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Logic of basic swarm reactions

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Abstract. In the paper, there are defined continuous-valued logical functions defined on $[0, 1]$ as natural reactions of swarm to several stimuli detected at one time step. In this model, the logical duality represented by squares or cubes of opposition is realizable by own swarm patterns based on lateral inhibition and lateral activation. Swarm reactions are considered not certain, but with a probability of their intensity in respect to the distance to an appropriate biologically active substance. At the same time, we can formalize cases when the swarm members are partly inhibited and partly activated. In other words, we assume that they can be less or more inhibited and less or more activated, etc. So, we can deal with a fuzzy mix of conjunction and disjunction at one time.

1. Introduction

Swarm intelligence as a branch of computer science [2] is focused on analyzing the collective behaviour of decentralized and self-organized multi-agent systems. There were developed many algorithms based on formalizing different swarms: ant colonies, bee colonies, fish schooling, bird flocking and horse herding, bacterial colonies with a kind of social behaviour, multinucleated giant amoebae *Physarum polycephalum* [1, 5, 6], etc.

So, swarm reactions are considered intelligent. It allows us to construct some computing devices from swarms such as *organic memristors* – organic devices with a memory. These devices can solve a wide range of computation tasks, including optimisation on graphs, computational geometry, decentralized robot control, logic and arithmetical computing, etc.

Hence, swarms can implement different logical and arithmetic functions through their typical reactions to outer stimuli. All these functions are defined due to artificial conditions completely controlled in the experiments. In this paper, I am going to consider basic swarm reactions as logical functions realized in natural conditions, not artificial ones. So, each swarm reacts to outer signals by means of decentralized reactions of its members and there are two kinds of reactions in respect to two types of outer signals: *attracting* swarm members in relation to attractants (food, pheromone) and *repelling* swarm members in relation to repellents (dangerous circumstances such as predators). Each attractant can have a different concentration of pheromone or other active substance. In this way, it attracts swarm members differently: stronger or weaker. At the same time, each repellent can have a different concentration of dangerous substance, too. Therefore, it repels swarm members differently also: stronger or weaker.

Thus, if we regard attractants as logical variables, they will have values from the interval $[0, 1]$. Let p be a logical variable that denotes an attractant. Then its value $m(p)$ is equal to 0 if the signal from an appropriate attractant cannot be perceived by the swarm members (e.g. the

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attractant is located so far from the swarm). The value $m(p)$ is equal to 1 if the attractant is occupied by the swarm members. In case the attractant just attracts the swarm, the value $m(p)$ belongs to $(0, 1)$. In the meanwhile, if the attractant q attracts stronger then the attractant p, then we have $m(q) > m(p)$.

Repellents can be examined as negations of logical variables. Let $\neg p$ be a logical formula that denotes a repellent. Then its value $m(\neg p)$ is equal to 0 if the signal from an appropriate repellent cannot be perceived by the swarm members (e.g. the repellent is located so far from the swarm). The value $m(\neg p)$ is equal to 1 if the repellent is placed at the closest distance from the swarm members. In case the repellent just repels the swarm, the value $m(p)$ belongs to $(0, 1)$. Thereby, if the repellent q repels stronger then the repellent p, then we have $m(q) > m(p)$.

The conjunction $p \wedge q$ of two biologically active substances p and q means a simultaneous action of both substances (attracting and/or repelling) in a different degree of their activity belonging to $[0, 1]$ depending on a concentration of active ingredients. The disjunction $p \vee q$ of two biologically active substances p and q means that p or q act (attract and/or repel) in respect to their concentration expressed numerically as a number from $[0, 1]$. Hence, a localization of different biologically active substances (attractants and repellents) can be represented as a logical function defined on these active substances with their different activity.

In section 2, different logical functions are defined on swarm patterns in the way mentioned above. In section 3, there are considered some natural limits of complex logical functions because of the following two main effects in networking reactions: lateral inhibition and lateral activation [5]. All the constructions of the paper are presented for the first time. Some real-time experiments were carried out within the project *Physarum Chip: Growing Computers from Slime Mould* funded by the Seventh Framework Programme (FP7) by the European Commission within CORDIS and the FET Proactive scheme (Grant agreement ID: 316366). In the paper there are presented just theoretical results.

2. Logical functions on swarm patterns

Let us take $\mathbf{F}$, a standard set of propositional formulas containing propositional variables $p_1, \ldots, p_k$ and their propositional superpositions by using negation $\neg$, conjunction $\wedge$, and disjunction $\vee$. A *two-valued logical meaning* m is defined as a mapping from the set of propositional formulas $\mathbf{F}$ to $\{0, 1\}$, where 1 means 'truth' and 0 means 'falsehood'. A *infinite-valued logical meaning* m' is defined as a mapping from the set of propositional formulas $\mathbf{F}$ to $[0, 1] \subset \mathbf{R}$, where numbers from the interval $(0, 1]$ mean a degree of 'truth' up to the highest degree 1, 0 means 'falsehood'. An n-place *two-valued* (*infinite-valued*) *logical function* f is defined as a mapping from $\mathbf{F}^n$ to $\mathbf{F}$ such that $m(f(\varphi_1, \ldots, \varphi_n)) = f(m(\varphi_1), \ldots, m(\varphi_n))$ (respectively, $m'(f(\varphi_1, \ldots, \varphi_n)) = f(m'(\varphi_1), \ldots, m'(\varphi_n))$). From this definition it follows that $\neg$ is an unary logical function and $\vee$, $\wedge$ are binary logical functions (two-valued or infinite-valued).

Let us take some unary predicates $P_1, \ldots, P_k$ verified on the domain of D and let us take some variables $x, y, \ldots$ that are interpreted as members of D. Then we can extend $\mathbf{F}$ to $\mathbf{F}_P$ by adding (i) atomic formulas $P_1(x), \ldots, P_k(x)$, (ii) their propositional superpositions constructed by using negation $\neg$, conjunction $\wedge$, and disjunction $\vee$; (iii) quantified formulas $\forall x \varphi$ (to read 'φ for all x') and $\exists x \varphi$ (to read 'φ for some x'), where φ is an atomic formula $P(x)$ or a propositional superpositions of atomic formulas $P_1(x), \ldots, P_l(x)$.

Now, let us take two unary modal operators: $\Box$ and $\diamond$, defined on formulas φ from $\mathbf{F}_P$. Let us introduce the so-called modal formulas: $\Box\varphi$ (to read: 'it is necessary that φ') and $\diamond\varphi$ (to read: 'it is possible that φ'). The set $\mathbf{F}_P$ is extended again to $\mathbf{F}_\Box$ by adding modal formulas. Assume that X is a set of indices and there is a distinguished index a. Let R be a binary relation on the indices. The distinguished index a is to represent an actual time. The relation R is to be said a possibility since a. Let us take x from X such that aRx. Then this aRx shows us a possibility since a at an index x.

Thus, finally, an n-place *two-valued* (*infinite-valued*) *logical function* f is defined as a mapping from $\mathbf{F}_{\Box}^{n}$ to $\mathbf{F}_{\Box}$ such that $m(f(\varphi_1, \ldots, \varphi_n)) = f(m(\varphi_1), \ldots, m(\varphi_n))$ (respectively, $m'(f(\varphi_1, \ldots, \varphi_n)) = f(m'(\varphi_1), \ldots, m'(\varphi_n)))$. So, $\forall$ and $\exists$ as well as $\Box$ and $\diamond$ are unary logical functions.

Let us introduce a semantics for $\mathbf{F}_{\Box}$ defined on the swarm patterns for infinite-valued logical functions. As we know, each swarm may be regarded as a *multi-agent system* consisting of n actors (members of this swarm) $A_1, \ldots, A_n$. All the behaviors of the actors are influenced by biologically active substances of two types: (i) *repellents* (such as hazardous substances or predators), which are always avoided by the actors, and (ii) *attractants* (such as food or sexual pheromone) which attract them. Each member of swarm can detect an attractant (repellent) in a radius not longer than r. In other words, there is a minimal r such that at each distance $d \in [0, r)$ an active substance is detected by the swarm. Hence, assume that this attractant (repellent) X is placed at the distance $d \in [0, r)$ from an agent A. Let us define a probability $\mathbf{P}_t^A(X) = 1 - \frac{d}{r}$ for the agent A at the time step t in detecting the attractant (repellent) X. It means that this X attracts (repels) the swarm member A with a probability $1 - \frac{d}{r}$. Suppose that $d = 0$. Then $\mathbf{P}_t^A(X) = 1$. It is just a case when the attractant (repellent) X is occupied by the agent A. Now, suppose that $d = r$. Then $\mathbf{P}_t^A(X) = 0$. Beginning from the distance r the swarm member cannot detect an attractant (repellent). Assume that for all $d > r$, we obtain $\mathbf{P}_t^A(X) = 0$, too:

$$\mathbf{P}_t^A(X) = \begin{cases} 1 - \frac{d}{r}, & \text{if } d \in [0, r); \\ 0, & \text{if } d \in [r, +\infty). \end{cases}$$

This property allows us to interpret propositional variables $p \in \mathbf{F}_{\Box}$ as biological stimuli:

true variable: $m_{X,t}^A(p) = \mathbf{P}_t^A(X) \in (0, 1]$ for the agent A and a biologically active thing X at the time step t if and only if A is attracted (repelled) by X at t with a probability $\mathbf{P}_t^A(X) > 0$;

false variable: $m_{X,t}^A(p) = 0$ for the agent A and a biologically active thing X at the time step t if and only if A is not attracted (repelled) by X at t, i.e. $\mathbf{P}_t^A(X) = 0$.

Let us interpret the negation $\neg p$ of propositional variables $p \in \mathbf{F}_{\Box}$ as a biological stimulus. Let X be an appropriate attractant (repellent) realizing p which attracts (repels) A at t with a probability $\mathbf{P}_t^A(X) \in [0, 1]$. Then $\neg p$ means that $\mathbf{P}_t^A(\neg X) = 1 - \mathbf{P}_t^A(X)$. For instance, if $\mathbf{P}_t^A(X) = 0$, then it means that X is not detected and then $\mathbf{P}_t^A(\neg X) = 1$. If $\mathbf{P}_t^A(X) = 1$, then it means that X is occupied by A and then $\mathbf{P}_t^A(\neg X) = 0$. In cases $\mathbf{P}_t^A(X) \in (0, 1)$, the situation is more advanced and means that X is not occupied with a degree $1 - \mathbf{P}_t^A(X)$. Thus, our definition of negation is as follows:

true negation: $m_{X,t}^A(\neg p) = \mathbf{P}_t^A(X) \in [0, 1)$ for the agent A and biologically active element X at the time step t if and only if X is not occupied by A at t, in other words $\mathbf{P}_t^A(X) < 1$;

false negation: $m_{X,t}^A(\neg p) = 0$ for the agent A and biologically active element X at the time step t if and only if $(m_{X,t}^A(p)) = 1$, i.e. X maximally actively works on A at t.

Now, let us assume the existence of k agents $A_1, \ldots, A_k$ with the radii of sensitivity $r_{A_1}, \ldots, r_{A_k}$, respectively. So, $r_{A_1}, \ldots, r_{A_k}$ are minimal distance where an active ingredient cannot be detected by the members $A_1, \ldots, A_k$. Suppose that there are two different active things X_1 and X_2 located at the distance d such that $d < r_{A_1}, \ldots, d < r_{A_k}$. There are two possibilities, provided that X_1 and X_2 are attractants: (i) both X_1 **and** X_2 are occupied by the members $A_1, \ldots, A_k$ (for instance, X_1 is occupied by A_1 and X_2 is occupied by A_2, or X_1 is occupied by A_2 and X_2 is occupied by A_1, etc.); (ii) X_1 **or** X_2 are occupied by the members $A_1, \ldots,$

A_k (for instance, X_1 is occupied by A_1 and A_2 simultaneously, or X_2 is occupied by A_1 and A_2 simultaneously, etc.). Also, there are two possibilities, provided that X_1 and X_2 are repellents: (iii) both X_1 **and** X_2 are avoided by the members $A_1, \ldots, A_k$ (e.g. X_1 is avoided by A_1 and X_2 is avoided by A_2, or X_1 is avoided by A_2 and X_2 is avoided by A_1, etc.); (iv) X_1 **or** X_2 are avoided by the members $A_1, \ldots, A_k$ (e.g. X_1 is avoided by A_1 and A_2 simultaneously, or X_2 is avoided by A_1 and A_2 simultaneously, etc.). In the third situation that one of X_1 and X_2 is a repellent and another active element is an attractant, we face the following possibilities: (v) both X_1 **and** X_2 affect the members $A_1, \ldots, A_k$; (iv) X_1 **or** X_2 affect the members $A_1, \ldots, A_k$.

Thus, we can interpret the conjunction $p \wedge q$ of propositional variables $p, q \in \mathbf{F}_\Box$ as a biological stimulus, too:

true conjunction: $m^{A_1,\ldots,A_k}_{X_1,X_2,t}(p \wedge q) = \mathbf{P}^{A_1,\ldots,A_k}_t(X_1 \wedge X_2) = \min(\mathbf{P}^{A_1,\ldots,A_k}_t(X_1), \mathbf{P}^{A_1,\ldots,A_k}_t(X_2)) > 0$ for the agents $A_1, \ldots, A_k$ and biologically active elements X_1 and X_2 at the time step t if and only if both X_1 and X_2 attract (repel) $A_1, \ldots, A_k$ at t with a probability $\mathbf{P}^{A_1,\ldots,A_k}_t(X_1) > 0$ and $\mathbf{P}^{A_1,\ldots,A_k}_t(X_2) > 0$;

false conjunction: $m^{A_1,\ldots,A_k}_{X_1,X_2,t}(p \wedge q) = \mathbf{P}^{A_1,\ldots,A_k}_t(X_1 \wedge X_2) = \min(\mathbf{P}^{A_1,\ldots,A_k}_t(X_1), \mathbf{P}^{A_1,\ldots,A_k}_t(X_2)) = 0$ for the agents $A_1, \ldots, A_k$ and biologically active elements X_1 and X_2 at the time step t if and only if X_1 or X_2 do not affect $A_1, \ldots, A_k$ at t, i.e. $\mathbf{P}^{A_1,\ldots,A_k}_t(X_1) = 0$ or $\mathbf{P}^{A_1,\ldots,A_k}_t(X_2) = 0$.

The same concerns the disjunction $p \vee q$ of propositional variables $p, q \in \mathbf{F}_\Box$:

true disjunction: $m^{A_1,\ldots,A_k}_{X_1,X_2,t}(p \vee q) = \mathbf{P}^{A_1,\ldots,A_k}_t(X_1 \vee X_2) = \max(\mathbf{P}^{A_1,\ldots,A_k}_t(X_1), \mathbf{P}^{A_1,\ldots,A_k}_t(X_2)) > 0$ for the agents $A_1, \ldots, A_k$ and biologically active elements X_1 and X_2 at the time step t if and only if X_1 or X_2 attract (repel) $A_1, \ldots, A_k$ at t with a probability $\mathbf{P}^{A_1,\ldots,A_k}_t(X_1) > 0$ or $\mathbf{P}^{A_1,\ldots,A_k}_t(X_2) > 0$;

false disjunction: $m^{A_1,\ldots,A_k}_{X_1,X_2,t}(p \vee q) = \mathbf{P}^{A_1,\ldots,A_k}_t(X_1 \vee X_2) = \max(\mathbf{P}^{A_1,\ldots,A_k}_t(X_1), \mathbf{P}^{A_1,\ldots,A_k}_t(X_2)) = 0$ for the agents $A_1, \ldots, A_k$ and biologically active elements X_1 and X_2 at the time step t if and only if X_1 and X_2 do not affect $A_1, \ldots, A_k$ at t, i.e. $\mathbf{P}^{A_1,\ldots,A_k}_t(X_1) = 0$ and $\mathbf{P}^{A_1,\ldots,A_k}_t(X_2) = 0$.

Let l be a maximal number of different active things which can be detected at the time t by all the members $A_1, \ldots, A_k$. Then we can generalize our new semantics as follows:

true l-place logical function: $m^{A_1,\ldots,A_k}_{X_1,\ldots,X_l,t}(f(p_1, \ldots, p_l) = \mathbf{P}^{A_1,\ldots,A_k}_t(f(X_1, \ldots, X_l)) = f(\mathbf{P}^{A_1,\ldots,A_k}_t(X_1), \ldots, \mathbf{P}^{A_1,\ldots,A_k}_t(X_l)) > 0$ for the agents $A_1, \ldots, A_k$ and biologically active elements $X_1, \ldots, X_l$ at the time step t if and only if $f(\mathbf{P}^{A_1,\ldots,A_k}_t(X_1), \ldots, \mathbf{P}^{A_1,\ldots,A_k}_t(X_l)) > 0$;

false l-place logical function: $m^{A_1,\ldots,A_k}_{X_1,\ldots,X_l,t}(f(p_1, \ldots, p_l) = \mathbf{P}^{A_1,\ldots,A_k}_t(f(X_1, \ldots, X_l)) = f(\mathbf{P}^{A_1,\ldots,A_k}_t(X_1), \ldots, \mathbf{P}^{A_1,\ldots,A_k}_t(X_l)) = 0$ for the agents $A_1, \ldots, A_k$ and biologically active elements $X_1, \ldots, X_l$ at the time step t if and only if $f(\mathbf{P}^{A_1,\ldots,A_k}_t(X_1), \ldots, \mathbf{P}^{A_1,\ldots,A_k}_t(X_l)) = 0$.

Hence, in our definition of realizing l-place logical function on the swarm behavior, we assume a bijection between the set of propositional variables and the set of active elements affecting at the time step t.

Let us show that $m^{A_1,\ldots,A_k}_{X,t}(p \vee \neg p) = 1$ if $m^{A_1,\ldots,A_k}_{X,t}(p) = 0$ or $m^{A_1,\ldots,A_k}_{X,t}(p) = 1$. Indeed, $m^{A_1,\ldots,A_k}_{X,t}(p \vee \neg p) = (m^{A_1,\ldots,A_k}_{X,t}(p) \vee \neg m^{A_1,\ldots,A_k}_{X,t}(p)) = (\mathbf{P}^{A_1,\ldots,A_k}_{X,t}(p) \vee \neg \mathbf{P}^{A_1,\ldots,A_k}_{X,t}(p)) = \max(\mathbf{P}^{A_1,\ldots,A_k}_{X,t}(p), 1 - \mathbf{P}^{A_1,\ldots,A_k}_{X,t}(p)) = 1$ if $\mathbf{P}^{A_1,\ldots,A_k}_{X,t}(p) = 0$ or $\mathbf{P}^{A_1,\ldots,A_k}_{X,t}(p) = 1$.

Also, we can behaviorally interpret unary predicates $P_1, \ldots, P_k$ verified on the domain of D as well as variables $x, y, \ldots$ understood as members of D. Let D_t mean all the biologically active elements reachable for $A_1, \ldots, A_k$ at the time t. Then $D = \bigcup_{t=0}^{n} D_t$, where n is a time step denoting one life cycle of a swarm.

True atomic formula:

$$m_{D,t}^{A_1,\dots,A_k}(P(x)) = \mathbf{P}_{D,t}^{A_1,\dots,A_k}(P(x)) > 0$$

for $A_1, \dots, A_k$ and biologically active elements of D at the time step t if and only if $P(m_{D,t}^{A_1,\dots,A_k}(x)) \subseteq D_t$ with a probability $\mathbf{P}_{D,t}^{A_1,\dots,A_k}(P(x)) > 0$, where $m_{D,t}^{A_1,\dots,A_k}(x)$ means an element from D_t. For instance, let P mean 'neighbors for the attractant X (i.e. elements reachable from X at one step of agents $A_1, \dots, A_k$)'. Then $m_{D,t}^{A_1,\dots,A_k}(P(x)) > 0$ if and only if there are neighbors for X in D at t and these neighbors affect with a probability $\mathbf{P}_{D,t}^{A_1,\dots,A_k}(P(x)) > 0$.

False atomic formula:

$$m_{D,t}^{A_1,\dots,A_k}(P(x)) = \mathbf{P}_{D,t}^{A_1,\dots,A_k}(P(x)) = 0$$

for $A_1, \dots, A_k$ and biologically active elements of D at the time step t if and only if $P(m_{D,t}^{A_1,\dots,A_k}(x)) \nsubseteq D_t$, where $m_{D,t}^{A_1,\dots,A_k}(x)$ means an element from D_t.

Let $f(\varphi_1, \dots, \varphi_i)$ be a logical function defined on atomic formulas or their propositional superpositions $\varphi_1, \dots, \varphi_i$ from $\mathbf{F}_P$, then $m_{D,t}^{A_1,\dots,A_k}(f(\varphi_1, \dots, \varphi_i)) > 0$ if and only if $f(m_{D,t}^{A_1,\dots,A_k}(\varphi_1), \dots, m_{D,t}^{A_1,\dots,A_k}(\varphi_i)) > 0$, otherwise $f(m_{D,t}^{A_1,\dots,A_k}(\varphi_1), \dots, m_{D,t}^{A_1,\dots,A_k}(\varphi_i)) = 0$.

Suppose, $\forall x\varphi$ is a quantified formula of $\mathbf{F}_P$, then $m_{D,t}^{A_1,\dots,A_k}(\forall x\varphi) > 0$ if and only if for every element a in the domain of D_t, $m_{D,t}^{A_1,\dots,A_k}(x) = a$ and we have $m_{D,t}^{A_1,\dots,A_k}(\varphi) > 0$, otherwise $m_{D,t}^{A_1,\dots,A_k}(\forall x P(x)) = 0$. Let $\exists x\varphi$ be a quantified formula of $\mathbf{F}_P$, then $m_{D,t}^{A_1,\dots,A_k}(\exists x\varphi) > 0$ if and only if there is an element a in the domain of D_t, such that $m_{D,t}^{A_1,\dots,A_k}(x) = a$ and we have $m_{D,t}^{A_1,\dots,A_k}(\varphi) > 0$, otherwise $m_{D,t}^{A_1,\dots,A_k}(\exists x P(x)) = 0$.

Modal formulas of $\mathbf{F}_\Box$ can be behaviorally interpreted, too. Let the index set X consist of indices denoting different time steps within one life cycle of a swarm.

- for a formula $\varphi \in \mathbf{F}_\Box$ without modal operators, $m_{\Box,t}^{A_1,\dots,A_k}(\varphi) > 0$ at $t \in X$ if and only if $m_{D,t}^{A_1,\dots,A_k}(\varphi) > 0$ or $m_{X_1,\dots,X_l,t}^{A_1,\dots,A_k}(\varphi) > 0$;
- for a formula $\varphi \in \mathbf{F}_\Box$ without modal operators, $m_{\Box,t}^{A_1,\dots,A_k}(\varphi) = 0$ at $t \in X$ if and only if $m_{D,t}^{A_1,\dots,A_k}(\varphi) = 0$ or $m_{X_1,\dots,X_l,t}^{A_1,\dots,A_k}(\varphi) = 0$;
- let $\Box\varphi$ be a modal formula of $\mathbf{F}_\Box$, then $m_{\Box,t}^{A_1,\dots,A_k}(\Box\varphi) > 0$ at $x \in X$ if and only if for all $y \in X$ with xRy, $m_{\Box,t}^{A_1,\dots,A_k}(\varphi) > 0$ at y, otherwise $m_{\Box,t}^{A_1,\dots,A_k}(\Box\varphi) = 0$ at x;
- let $\diamond\varphi$ be a modal formula of $\mathbf{F}_\Box$, then $m_{\Box,t}^{A_1,\dots,A_k}(\diamond\varphi) > 0$ at $x \in X$ if and only if for some $y \in X$ with xRy, $m_{\Box,t}^{A_1,\dots,A_k}(\varphi) > 0$ at y, otherwise $m_{\Box,t}^{A_1,\dots,A_k}(\diamond\varphi) = 0$ at x;
- let $\forall x\varphi$ be a quantified formula of $\mathbf{F}_\Box$, then $m_{\Box,t}^{A_1,\dots,A_k}(\forall x\varphi) > 0$ at $t \in X$ if and only if for every element a in the domain of D_t, $m_{D,t}^{A_1,\dots,A_k}(x) = a$ and we have $m_{\Box,t}^{A_1,\dots,A_k}(\varphi) > 0$ at t, otherwise $m_{\Box,t}^{A_1,\dots,A_k}(\forall x P(x)) = 0$ at t;
- let $\exists x\varphi$ be a quantified formula of $\mathbf{F}_\Box$, then $m_{\Box,t}^{A_1,\dots,A_k}(\exists x\varphi) > 0$ at $t \in X$ if and only if there is an element a in the domain of D_t, such that $m_{D,t}^{A_1,\dots,A_k}(x) = a$ and we have $m_{\Box,t}^{A_1,\dots,A_k}(\varphi) > 0$ at t, otherwise $m_{\Box,t}^{A_1,\dots,A_k}(\exists x P(x)) = 0$ at t.

3. Logical duality on swarm patterns

Suppose that f is an n-place two-valued logical function. Another n-place two-valued logical function f' is said to be *dual* (or *logically dual*) to f if and only if

$$f'(\varphi_1, \dots, \varphi_n) \equiv \neg f(\neg\varphi_1, \dots, \neg\varphi_n),$$

where $\equiv$ is a sign for the equivalence relation: $f'(\varphi_1, \ldots, \varphi_n)$ is true if and only if $\neg f(\neg\varphi_1, \ldots, \neg\varphi_n)$ is true and $f'(\varphi_1, \ldots, \varphi_n)$ is false if and only if $\neg f(\neg\varphi_1, \ldots, \neg\varphi_n)$ is false.

According to this definition, if f' is dual to f, then f is dual to f'. So, the duality is always mutual.

Let us notice that conjunction and disjunction are dual to each other:

$$(\varphi \wedge \psi) \equiv \neg(\neg\varphi \vee \neg\psi);$$

$$(\varphi \vee \psi) \equiv \neg(\neg\varphi \wedge \neg\psi).$$

The universal quantifier $\forall$ and the existential quantifier $\exists$ are dual to each other, too:

$$\forall x\varphi \equiv \neg\exists x\neg\varphi;$$

$$\exists x\varphi \equiv \neg\forall x\neg\varphi.$$

The necessity modal operator $\Box$ and the possibility modal operator $\diamond$ are another example of logical duality:

$$\Box\varphi \equiv \neg \diamond \neg\varphi;$$

$$\diamond\varphi \equiv \neg\Box\neg\varphi.$$

The *logical duality* is a significant concept of logic, because it allows us to define a lattice, i.e. it can be used for ordering logical functions. Indeed, if f and f' are dual to each other, then either $(f \Rightarrow f') \equiv 1$ or $(f' \Rightarrow f) \equiv 1$, where $\Rightarrow$ is a sign for implication (a two-place two-valued logical function "if ..., then ...') such that

$$(\varphi \Rightarrow \psi) \equiv (\neg\varphi \vee \psi).$$

It means that either $m(f) \leq m(f')$ or $m(f') \leq m(f)$. So, we have the following true implications:

$$(\varphi \wedge \psi) \Rightarrow (\varphi \vee \psi);$$

$$\forall x\varphi \Rightarrow \exists x\varphi.$$

Let us check the first claim. Assume that $1 \equiv (\neg\varphi \vee \varphi)$. Then $((\varphi \wedge \psi) \Rightarrow (\varphi \vee \psi)) \equiv (\neg(\varphi \wedge \psi) \vee (\varphi \vee \psi)) \equiv (\neg\varphi \vee \neg\psi \vee \varphi \vee \psi) \equiv ((\neg\varphi \vee \varphi) \vee (\neg\psi \vee \psi)) \equiv 1 \vee 1 \equiv 1$. In the modal logic **D**, the true implication is as follows:

$$\Box\varphi \Rightarrow \diamond\varphi.$$

On the basis of logical duality, we can define contrary, subcontrary, subaltern, and contradictory logical functions:

contrary: two functions h and h' are *contrary* if and only if $(h \wedge h') \equiv 0$, but not always $(h \vee h') \equiv 1$;

subcontrary: two functions h and h' are *subcontrary* if and only if $(h \vee h') \equiv 1$, but not always $(h \wedge h') \equiv 0$;

subaltern: a function h is *subaltern* to h' if and only if $(h' \Rightarrow h) \equiv 1$;

contradictory: two functions h and h' are *contradictory* if and only if $(h \vee h') \equiv 1$ and $(h \wedge h') \equiv 0$.

Let us assume that $f(\varphi_1, \ldots, \varphi_n)$ and $f'(\varphi_1, \ldots, \varphi_n)$ are dual and $(f(\varphi_1, \ldots, \varphi_n) \Rightarrow f'(\varphi_1, \ldots, \varphi_n)) \equiv 1$, see figure 1:

- $f'(\varphi_1, \ldots, \varphi_n)$ is *subaltern* to $f(\varphi_1, \ldots, \varphi_n)$;

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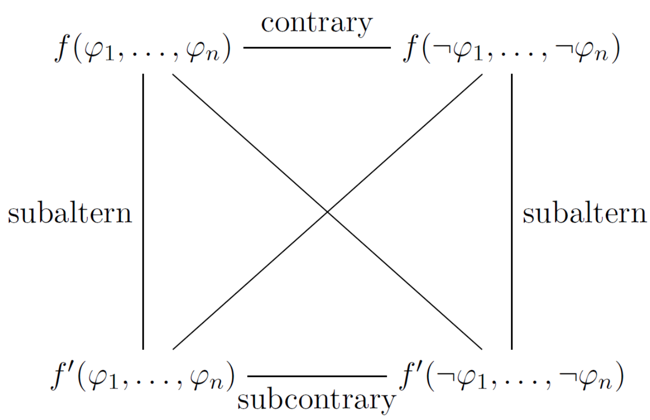

Figure 1. The square of opposition for the dual logical functions $f(\varphi_1, \ldots, \varphi_n)$ and $f'(\varphi_1, \ldots, \varphi_n)$.

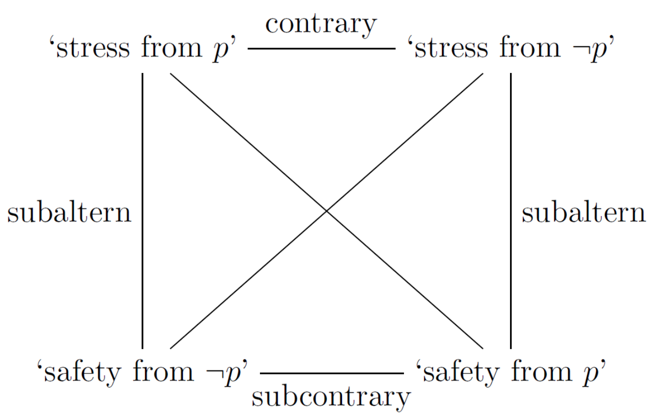

Figure 2. The square of opposition for the expressions 'relax from p' and 'stress from $\neg p$'.

- $f(\varphi_1, \ldots, \varphi_n)$ and $f(\neg\varphi_1, \ldots, \neg\varphi_n)$ are *contrary*;
- $f'(\varphi_1, \ldots, \varphi_n)$ and $f'(\neg\varphi_1, \ldots, \neg\varphi_n)$ are *subcontrary*;
- $f(\varphi_1, \ldots, \varphi_n)$ and $f'(\neg\varphi_1, \ldots, \neg\varphi_n)$ (as well as $f'(\varphi_1, \ldots, \varphi_n)$ and $f(\neg\varphi_1, \ \ldots, \ \neg\varphi_n)$) are *contradictory*.

The logical duality is observed in swarm patterns. So, the swarm members can move either under a stress or with a sense of safety. For example, if some swarm individuals face two attractants, they experience rather a safety and if they face two repellents, they experience rather a stress. Also, a stress can be caused by a very high concentration of attractive pheromone. In any case, there is a natural duality in the swarm reactions: these reactions are carried out either under stress or with sense of safety [5].

Let us define two unary predicates on propositional variables $p \in \mathbf{F}_\square$: 'stress from p' and 'safety from p'. The variables p are interpreted as biologically active elements (attractants or repellents). Standardly, an attractant with a usual concentration of pheromone causes safety and a repellent with a usual concentration of dangerous matter causes stress. Let $\neg p$ be interpreted as a complement to the active element of p in the class of all active elements. Thus, we can construct a square of opposition, see figure 2.

In this square of opposition, the predicates 'stress from p' and 'safety from $\neg p$' are considered dual: if 'stress from p' holds true, then 'safety from $\neg p$' holds true. However, it is possible

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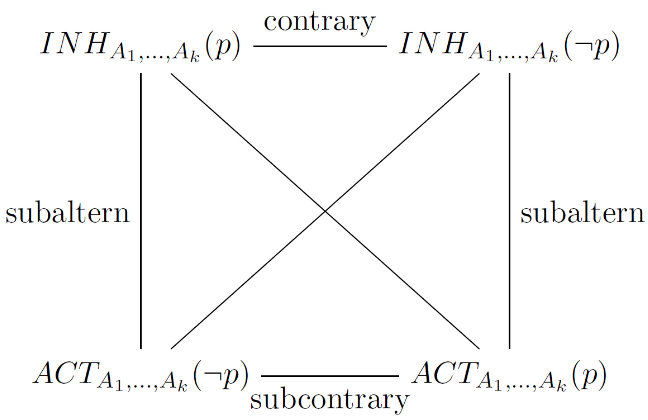

Figure 3. The square of opposition for the unary operators $INH_{A_1,\ldots,A_k}$ and $ACT_{A_1,\ldots,A_k}$.

to claim that the feeling of stress and the feeling of safety are two psychological phenomena which concern individuals of birds and mammals and they do not concern insects or unicellular organisms surely. But how to explain the same swarm reactions of social insects and social bacteria? They also prefer either conjunction or disjunction under different conditions. The point is that 'stress from p' and 'safety from p' can be realized even by an individual such as an insect or bacterium without any mechanism of awareness. In each networking, including networks of actin filaments in one cell, there are two basic reactions to outer stimuli: *lateral activation* (a reaction under safety) and *lateral inhibition* (a reaction under stress). Both types of reactions can be realized without awareness and explained rather chemically. For instance, these types of reactions are observed in plasmodia of *Physarum polycephalum* – an amoeboid multinucleated organism. Evidently that this organism does not have feelings at all.

The *lateral activation* is a reaction of particles within one network to outer stimuli, according to which different particles are not concentrated on the same stimuli. As a result, we observe a decreasing of the intensity of the outer signals and the contrast of the signals is made less visible. The *lateral inhibition* is a reaction of particles within one network to outer stimuli, according to which different particles are concentrated on the same stimuli. This has led us to an increasing of the intensity of the outer signals and the contrast of the signals is made more visible. The plasmodia of *Physarum polycephalum* follow the lateral activation if they detect normal attractants and they follow the lateral inhibition if they face standard repellents [5].

Thus, let us define two unary operators: (i) $INH_{A_1,\ldots,A_k}(p)$ ::= 'the swarm individuals $A_1,\ldots,A_k$ are laterally inhibited by an outer signal p'; $m_{D,t}^{A_1,\ldots,A_k}(INH_{A_1,\ldots,A_k}(p)) > 0$ if and only if $m_{D,t}^{A_1,\ldots,A_k}(\neg p) > 0$, otherwise $m_{D,t}^{A_1,\ldots,A_k}(INH_{A_1,\ldots,A_k}(p)) = 0$; (ii) $ACT_{A_1,\ldots,A_k}(p)$ ::= 'the swarm individuals $A_1,\ldots,A_k$ are laterally activated by an outer signal p'; $m_{D,t}^{A_1,\ldots,A_k}(ACT_{A_1,\ldots,A_k}(p)) > 0$ if and only if $m_{D,t}^{A_1,\ldots,A_k}(p) > 0$, otherwise $m_{D,t}^{A_1,\ldots,A_k}(ACT_{A_1,\ldots,A_k}(p)) = 0$. There is a logical duality between $INH_{A_1,\ldots,A_k}(p)$ and $ACT_{A_1,\ldots,A_k}(\neg p)$:

$$(INH_{A_1,\ldots,A_k}(p) \Rightarrow ACT_{A_1,\ldots,A_k}(\neg p)) \equiv 1.$$

As a consequence, we can introduce a logical square of opposition for $INH_{A_1,\ldots,A_k}(p)$ and $ACT_{A_1,\ldots,A_k}(\neg p)$, please see figure 3. It is the same as in figure 2, but it is defined more correctly from the point of view of cognitive science, because 'stress' and 'safety' are not precise terms.

Now, we can define binary operators: (i) $INH_{A_1,\ldots,A_k}(p,q)$ ::= 'the swarm individuals $A_1,\ldots,A_k$ are laterally inhibited by two outer signals p and q'; $m_{D,t}^{A_1,\ldots,A_k}(INH_{A_1,\ldots,A_k}(p,q)) >$

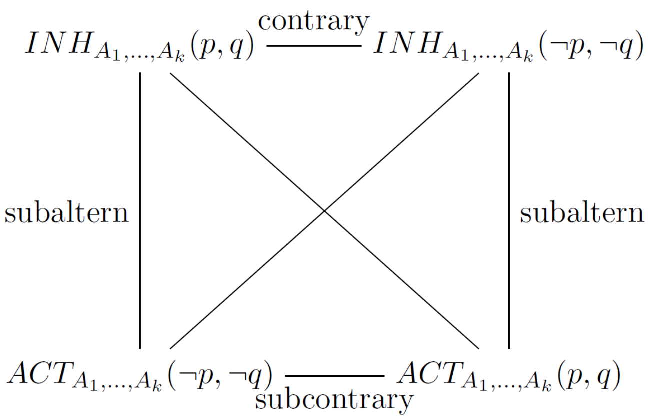

Figure 4. The square of opposition for the binary operators $INH_{A_1,\ldots,A_k}$ and $ACT_{A_1,\ldots,A_k}$.

0 if and only if $m_{D,t}^{A_1,\ldots,A_k}(p \wedge q) > 0$, otherwise $m_{D,t}^{A_1,\ldots,A_k}(INH_{A_1,\ldots,A_k}(p,q)) = 0$; (i) $ACT_{A_1,\ldots,A_k}(p,q)$ ::= 'the swarm individuals $A_1,\ldots,A_k$ are laterally activated by two outer signals p and q'; $m_{D,t}^{A_1,\ldots,A_k}(ACT_{A_1,\ldots,A_k}(p,q)) > 0$ if and only if $m_{D,t}^{A_1,\ldots,A_k}((\neg(p \wedge q))) = m_{D,t}^{A_1,\ldots,A_k}((\neg p \vee \neg q) > 0$, otherwise $m_{D,t}^{A_1,\ldots,A_k}(ACT_{A_1,\ldots,A_k}(p,q)) = 0$. From these definitions it follows that

$$(INH_{A_1,\ldots,A_k}(p,q) \Rightarrow ACT_{A_1,\ldots,A_k}(\neg p,\neg q)) \equiv 1.$$

Hence, we obtain a square of opposition for $INH_{A_1,\ldots,A_k}(p,q)$ and $ACT_{A_1,\ldots,A_k}(\neg p, \neg q)$, see figure 4.

Also, we can obtain the cube of opposition for the binary operators $INH_{A_1,\ldots,A_k}$ and $ACT_{A_1,\ldots,A_k}$, see figure 5.

To sum up, we see that swarms realize a kind of logical duality in their reactions towards outer stimuli p and q, since either they behave under lateral activation and realize the false conjunction of p and q or they can behave under lateral inhibition and realize the true conjunction of p and q, see figure 4–5. For more details about the logical duality, please see [3, 4].

From the experiments [5], we know that lateral activation and lateral inhibition in swarm reactions have a continuous smooth transition between them: to be more laterally activated and less laterally inhibited or to be less laterally activated and more laterally inhibited. We can express this property as follows. Let $X_1, \ldots, X_l$ be active substances. Let us consider

$$\mathbf{Q}_t^{A_1,\ldots,A_k}(X_1,\ldots,X_l) = (\max(\mathbf{P}_t^{A_1,\ldots,A_k}(X_1),\ldots,
\mathbf{P}_t^{A_1,\ldots,A_k}(X_l)) - \min(\mathbf{P}_t^{A_1,\ldots,A_k}(X_1),\ldots,
\mathbf{P}_t^{A_1,\ldots,A_k}(X_l)))/2.$$

Then

$$ACT_{A_1,\ldots,A_k}(X_1,\ldots,X_l) \in [\max(\mathbf{P}_t^{A_1,\ldots,A_k}(X_1),\ldots,
\mathbf{P}_t^{A_1,\ldots,A_k}(X_l)) - \mathbf{Q}_t^{A_1,\ldots,A_k}(X_1,\ldots,X_l),
\max(\mathbf{P}_t^{A_1,\ldots,A_k}(X_1),\ldots,\mathbf{P}_t^{A_1,\ldots,A_k}(X_l))].$$

As a consequence,

$$INH_{A_1,\ldots,A_k}(X_1,\ldots,X_l) = 1 - ACT_{A_1,\ldots,A_k}(X_1,\ldots,X_l).$$

Thus, we can simulate different degrees of lateral activation and lateral inhibition in the swarm reactions.

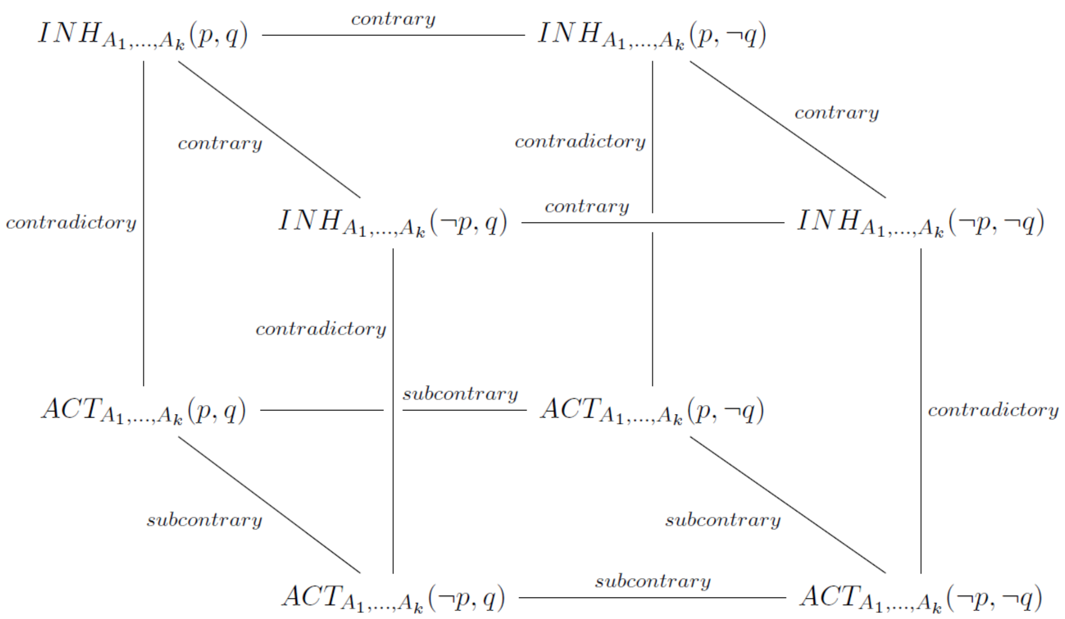

Figure 5. The cube of generalized Post duality for the binary operators $INH_{A_1,\ldots,A_k}$ and $ACT_{A_1,\ldots,A_k}$.

4. Conclusion and Discussion

We have just considered an abstract model of swarms, where we have defined logical functions of $\mathbf{F}_\square$ as natural reactions of swarm members to several stimuli detected at one time step. In this model, the logical duality represented by squares or cubes of opposition is realizable by own swarm patterns based on lateral inhibition and lateral activation. Swarm reactions are considered not certain, but with a probability of their intensity in respect to the distance to an appropriate biologically active substance. At the same time, we can formalize cases when the swarm members are partly inhibited and partly activated. In other words, we assume that they can be less or more inhibited and less or more activated, etc. So, we can deal with a fuzzy mix of conjunction and disjunction at one time. And this mix of effects from $X_1, \ldots, X_l$ can have continuous modifications. For the disjunction of $X_1, \ldots, X_l$, the values can be defined as running over the interval from $\max(\mathbf{P}_t^{A_1,\ldots,A_k}(X_1), \ldots, \mathbf{P}_t^{A_1,\ldots,A_k}(X_l)) - \mathbf{Q}_t^{A_1,\ldots,A_k}(X_1, \ldots, X_l)$ to $\max(\mathbf{P}_t^{A_1,\ldots,A_k}(X_1), \ldots, \mathbf{P}_t^{A_1,\ldots,A_k}(X_l))$. For the conjunction of $X_1, \ldots, X_l$, the values can be defined as running over the interval from $\min(\mathbf{P}_t^{A_1,\ldots,A_k}(X_1), \ldots, \mathbf{P}_t^{A_1,\ldots,A_k}(X_l))$ to $\min(\mathbf{P}_t^{A_1,\ldots,A_k}(X_1), \ldots, \mathbf{P}_t^{A_1,\ldots,A_k}(X_l)) + \mathbf{Q}_t^{A_1,\ldots,A_k}(X_1, \ldots, X_l)$.

References

[1] Adamatzky A, Erokhin V, Grube M, Schubert T and Schumann A 2012 Physarum chip project: growing computers from slime mould. *International Journal of Unconventional Computing* **8(4)** 319–323

[2] Carlos A, Coello Coello, Rosa Laura G Zavala, Benito Mendoza García and Arturo Hernández Aguirre 2000 Ant Colony System for the Design of Combinational Logic Circuits. In J. Miller, A. Thompson, P. Thomson, T. C. Fogarty, editors, Evolvable Systems: From Biology to Hardware. ICES 2000. Lecture Notes in Computer Science, **1801** Springer, Berlin, Heidelberg, 21–30

[3] Demey L and Smessaert H 2018 Aristotelian and Duality Relations Beyond the Square of Opposition. In: Chapman P., Stapleton G., Moktefi A., Perez-Kriz S., Bellucci F. (eds) Diagrammatic Representation and Inference. Diagrams 2018. Lecture Notes in Computer Science, vol 10871. Springer, Cham, 640–656

[4] Demey L and Smessaert H Duality in Logic and Language *In Internet Encyclopedia of Philosophy* https://www.iep.utm.edu/dual-log/

[5] Schumann A 2019 Behaviourism in Studying Swarms: Logical Models of Sensing and Motoring. *Emergence, Complexity and Computation* **33** Springer, Cham
[6] Schumann A and Pancerz K 2019 High-Level Models of Unconventional Computations: A Case of Plasmodium. Springer International Publishing