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Assessment of the behavior uncertainty level of the short time series

L A Demidova[1] and M A Stepanov[2]

[1]MIREA Russian Technological University, Moscow, Russia
[2]Ryazan State Radio Engineering University, Ryazan, Russia

E-mail: smastefan@gmail.com

Abstract. The article considers the problem of assessing the level of uncertainty in the behavior of the short time series. The approach to solving this problem, based on the use of the fuzzy linear regression equation with the asymmetric parameters has been proposed. The methods for assessing the level of uncertainty in the behavior of the short time series using the fuzzy similarity measures have been considered. A herewith, the time series which describes the equation of the classical linear regression is used as the standard time series. The presented results of experimental studies confirm the effectiveness of using the proposed approach to assess the level of uncertainty in the behavior of the short time series.

1. Introduction

The problem of the time series (TS) forecasting demands a performance of the preliminary evaluation of the possibility to apply certain technologies of forecasting to the specific time series [1, 2]. In addition to the standard (classical) toolkit of such evaluation it is offered to apply indicators that allow us to characterize the uncertainty level of the time series behavior. At the same time it is especially vital to apply such indicators, when working with short time series, for which appropriate calculation of the classical statistics is fundamentally impossible, due to the limited amount of the available information.

To calculate the indicators of the uncertainty level of the short TS behavior, it is proposed to ap-ply toolkit of the fuzzy set theory [3, 4] that is generally applied in the sphere of the TS analysis in terms of the development of the fuzzy forecasting models [5 – 14] both for the value fuzzyfication of the TS elements [5 – 7], and for calculation of the degree of membership of TS elements to certain subintervals, obtained by partition of the sphere of determination of its elements value [7 – 14]. Also this toolkit is quite successfully applied [15] to form fuzzy sets of TS memory depth that is further applied to evaluate the order of the forecasting model. A number of authors study the aspects of developing of fuzzy regression models, including linear, and choice of their parameters value [16 – 18]. In [19] it is analyzed a problem of the application of symmetric and asymmetric triangular fuzzy numbers as a parameter of the fuzzy regression equation, and it is concluded, that asymmetric triangular fuzzy numbers have a clear advantage. At the same time the solution of the task of evaluation of correspondence of the analyzed TS to the chosen regression equation is particularly interesting in terms of the fuzzy sets theory.

The approach to the solution of the task of evaluation of the correspondence of the analyzed TS to the chosen regression equation, studied in the article, is author's. It is based on the application of the term "fuzzy linear regression" [19], that was not earlier used in such a context. The evaluation of the

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uncertainty level of the TS behavior is carried out in comparison to some standard TS, characterizing the linear trend line.

2. Theoretical part

Describing the short TS behavior can be implemented on the basis of the linear regression model with application of the classical linear regression equation (CLR) by the solution of the task of parameters selection k and b of the equation:

$$y = k \cdot x + b \tag{1}$$

with following calculation of the standard deviation (SD) for residual, that can be applied to evaluate the uncertainty level of the TS and the coefficient of determination, that allows us to estimate the relevance of the applied linear regression model.

If we use only one indicator to evaluate the uncertainty level of the TS behavior, for example, such as SD, than in a number of cases some TS, having approximately equal value of SD, can be referred to the same class (group), though the structure of these TS is different.

As a result it is relevant to apply additional indicators for evaluation of the uncertainty level of the TS behavior. These indicators can be obtained on the basis of the fuzzy linear regression model with application of the fuzzy linear regression equation.

In [19] it's shown, that application of the fuzzy linear regression equation (FLR) with asymmetric fuzzy parameters is the most relevant, as such an equation provides us with calculation of SD value, that is equal or close to the value, obtained by the means of the classical linear regression equation (unlike the fuzzy linear regression equation with symmetric fuzzy parameters):

$$Y(x) = A_1 \cdot x + A_0 , \tag{2}$$

where $A_1 = (a_1, c_1, d_1)$ and $A_0 = (a_0, c_0, d_0)$ are the triangular fuzzy numbers (TFN) corresponding to the parameters k and b of the classical linear regression equation (1), represented with the help of triangular membership functions, and that are asymmetric fuzzy parameters of the fuzzy linear regression equation (2).

To define the parameters value of the fuzzy linear regression equation, it is necessary to solve the task of the quadratic programming, formulated as [19, 20]:

$$\begin{aligned} F_{QPP} &= k_1 \cdot \sum_{j=1}^{n} (y_j - \sum_{i=0}^{m} a_i \cdot x_{ji})^2 + \\ &+ k_2 \cdot (1-\alpha) \cdot \sum_{j=1}^{n} \sum_{i=0}^{m} (c_i + d_i) \cdot x_{ji} + \\ &+ \xi \cdot \sum_{i=0}^{m} (c_i^2 + d_i^2) \to \min_{a,c,d} \end{aligned} \tag{3}$$

under restrictions:

$$\sum_{i=0}^{m} a_i \cdot x_{ji} + (1-\alpha) \cdot \sum_{i=0}^{m} d_i \cdot x_{ji} \geq y_j ; \tag{4}$$

$$\sum_{i=0}^{m} a_i \cdot x_{ji} - (1-\alpha) \cdot \sum_{i=0}^{m} c_i \cdot x_{ji} \leq y_j ; \tag{5}$$

$$c_i \geq 0; d_i \geq 0 \ \ (j = \overline{1,n};\ i = \overline{0,m};\ m = 1) , \tag{6}$$

where j is the number of the TS element ($j=\overline{1,n}$); n is the number of time moments (number of TS elements); i is the number of the TFN ($i=\overline{0,1}$ for $m=1$); all $x_{j0}=1$ ($j=\overline{1,n}$), as they correspond to the b (i.e. free term) of the equation of the classical linear regression (1); x_{j1} is the value of the j-th time moment; y_j is the value of TS element for the j-th time moment; α is the value of the TFN level, characterizing the width of the corridor of the fuzzy linear regression equation ($\alpha\in[0,1]$); k_1,k_2 are the weighting factors, characterizing contribution of the first and second summand to the efficiency function (3); ξ is the small positive number, where $k_1,k_2>>\xi$ (the third summand is inserted into the efficiency function, in order it to become quadratic, and to allow us to form the task of the quadratic programming, when searching the TFN values [19]).

When solving the task of the quadratic programming (3) – (6) it is assumed that $k_1=k_2=1$ (in [19] it is shown, that the choice of the parameters values k_1,k_2 doesn't influence the solution of the task a lot); $\xi=0{,}001$.

If $m=1$ and assuming that $x_j=x_{j1}$, system (3) – (6) can be written down as:

$$\begin{aligned} F_{QPP} &= k_1\cdot\sum_{j=1}^{n}(y_j-(a_0+a_1\cdot x_j))^2+ \\ &+k_2\cdot(1-\alpha)\cdot(n\cdot(c_0+d_0)+\sum_{j=1}^{n}(c_1+d_1)\cdot x_j)+ \\ &+\xi\cdot(c_0^2+d_0^2+c_1^2+d_1^2)\to\min_{a,c,d} \end{aligned} \tag{7}$$

under restrictions:

$$a_0+a_1\cdot x_j+(1-\alpha)\cdot(d_0+d_1\cdot x_j)\geq y_j; \tag{8}$$

$$a_0+a_1\cdot x_j-(1-\alpha)\cdot(c_0+c_1\cdot x_j)\leq y_j; \tag{9}$$

$$c_0\geq 0; d_0\geq 0; c_1\geq 0; d_1\geq 0\ \ (j=\overline{1,n}). \tag{10}$$

Membership function of the data point (x',y') ($x'=(x_0',x_1',\ldots,x_m')$; $x_0'=1$) to the fuzzy linear regression equation can be determined as:

$$u(x',y')=\begin{cases} 1-\dfrac{\sum_{i=0}^{m}a_ix_i'-y'}{\sum_{i=0}^{m}c_ix_i'}, & \sum_{i=0}^{m}(a_i-c_i)x_i'\leq y'\leq\sum_{i=0}^{m}a_ix_i'; \\ 1-\dfrac{y-\sum_{i=0}^{m}a_ix_i'}{\sum_{i=0}^{m}d_ix_i'}, & \sum_{i=0}^{m}a_ix_i'\leq y'\leq\sum_{i=0}^{m}(a_i+d_i)x_i'; \\ 0, & \text{otherwise.} \end{cases} \tag{11}$$

If $m=1$, membership function of the data point (x',y') ($x'=(x_0',x_1')$; $x_0'=1$) to the fuzzy linear regression equation is determined as:

$$u(x',y')=\begin{cases}1-\dfrac{a_0+a_1\cdot x_1'-y'}{c_0+c_1\cdot x_1'}, \text{if} \\ a_0+a_1\cdot x_1'-c_0-c_1\cdot x_1'\le y'\le a_0+a_1\cdot x_1'; \\ 1-\dfrac{y'-a_0-a_1\cdot x_1'}{d_0+d_1\cdot x_1'}, \text{if} \\ a_0+a_1\cdot x_1'\le y'\le a_0+a_1\cdot x_1'+d_0+d_1\cdot x_1'; \\ 0, \text{otherwise};\end{cases} \quad (12)$$

where $a_0, c_0, d_0, a_1, c_1, d_1$ are the parameters of the TFN, calculated when solving the problem of the quadratic programming.

Fuzzy linear regression equation (2) for the analyzed TS is based on the task solving of the quadratic programming (7) – (10). At the same time on the basis of the calculated TFN $A_1=(a_1,c_1,d_1)$ and $A_0=(a_0,c_0,d_0)$ of the fuzzy linear regression equation (2) for the values of TS elements x_j ($j=\overline{1,n}$) the characteristic points of the classical linear regression equation $Y_{FLR}^{CLR}(x_j)$ are determined:

$$Y_{FLR}^{CLR}(x_j)=a_0+a_1\cdot x_j\ , \quad (13)$$

and also equations of the upper $Y_{FLR}^{UP}(x_j)$ and lower $Y_{FLR}^{LOW}(x_j)$ boundaries of the corridor of the fuzzy linear regression equation:

$$Y_{FLR}^{UP}(x_j)=a_0+d_0+(a_1+d_1)\cdot x_j\ , \quad (14)$$

$$Y_{FLR}^{LOW}(x_j)=a_0-c_0+(a_1-c_1)\cdot x_j\ . \quad (15)$$

Due to the asymmetry of the TFN $A_1=(a_1,c_1,d_1)$ and $A_0=(a_0,c_0,d_0)$, the corridor of the fuzzy linear regression equation of the analyzed TS will be also asymmetric. As a result the elements of TS can be divided into 2 subsets; subset of points, situated in the upper part *UP* of the corridor of the fuzzy linear regression equation (between the line of the upper boundary of the corridor of the fuzzy linear regression equation and the line of the classical linear regression equation, determined by equations (14) and (13) correspondingly); and subset of points, situated in the lower part *LOW* of the corridor of the fuzzy linear regression equation (between the line of the lower boundary of the fuzzy linear regression equation and the line of the classical linear regression equation, determined by the equations (15) and (13) correspondingly).

For the points of the analyzed TS, situated in the upper UP and the lower LOW parts of the corridors of the fuzzy linear regression equation, and TS, corresponding to the classical linear regression, it is possible to calculate the value of the fuzzy similarity measures F^{UP} and F^{LOW}, applying, for example, following measures of the similarity [21]:

$$f_1=1-\frac{\sum_{j=1}^{n}|u_S(x_j,y_j')-u_{TS}(x_j,y_j'')|}{\sum_{j=1}^{n}(u_S(x_j,y_j')+u_{TS}(x_j,y_j''))}, \quad (16)$$

$$f_2 = \frac{1}{n} \cdot \sum_{j=1}^{n} \frac{min(u_S(x_j, y'_j), u_{TS}(x_j, y''_j))}{max(u_S(x_j, y'_j), u_{TS}(x_j, y''_j))}, \tag{17}$$

where $u_S(x_j, y''_j)$ is the value of the membership function of the fuzzy set S (Standard) of the TS corresponding to the classical linear regression equation for the value of TS element y''_j, corresponding to the time moment x_j ($j = \overline{1,n}$); $u_{TS}(x_j, y'_j)$ is the value of the membership function of the fuzzy set TS (Time Series) of the analyzed TS to the fuzzy linear regression equation of this TS for the value of the TS element y'_j, corresponding to the time moment x_j ($j = \overline{1,n}$); n is the number of the time moments.

If the similarity of the analyzed TS in respect to the TS, corresponding to the classical linear regression equation, is estimated, than $u_S(x_j, y''_j) \equiv 1$ under all the time moments j ($j = \overline{1,n}$).

Fuzzy similarity measures (16) and (17) were chosen out of variety of the known fuzzy similarity measures, as they showed the highest quality of the results during the performing of the similarity evaluation of the test data sets, described by the means of the fuzzy linear regression equation. At the same time the data set, corresponding to the points of the classical linear regression equation, was chosen as a model.

In the context of work with the fuzzy linear regression equation it is irrelevant to apply classic similarity measures to estimate the uncertainty level of the TS behavior (for example, such as similarity measure based on the Euclidean distance or angular similarity measure [20]). This is due to the fact, that the parameters of the fuzzy linear regression equation (2), which values are determined on the basis of the TFN $A_1 = (a_1, c_1, d_1)$ and $A_0 = (a_0, c_0, d_0)$, are involved, when we calculate the values of the membership function (12), and at the same time on the basis of these values the similarity in the formulas (16) and (17) is estimated. We should mention, that the standard similarity measures can be applied as additional toolkit to form a consolidated decision on the basis of a few similarity measures [20], when solving the task of the standard TS search out of a certain data base with the TS for the certain analyzed TS.

If we apply the standard TS, formed on the basis of the point of the classical linear regression equation, we can note that: $u_S(x_j, y''_j) \equiv 1$ at all the time moments j ($j = \overline{1,n}$). Then the formulas (16) and (17) can be rewritten as:

$$f_1 = 1 - \frac{\sum_{j=1}^{n}(1 - u_{TS}(x_j, y'_j))}{\sum_{j=1}^{n}(1 + u_{TS}(x_j, y'_j))}, \tag{18}$$

$$f_2 = \frac{1}{n} \cdot \sum_{j=1}^{n} \frac{min(1, u_{TS}(x_j, y'_j))}{max(1, u_{TS}(x_j, y'_j))}. \tag{19}$$

Fuzzy measures (18) and (19) can be applied as indicators, characterizing the uncertainty level of the TS behavior.

It is obvious, that if all the points of the fuzzy linear regression equation comply with the corresponding points of the classical linear regression equation, the identity law will be fulfilled: $u_{TS}(x_j, y'_j) \equiv 1$ at all the time moments j ($j = \overline{1,n}$). Then the numerator of the formula (18) will be identically equal to zero, and the value of the fuzzy measure (18) will be identically equal to 1. In the formula (19) every summand under the summation symbol will be identically equal to 1, at the same time the value of the fuzzy measure (19) will be equal to 1 itself.

The closer the indicator value (18) or (19) is to 1, the more obvious the compliance of the analyzed TS is to the classical linear regression equation.

The closer the indicator value (18) or (19) is to 0, the more certain we can be about the high uncertainty level of the TS behavior (in the context of its representation on the basis of classical linear regression).

Application of the fuzziness indices, for example, such as the fuzziness index of the sets based on the linear distance (Hamming distance) and a fuzziness index of the sets based on the Euclidean distance [22], is irrelevant for the task of the evaluation of the uncertainty level of the TS behavior, due to the fact, that these indexes estimate fuzziness of the set A in relation to the closest crisp set $\tilde{A}$, formed, for example, under conditions: if $u_A(x) \geq 0.5$, then $u_{\tilde{A}}(x)=1$, otherwise $u_{\tilde{A}}(x)=0$, where $u_A(x)$ and $u_{\tilde{A}}(x)$ are the membership function of the fuzzy set A and characteristic function of the crisp set $\tilde{A}$; x is the element of the universe X, where the functions $u_A(x)$ and $u_{\tilde{A}}(x)$ are defined. The comparison of the value of the membership function, calculated by the formula (12), for certain time moments for the fuzzy linear regression equation, is carried out without the values of the characteristic function, but with the values of the membership function for the same time moments for the classical linear regression equation in the examined problem.

For the analyzed TS it is relevant to calculate the resulting fuzzy similarity measure, defined at least by the fuzzy similarity measures F^{UP} and F^{LOW}:

$$F = min(F^{UP}, F^{LOW}). \tag{20}$$

The analysis of the areas and centroids of the lower and upper parts of the "corridors" of the fuzzy linear regression equation corridor of the analyzed TS is quite interesting. At the same time the areas can be calculated by the application of the parameters $T_0 = (a_0, c_0, d_0)_{TFN}$ and $T_1 = (a_1, c_1, d_1)_{TFN}$ of the fuzzy linear regression equation and a formula of measuring an area of a triangle, and centroids can be calculated with application of a formula of measuring of the centroid of gravity of singleton:

$$C = \frac{\sum_{j=1}^{n} x_i \cdot u_{TS}(x_j, y'_j)}{\sum_{j=1}^{n} u_{TS}(x_j, y'_j)}. \tag{21}$$

Fuzzy measures (18) and (19), as well as the indicators, characterizing areas and centroids of the upper and lower part of the corridors of the fuzzy linear regression equation of the analyzed TS, can be applied as additional characteristics of the TS alongside with the standard characteristics (for, example, alongside with the classical average value of TS elements and the standard deviation of the TS elements) in order to form expanded set of characteristic when solving the problem of the TS clustering [23 – 25].

It is obvious, that in this case we can obtain the more accurate results of clustering, as we will take into account some distinctive features of the analyzed TS behavior additionally, that were not reflected in standard characteristics.

3. Experimental part

Approbation of the proposed approach to the evaluation of the uncertainty level of the short TS behavior was carried out on the great number of the social and economic TS and TS, characterizing climate events. Particularly two TS [26], characterizing during 20 time moments, were examined:

- annual flood stage on the Amur river in the alignment of Khabarovsk (cm):

d_1=[371; 384; 620; 550; 349; 426; 435; 497; 446; 564; 330; 419; 408; 337; 414; 366; 523; 294; 322; 261];

- monthly amount of the rainfall in the river basin of the Kuban River (mm):

d_2=[10.8; 3.8; 7.7; 5.2; 5.5; 3; 9.1; 11.3; 8.6; 12; 17.4; 9.9; 11.5; 7.9; 13.6; 10.5; 15.1; 10.6; 17.7; 10.4].

In this case all the calculations were carried out with the application of the fuzzy linear regression equation with symmetric parameters, as well as with asymmetric ones.

The parameters of the fuzzy linear regression (in the symmetric and asymmetric cases) for the analyzed TSs are shown in tables 1 and 2 correspondingly.

The results of the calculations of the different indicators are shown in tables 3 and 4. They were received in the process of the formulation of the fuzzy linear regression equation with symmetric fuzzy parameters and the fuzzy linear regression equation with asymmetric fuzzy parameters.

Figures 1 – 4 show the graphical depiction of the fuzzy regression corridors for the analyzed TS.

Analysis of the obtained results allows us to conclude, that the fuzzy linear regression equation with asymmetric parameters allows us to describe the fuzzy regression corridor more precisely and get more accurate evaluation of the uncertainty level of the TS behavior.

Table 1. Parameters of the fuzzy linear regression (symmetric case).

River	a_0	c_0	d_0	a_1	c_1	d_1
Amur	487.87	152.909	152.909	-6.929	0	0
Kuban	4.9843	5.156	5.156	0.594	0.0657	0.0657

Table 2. Parameters of the fuzzy linear regression (asymmetric case).

River	a_0	c_0	d_0	a_1	c_1	d_1
Amur	491.299	113.109	149.487	-7.190	0	0.262
Kuban	5.039	4.891	5.101	0.475	0	5.039

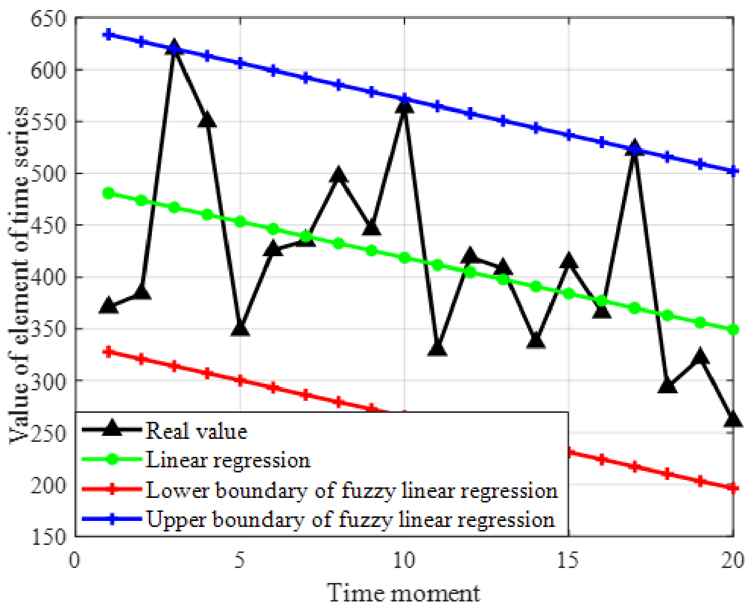

Figure 1. Fuzzy linear regression with symmetric corridor (Amur River).

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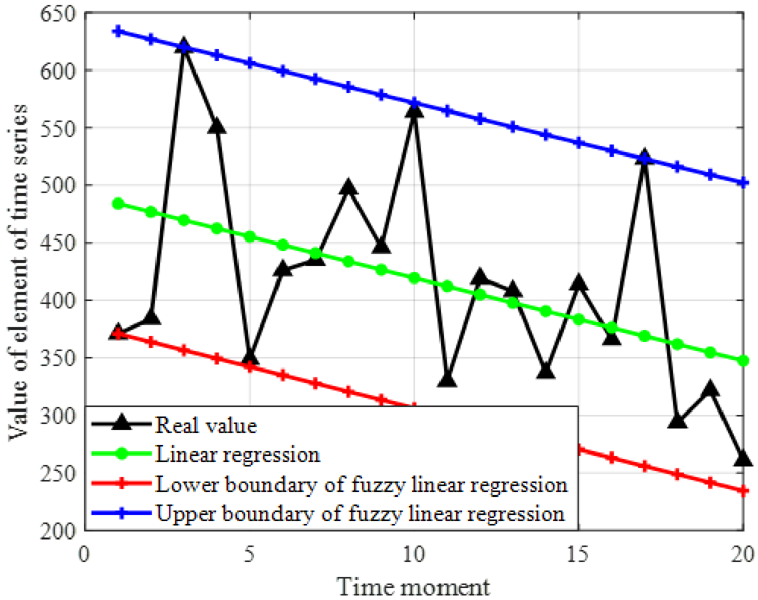

Figure 2. Fuzzy linear regression with asymmetric corridor (Amur River).

So, in particular, it is evident, that TS for the Amur River is characterized by the greater uncertainty level than for the Kuban River (in the asymmetric case, the values of the parameters f_1 and f_2 for the Amur River are greater than the values of the corresponding indicators of the Kuban River). The same conclusions we can draw from the visual analysis of the fuzzy regression corridors in figure 2 and 4 (while conclusions, formed with application of the fuzzy linear regression equation with symmetric parameters are questionable).

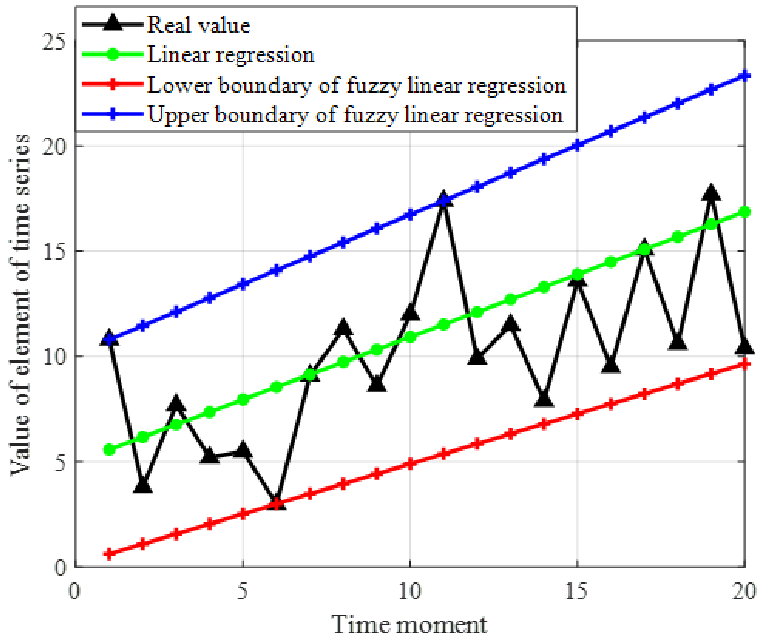

Figure 3. Fuzzy linear regression with symmetric corridor (Kuban River).

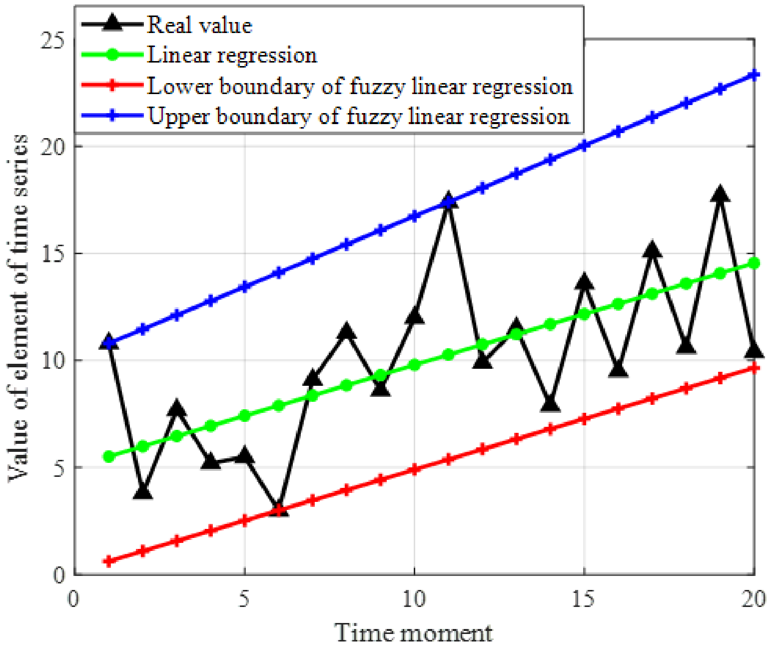

Figure 4. Fuzzy linear regression with asymmetric corridor (Kuban River).

Table 3. Indicators on the basis of the fuzzy linear regression equation with the symmetric fuzzy parameters.

River	f_1	f_2	C_{LOW}	C_{UP}	C	S_{LOW}	S_{UP}	S
Amur	0.717	0.587	11.570	10.909	11.301	2905.261	2905.261	5810.522
Kuban	0.685	0.547	9.547	11.631	10.375	111.069	111.069	222.137

Table 4. Indicators on the basis of the fuzzy linear regression equation with the asymmetric fuzzy parameters.

River	f_1	f_2	C_{LOW}	C_{UP}	C	S_{LOW}	S_{UP}	S
Amur	0.649	0.505	12.379	10.891	11.672	2892.444	2149.069	5041.513
Kuban	0.697	0.562	9.380	11.333	10.492	112.264	92.933	226.693

Indicators f_1 and f_2, and the indicators on the basis of the centroids and the areas of the fuzzy regression corridors can be applied as additional characteristics of the analyzed TS, when solving the task of the TS group clustering.

Conclusion

The results of the empirical research prove the efficiency and relevance of the application of the fuzzy linear regression equation with asymmetric parameters for the evaluation of the uncertainty level of the TS behavior.

The aim of the further research is to develop an approach to the evaluation of the uncertainty level of the TS behavior with application of “uncertainty imprints”, formed on the basis of the interval fuzzy sets of the second type, and carrying out of the comparative analysis of two approaches to the evaluation of the uncertainty level of the TS behavior. The first approach is based on the fuzzy linear regression with asymmetric parameters; the second one is based on the “uncertainty imprints”.

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