\cos\varphi - f \cdot \sin\varphi) > \gamma$$

or

$$\beta > \gamma_-(\varphi)\ , \tag{8}$$

where

$$\gamma_-(\varphi) = \frac{\gamma}{\cos\varphi - f \cdot \sin\varphi}\,.$$

The function $\gamma_+(\varphi)$ has a minimum equal to $\dfrac{\gamma}{\sqrt{1+f^2}}$ at $\varphi = \operatorname{arctg}\dfrac{1}{f}$ and the graph of the function $\gamma_+(\varphi)$ has a vertical asymptote $\varphi = -\operatorname{arctg}\dfrac{1}{f}$, i.e., function $\gamma_+(\varphi)$ is defined for

$$-\operatorname{arctg}\frac{1}{f} < \varphi < \frac{\pi}{2}\,. \tag{9}$$

The function $\gamma_-(\varphi)$ has the same minimum $\dfrac{\gamma}{\sqrt{1+f^2}}$ at $\varphi = -\operatorname{arctg}\dfrac{1}{f}$ and the graph of the function $\gamma_-(\varphi)$ has a vertical asymptote $\varphi = \operatorname{arctg}\dfrac{1}{f}$, i.e., function $\gamma_-(\varphi)$ is defined for

$$-\frac{\pi}{2} < \varphi < \operatorname{arctg}\frac{1}{f}\,. \tag{10}$$

It was found in [10] that if

$$\beta > \frac{\gamma}{\sqrt{1+f^2}} \tag{11}$$

and $\varphi \in \overrightarrow{\Phi}(\beta) \equiv (\overrightarrow{\varphi}_1; \overrightarrow{\varphi}_2)$, where

$$\overrightarrow{\varphi}_1(\beta) = 2\,\mathrm{arctg}\,\frac{\beta \cdot f - \sqrt{\beta^2(1+f^2) - \gamma^2}}{\beta + \gamma}\,,$$

$$\overrightarrow{\varphi}_2(\beta) = 2\,\mathrm{arctg}\,\frac{\beta \cdot f + \sqrt{\beta^2(1+f^2) - \gamma^2}}{\beta + \gamma}\,,$$

then the carrier can move from a state of rest in the positive direction of the axis Ox, i.e., CMDE (1) takes place in the real dynamics of the carrier.

If (11) is satisfied, and $\varphi \in \overleftarrow{\Phi}(\beta) \equiv (\overleftarrow{\varphi}_1; \overleftarrow{\varphi}_2)$, where $\overleftarrow{\varphi}_1 = -\overrightarrow{\varphi}_2$, $\overleftarrow{\varphi}_2 = -\overrightarrow{\varphi}_1$, then the carrier can move from the state of rest in the negative direction of the axis Ox, i.e., CMDE (2) takes place in the real dynamics of the carrier.

According [10], if

$$\beta > \gamma \tag{12}$$

takes place, and the channel setting angle is such that $\varphi \in \overleftrightarrow{\Phi}(\beta) \equiv (\overrightarrow{\varphi}_1; \overleftarrow{\varphi}_2)$, then the carrier can move from the state of rest both in the positive and negative directions of the axis Ox, i.e., CMDE (1) and (2) can be realized.

Applying conditions (7) and (8) and taking into account (6) some subdomains can be selected in the plane of parameters $(\varphi; \beta)$. In figures 2 and 3 the following subdomains are presented: the carrier state of rest domain S_0; the carrier one-way motion domains S_1^+ and S_1^- (in positive and negative directions of the axis Ox, respectively); the carrier bilateral motions from the state of rest domain S_2.

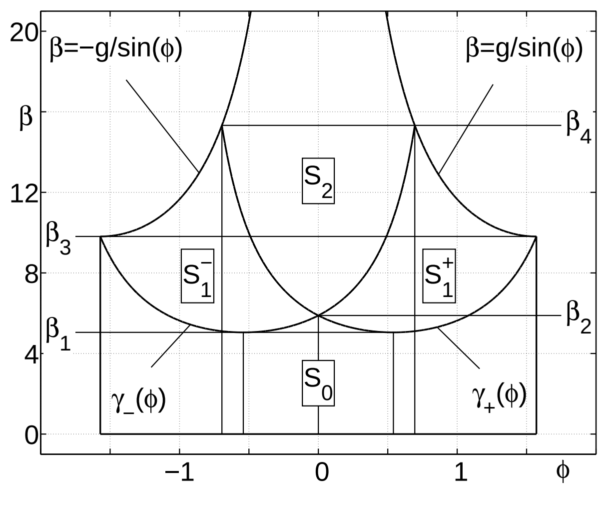

Figure 2. The domains of possible carrier motions (case $f = 0.6$)

In figure 2 for $f = 0.6$ and $\beta_3 = g \approx 9.8100$ the values of the others β are

$$\beta_1 = \frac{g \cdot f}{\sqrt{1+f^2}} \approx 5.0472\,; \quad \beta_2 = g \cdot f \approx 5.8860\,; \quad \beta_4 = g \cdot \sqrt{1 + 4 \cdot f^2} \approx 15.3237\,.$$

Here β_4 is the root of equation

$$2\,\mathrm{arctg}\,\frac{-\beta_4 \cdot f + \sqrt{\beta_4^2\,(1+f^2) - \gamma^2}}{\beta_4 + \gamma} = \arcsin\left(\frac{g}{\beta_4}\right).$$

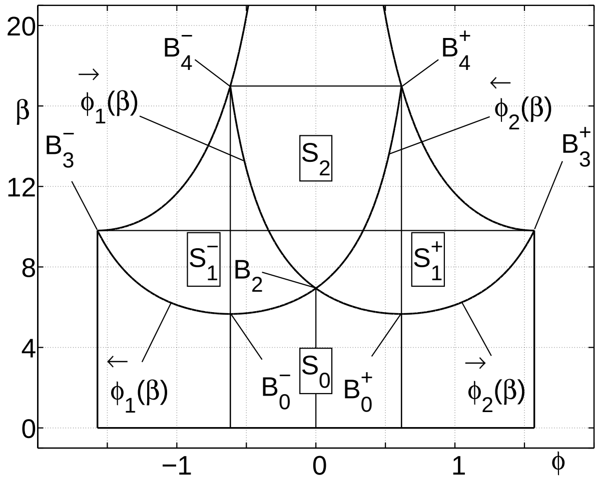

Figure 3. The domains of possible carrier motions (case $f = \sqrt{2}/2$)

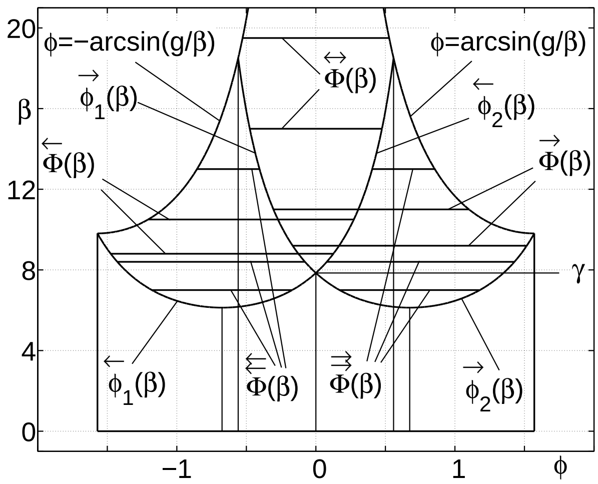

Figure 4. The diapasons of angle of channel setting (case $f = 0.8$)

The specific points

$$B_0^+\left(\operatorname{arctg} f; g\cdot\frac{f}{\sqrt{1+f^2}}\right);\ B_0^-\left(-\operatorname{arctg} f; g\cdot\frac{f}{\sqrt{1+f^2}}\right);\quad B_3^+\left(\frac{\pi}{2}; g\right);\ B_3^-\left(-\frac{\pi}{2}; g\right);$$

$$B_2\,(0; g\cdot f);\quad B_4^+\left(\operatorname{arctg}\frac{1}{2\cdot f}; g\cdot\sqrt{1+4\cdot f^2}\right);\ B_4^-\left(-\operatorname{arctg}\frac{1}{2\cdot f}; g\cdot\sqrt{1+4\cdot f^2}\right)$$

of boundary of these domains are presented in figure 3.

In figure 3 for $f = \sqrt{2}/2$ and $g \approx 9.8100$ the specific points are

$$B_0^+\,(0.6155; 5.6638);\ B_0^-\,(-0.6155; 5.6638);\quad B_3^+\,(1.5708; 9.8100);\ B_3^-\,(-1.5708; 9.8100);$$

$$B_2\,(0; 6.9367);\quad B_4^+\,(0.6155; 16.9914);\ B_4^-\,(-0.6155; 16.9914).$$

The diapasons of angle of channel setting $\overrightarrow{\Phi}$, $\overleftarrow{\Phi}$, $\overset{\Longrightarrow}{\Phi}$, $\overset{\Longleftarrow}{\Phi}$ and $\overleftrightarrow{\Phi}$ corresponding to subdomains S_1^+, S_1^- and S_2 are presented in figure 4 taking into account restrictions (6), where (according [8, 10, 14]) $\overset{\Longrightarrow}{\Phi}(\beta) \equiv (\max\{\overleftarrow{\varphi}_2; \overrightarrow{\varphi}_1\}; \overrightarrow{\varphi}_2)$ and $\overset{\Longleftarrow}{\Phi}(\beta) \equiv (\overleftarrow{\varphi}_1; \min\{\overleftarrow{\varphi}_2; \overrightarrow{\varphi}_1\})$.

So, bilateral motions of a carrier from the state of rest are possible for the systems "carrier–load" with the parameters $(\varphi;\beta) \in S_2$, i.e., when

$$\varphi \in (\overrightarrow{\varphi}_1(\beta); \overleftarrow{\varphi}_2(\beta)) \qquad \text{at} \qquad \beta_2 < \beta < \beta_4; \tag{13.1}$$

$$\varphi \in \left(-\arcsin\left(\frac{g}{\beta}\right); \arcsin\left(\frac{g}{\beta}\right)\right) \qquad \text{at} \qquad \beta_4 \le \beta, \tag{13.2}$$

where $\beta_2 = g \cdot f$; $\beta_4 = g \cdot \sqrt{1 + 4 \cdot f^2}$.

Let's consider the carrier bilateral motions for $\varphi \ge 0$ only. Then the bilateral motions of a carrier from the state of rest are possible if

$$\varphi \in [0; \overleftarrow{\varphi}_2(\beta)) \qquad \text{at} \qquad \beta_2 < \beta < \beta_4; \tag{14.1}$$

$$\varphi \in \left[0; \arcsin\left(\frac{g}{\beta}\right)\right) \qquad \text{at} \qquad \beta_4 \le \beta. \tag{14.2}$$

Conditions (14) select the part S_2^+ from the domain S_2.

4. Proof of the property of the determining equations

Let the channel setting angle be such that the conditions (14) are satisfied. According [13] the determining expressions are introduced

$$I_1(\varphi;\beta) = \sqrt{\beta^2 - \gamma_+^2} + \sqrt{\beta^2 - \gamma_-^2} - \gamma_+ \cdot \left[\pi + \arcsin\left(\frac{\gamma_-}{\beta}\right) - \arcsin\left(\frac{\gamma_+}{\beta}\right)\right]; \tag{15}$$

$$I_2(\varphi;\beta) = \sqrt{\beta^2 - \gamma_+^2} + \sqrt{\beta^2 - \gamma_-^2} - \gamma_- \cdot \left[\pi + \arcsin\left(\frac{\gamma_+}{\beta}\right) - \arcsin\left(\frac{\gamma_-}{\beta}\right)\right]; \tag{16}$$

$$I_3(\varphi;\beta) = \sqrt{\beta^2 - \gamma_+^2} - \frac{\gamma\pi}{\cos\varphi} + \beta \cdot \cos\left[\arcsin\left(\frac{\gamma_+}{\beta}\right) + \pi \cdot f \cdot \operatorname{tg}\varphi\right]. \tag{17}$$

Using (15), (16) and (17) the determining equations can be written

$$I_1(\varphi;\beta) = 0; \tag{18}$$

$$I_2(\varphi;\beta) = 0; \tag{19}$$

$$I_3(\varphi;\beta) = 0. \tag{20}$$

4.1. Case $(I_1(\varphi;\beta) = 0, I_2(\varphi;\beta) = 0)$

Let's consider determining equations (18) and (19).

If $\varphi = 0$, then

$$\gamma_+ = \gamma_- = \gamma = g \cdot f,$$

$$I_1 = I_2 = I = 2\sqrt{\beta^2 - \gamma^2} - \gamma \cdot \pi,$$

equations (18) and (19) turn into

$$I = 0,$$

which can be satisfied at $\beta = \beta_0$, where

$$\beta_0 = \gamma \cdot \frac{\sqrt{\pi^2 + 4}}{2}. \tag{21}$$

That is, in the domain S_2^+ the curves given by equations (18) and (19) have a common point $M_0\,(0;\beta_0)$.

Now let $\varphi \neq 0$. Let's show, that there is no point

$$M_*\,(\varphi_*;\beta_*) \neq M_0\,(0;\beta_0)$$

in the domain S_2^+, where $I_1\,(\varphi_*;\beta_*) = 0$ and $I_2\,(\varphi_*;\beta_*) = 0$.

Suppose that such point exists, i.e., the system of determining equations (18) and (19) is compatible for $\varphi \neq 0$.

Let's replace the system of equations (18) and (19) by system

$$\arcsin\left(\frac{\gamma_-}{\beta}\right) - \arcsin\left(\frac{\gamma_+}{\beta}\right) = \pi\cdot f\cdot \operatorname{tg}\varphi\,; \tag{22}$$

$$\sqrt{\beta^2-\gamma_+^2} + \sqrt{\beta^2-\gamma_-^2} = \frac{\pi\cdot\gamma}{\cos\varphi}\,. \tag{23}$$

Equation (22) is obtained by subtracting (19) from (18). Equation (23) is obtained by substituting (22) in (18) or in (19).

Let's extend the domain S_2^+ by adding its lower boundary to it, i.e., we will assume that

$$\beta \geq \gamma_-\,.$$

Let $\beta = \gamma_-$, then equation (22) takes the form

$$\frac{\pi}{2} - \arcsin\left(\frac{1-f\cdot\operatorname{tg}\varphi}{1+f\cdot\operatorname{tg}\varphi}\right) = \pi\cdot f\cdot\operatorname{tg}\varphi\,.$$

Note that if the considered mechanical system is characterized by the parameter $f\cdot\operatorname{tg}\varphi = \dfrac{1}{3}$, then equation (22) is satisfied (this is verified by substitution).

Let's consider the function

$$W\,(\beta) = \arcsin\left(\frac{\gamma_-}{\beta}\right) - \arcsin\left(\frac{\gamma_+}{\beta}\right)$$

and obtain

$$\frac{dW}{d\beta} = \frac{1}{\beta}\cdot\frac{\gamma_+\cdot\sqrt{\beta^2-\gamma_-^2} - \gamma_-\cdot\sqrt{\beta^2-\gamma_+^2}}{\sqrt{\beta^2-\gamma_+^2}\cdot\sqrt{\beta^2-\gamma_-^2}}\,.$$

As $\gamma_+ < \gamma_-$ and $\sqrt{\beta^2-\gamma_-^2} < \sqrt{\beta^2-\gamma_+^2}$, then $\dfrac{dW}{d\beta} < 0$ is satisfied for $\beta > \gamma_-$. So, $W\,(\beta)$ f is a decreasing function. Hence, if $f\cdot\operatorname{tg}\varphi > \dfrac{1}{3}$, then equation (22) has no solutions. Therefore, the curves given by determining equations (18) and (19) have only one intersection point M_0.

Thus, it is shown that equation (22) can have a solution only for

$$0 < f\cdot\operatorname{tg}\varphi \leq \frac{1}{3}\,. \tag{24}$$

Assuming that (24) is satisfied, we transform (22). Let's denote

$$p = \arcsin\left(\frac{\gamma_-}{\beta}\right);\quad q = \arcsin\left(\frac{\gamma_+}{\beta}\right);\quad r = \pi\cdot f\cdot\operatorname{tg}\varphi\,.$$

Then the expression $\sin(p-q) = \sin r$ takes the form

$$\gamma_- \cdot \sqrt{\beta^2 - \gamma_+^2} - \gamma_+ \cdot \sqrt{\beta^2 - \gamma_-^2} = \beta^2 \cdot \sin r \tag{25}$$

and $\cos(p-q) = \cos r$ takes the form

$$\sqrt{\beta^2 - \gamma_+^2} \cdot \sqrt{\beta^2 - \gamma_-^2} + \gamma_+ \cdot \gamma_- = \beta^2 \cdot \cos r\,. \tag{26}$$

Using (25) and (26), we can show that

$$\beta^2 = \frac{\gamma_-^2 + \gamma_+^2 - 2\gamma_- \cdot \gamma_+ \cdot \cos\left(\pi \cdot f \cdot \mathrm{tg}\,\varphi\right)}{\sin^2\left(\pi \cdot f \cdot \mathrm{tg}\,\varphi\right)}\,. \tag{27}$$

Thus, being a solution to equation (22), relation (27) determines the explicit expression of the parameter β in terms of the parameters φ and f. For $\varphi \neq 0$, expression (27) gives

$$\gamma_+ \cdot \left[\pi + \arcsin\left(\frac{\gamma_-}{\beta}\right) - \arcsin\left(\frac{\gamma_+}{\beta}\right)\right] = \gamma_- \cdot \left[\pi + \arcsin\left(\frac{\gamma_+}{\beta}\right) - \arcsin\left(\frac{\gamma_-}{\beta}\right)\right].$$

That is, the values of the determining expressions are equal, in other words, $I_1 = I_2$.

Substituting (27) in equation (23) and taking into account that $\cos\varphi \neq 0$, we obtain

$$\frac{\gamma}{\cos\varphi} \cdot \left[\mathrm{ctg}\left(\frac{\pi z}{2}\right) \cdot \frac{2z}{1-z^2} - \pi\right] = 0\,, \quad \text{where} \quad z = f \cdot \mathrm{tg}\,\varphi \quad \text{for} \quad z \in (0; 1/3]\,.$$

If the curves given by equations (22) and (23) have an intersection for $\varphi \neq 0$, then

$$\mathrm{ctg}\left(\frac{\pi z}{2}\right) \cdot \frac{2z}{1-z^2} - \pi = 0\,,$$

since $\gamma = g \cdot f \neq 0$. Let's consider the function

$$F\left(z\right) = \mathrm{ctg}\left(\frac{\pi z}{2}\right) \cdot \frac{2z}{1-z^2} \quad \text{for} \quad z \in (0; 1/3]$$

and obtain

$$\lim_{z\to 0+} F\left(z\right) = \lim_{z\to 0+} \frac{\cos\left(\frac{\pi z}{2}\right)}{\sin\left(\frac{\pi z}{2}\right)} \cdot \frac{2z}{1-z^2} = \lim_{z\to 0} \frac{2z}{\sin\left(\frac{\pi z}{2}\right)} = \frac{4}{\pi} \approx 1.2732\,.$$

It can be shown that $\dfrac{dF}{dz} > 0$ for $z \in (0; 1/3]$. Hence, the increasing function $F\left(z\right)$ reaches its maximum value

$$F\left(z\right)\big|_{z=\frac{1}{3}} \approx 1.2990 \neq \pi$$

on the right boundary of the set $(0; 1/3]$. That is, equation (23) is not satisfied when substituting expression (27) into it. This proves the fact that the intersection points of the curves defined by determining equations (18) and (19) do not exist for $\varphi \neq 0$.

Conclusion

The proved property of the curves defined by the determining equations $I_1 = 0$ and $I_2 = 0$ guarantees the partition of S_2^+ into five subdomains, each of which will have its own combination of signatures of the determining expressions:

either $S_{2,1}^+$ $(I_1 < 0,\ I_2 < 0)$,
or $S_{2,2}^+$ $(I_1 = 0,\ I_2 < 0)$,
or $S_{2,3}^+$ $(I_1 > 0,\ I_2 < 0)$,
or $S_{2,4}^+$ $(I_1 > 0,\ I_2 = 0)$,
or $S_{2,5}^+$ $(I_1 > 0,\ I_2 > 0)$.

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4.2. Case $(I_2(\varphi;\beta)=0, I_3(\varphi;\beta)=0)$

The determining expression $I_3(\varphi;\beta)$ is used to determine the type of carrier motion when $I_2(\varphi;\beta)>0$.

Let's consider determining equations (19) and (20).

If $\varphi = 0$, then

$$\gamma_+ = \gamma_- = \gamma = g \cdot f\,,$$

$$I_2 = I_3 = I = 2\sqrt{\beta^2 - \gamma^2} - \gamma \cdot \pi\,,$$

equations (19) and (20) turn into

$$I = 0\,,$$

which can be satisfied at $\beta = \beta_0$, where β_0 is determined by (21). That is, in the domain S_2^+ the curves given by equations (19) and (20) have a common point $M_0(0;\beta_0)$.

Now let $\varphi \neq 0$. Let's show, that there is no point

$$M_*(\varphi_*;\beta_*) \neq M_0(0;\beta_0)$$

in the domain S_2^+, where $I_2(\varphi_*;\beta_*)=0$ and $I_3(\varphi_*;\beta_*)=0$.

Suppose that such point exists, i.e., the system of determining equations (19) and (20) is compatible for $\varphi \neq 0$.

The values of the mismatching terms of equations (19) and (20) at the intersection points of the curves $I_2(\varphi;\beta)=0$ and $I_3(\varphi;\beta)=0$ should be equal, i.e.,

$$\sqrt{\beta^2-\gamma_-^2} - \gamma_- \cdot \left[\pi + \arcsin\left(\frac{\gamma_+}{\beta}\right) - \arcsin\left(\frac{\gamma_-}{\beta}\right)\right] =$$
$$= \beta \cdot \cos\left[\arcsin\left(\frac{\gamma_+}{\beta}\right) + \pi \cdot f \cdot \operatorname{tg}\varphi\right] - \frac{\gamma\pi}{\cos\varphi}\,. \tag{28}$$

Equation (28) is satisfied for

$$\arcsin\left(\frac{\gamma_+}{\beta}\right) + \pi \cdot f \cdot \operatorname{tg}\varphi = \arcsin\left(\frac{\gamma_-}{\beta}\right),$$

which is verified by substitution, i.e., equation (28) is replaced by (22). Substituting (22) in (19) or in (20), we arrive to equation (23). In paragraph 4.1, it was shown that (22) and (23) are incompatible. This proves the fact that the intersection points of the curves defined by determining equations (19) and (20) do not exist for $\varphi \neq 0$.

Conclusion

The proved property of the curves defined by the determining equations $I_2 = 0$ and $I_3 = 0$ guarantees the partition of $S_{2,5}^+$ $(I_1 > 0,\, I_2 > 0)$ into three subdomains, each of which will have its own combination of signatures of the determining expressions:

either $S_{2,5,1}^+$ $(I_1 > 0,\, I_2 > 0,\, I_3 < 0)$,
or $S_{2,5,2}^+$ $(I_1 > 0,\, I_2 > 0,\, I_3 = 0)$,
or $S_{2,5,3}^+$ $(I_1 > 0,\, I_2 > 0,\, I_3 > 0)$.

4.3. Conclusion

The proved property of the curves defined by the determining equations $I_1 = 0$, $I_2 = 0$ and $I_3 = 0$ guarantees the partition of S_2^+ into seven subdomains, each of which will have its own combination of signatures of the determining expressions:

either $S_{2,1}^+$ $(I_1 < 0,\, I_2 < 0,\, I_3 < 0)$,
or $S_{2,2}^+$ $(I_1 = 0,\, I_2 < 0,\, I_3 < 0)$,

$$
\begin{array}{lll}
\text{or} & S^{+}_{2,3} & (I_1 > 0,\ I_2 < 0,\ I_3 < 0), \\
\text{or} & S^{+}_{2,4} & (I_1 > 0,\ I_2 = 0,\ I_3 < 0), \\
\text{or} & S^{+}_{2,5,1} & (I_1 > 0,\ I_2 > 0,\ I_3 < 0), \\
\text{or} & S^{+}_{2,5,2} & (I_1 > 0,\ I_2 > 0,\ I_3 = 0), \\
\text{or} & S^{+}_{2,5,3} & (I_1 > 0,\ I_2 > 0,\ I_3 > 0).
\end{array}
$$

The found partition of domain S_2^+ is used in an algorithm that determines the type of carrier motion [13].

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