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Students of the extended abstract in proving Lobachevsky’s parallel lines theorem

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Students of the extended abstract in proving Lobachevsky's parallel lines theorem

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Abstract. The highest quality of the students in proving the theorem was extended abstract. One of the theorems that was difficult to determine was Lobachevsky's parallel lines. The purpose of this study was to describe the characteristics of extended abstract level students in proving the Lobachevsky parallel lines theorem. This was part of research development. This stage was the student analysis phase. The subjects of this study were 21 Mathematics Education Pre-service teachers who were taking Geometry courses. The instrument of this study was the researchers themselves with the help of task sheets and interview guides. Data was collected through task-based interviews. Qualitative data analysis techniques with fixed comparison techniques. The results of this study were that students were able to understand the axiom "through a point outside the line g, there were at least two lines parallel to g". This axiom was used to prove the Nonmetrical Theorem. Also, students were able to prove that through point T which was not located at line g, there were not so many lines parallel to g. Students were able to compare the deductive structure of Lobachevsky's Geometry with Euclid's Geometry. The conclusion of this study was that extended abstract students were able to present several elements and pass interdependence between one another, so that it becomes an integrated entity. He can generalize to new structures.

1. Introduction

Geometry learning was often avoided by students. Each category of spatial intelligence has a different type of error in solving problems in geometry material. Errors were mostly carried out by students with low spatial intelligence because they have deficiencies in visual abilities [1]. The types of student errors in learning geometry were conceptual errors, principle errors and operational errors. Some of the causes of these errors were student learning motivation was very low, geometry has been considered not important, the ability to solve problems was very lacking, and students' reasoning abilities were still very low [2]. The fact was that students often complain when getting assignments to prove a theorem in geometry.

In learning geometry, students often make mistakes at the stage of transformation and process skills. That was because students do not understand the procedures that will be used to solve the problem [3]. An understanding of geometric concepts was often overlooked by students. Also, errors occur for pseudo-thinking's students in solving geometry problems [4]. That was something we must learn, so we can give suggestions for improvement. Difficulties were increasingly experienced when students study non-euclide geometry. Bolyai solved the problem of more than two thousand years of age in connection with Euclid's fifth postulate and discovered non-Euclidean geometry. The glory of this discovery was owned by Nicolai Ivanovich Lobachevskii about non-euclidean geometry. That was a geometry that fundamentally changes our view of geometry and mathematics, in general [5]. The most monumental axiom of Lobachevsky was the axiom of parallels. The point was through one point outside a line h, there were two lines that were parallel to the line h. It was a basic revelation that was very difficult for students. Therefore, we need to trigger students to respond to geometry problems. This was the determination of the quality of the response.

The quality of student responses can be determined using the SOLO taxonomy (structure of the observed learning outcome) [6]. The SOLO provides a systematic way to describe how student performance grows in the complexity and skills postulated. That sequence can be used to guide general sequence formulations in the growth of many structural complexity concepts when mastering many tasks, especially the types of tasks performed in school.

Basic data will be useful to show any changes in students 'learning to learn' abilities after you introduce learning interventions that target the intended learning outcomes [7]. According to Biggs&Collis [7], learning interventions were like high-level thinking about maps and rubrics, thinking skills and strategies, and ICTs that help students achieve different learning outcomes. The intended learning outcomes were SOLO Taxonomy. There were five taxonomic levels, namely structural, unistructural, multi-structural, relational and extended abstract. Biggs & Collis describes that prestigious structural students cannot perform tasks assigned or carry out tasks using irrelevant data. Unistructural students can use one piece of information in response to a task. Multi-structural students can use several pieces of information but cannot connect them together. Relational students can combine separate pieces of information for the completion of a task. Extended abstract students can produce general principles of integrated data that can be applied to new situations [6].

The SOLO taxonomy was suitable for measuring various types of learning outcomes. SOLO works for different subjects because students from S1 and S2 take practice-oriented studies versus conceptual studies. SOLO can be applied to students from various rankings [8]. SOLO helps lecturers set the right expectations, as stated in the results of learning, because it can be used to diagnose the level of learning expected and revise it, to consider the level at which students start and the level at which you want them to go. This means that lecturers can use it to encourage a deeper learning approach by ensuring that lecturers do not expect too much or too little and provide a framework for strategically managing the planned learning experience. If it was used as part of a constructively aligned course, SOLO will also help the lecturer to make an assessment assignment that helps him accurately assess the quality of student learning as well as an effective learning experience in it [9]. SOLO taxonomy can also trace the quality of pre-service mathematics teachers. The teacher's response in geometric achievement tests was related to spatial orientation skills. They were generally at a multi-level structure in accordance with the taxonomy of SOLO. That was the response of pre-service teachers who were in the low and middle levels, most of which were at the multi-level structure. While the response of teachers at high levels was at the relational level. Meanwhile, pre-service teacher responses from two dimensions to three dimensions were mostly at the relational level and responses from three dimensions to two dimensions were mostly at the multistructural level [10].

Based on the description of SOLO, the highest quality of the student in proving the theorem was extended abstract. One of the theorems that was difficult to prove was Lobachevsky's parallel lines. Thus, we were interested in describing the characteristics of extended abstract level students in proving the Lobachevsky parallel lines theorem.

2. Method

The research was a part of developmental research. We have been and were conducting a long-term study. It was a research master plan from the mathematics education graduate study program. The study program was under the auspices of the Teacher Training and Education Faculty at the University of Bengkulu, Indonesia. We apply development research design. This paper was taken from a study which was part of the development research. This stage was the student analysis phase. That was a needs analysis. We want to explore students' ability to respond to the tasks they carry out. The subjects of this study were 21 Mathematics Education students who were taking Geometry courses. It was chosen using a snowball technique. The instrument of this study was the researchers themselves with the help of task sheets and interview guides. To get accurate and complete data we use an audio-visual documentation tool. Data was collected through task-based interviews. Data were analyzed qualitatively and the constant comparative technique.

3. Results and Discussion

Our research this time was to explore the ability of students' responses in proving Lobachevsky's alignment theorem. We give several theorems that they have to respond to. That was a response to how to prove it. The accuracy of your data was maintained by documenting it through audio-visual media. Students were given the same assignment. The task was to examine statements about the properties in Lobachevsky's geometry. They were asked to prove it. Based on paper and pencil, we chose to be interviewed in depth. The results of the completion of student assignments can be categorized in the response quality of SOLO levels based on the assignment sheet. This was one step to determine the choice of the right interview subject. The percentage data of students at SOLO levels can be seen in Figure 1.

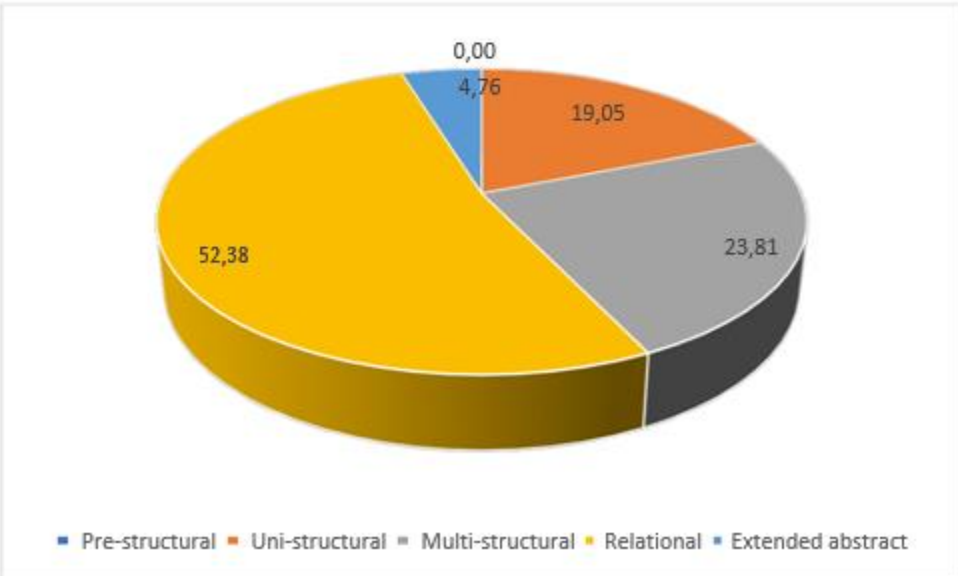

Figure 1. Percentage of many students at each SOLO level

Based on Figure 1, there were 0% of students at the pre-structural level, 4.76% of students who were at the highest level, that was as many as 1 person. Furthermore there were 19.05% (as many as 4 students) who were at the unstructural level, 38.81% (as many as 5 students) were at the multi-structural level, and 52.38% (11 students) were in the relational.

When listening to the results of the research in Figure 1, most students were at the relational level. There were no pre-structural students. Also, only one student gets the highest abstract title (i.e. extended abstract). It was reasonable, in another paper stating that each SOLO taxonomy level increases the demand for the amount of working memory. At a low level (i.e. uni-structural and multi-structural), a student only needs to encode the information provided and can use a remembering strategy to provide answers. as for the higher level (i.e. relational or expanded abstract), a student needs to think not only about more things at once, but also how things were related. Also, at the uni-structural level, students often use direct withdrawal information, but at the extended abstract level students must integrate potentially inconsistent ideas and must tolerate possible inconsistencies in all contexts. [11]. Therefore, we were interested in 1 person who was on the extended abstract level. We will present the interview results of the researcher (= Q) and a student who is at the level of extended abstract (= Nil).

Footage 1:
Q: What do you know about the basic concept in geometry?
Nil.01: ... yes I know ... it was points, lines and fields ...
Q: Why was it called the base concept?
Nil.02: Because it was a concept ... but it was not defined ... it was to avoid the spinning of definitions.
Q: Alright ... good ... then ... do you know a collection of axioms in geometry?
Nil. 03: I understand a number of incident axioms ... such as "through two different points there was exactly one line that contains it", ... "through three different points and not collinear, there was exactly one field that contains it." ... "If there were two points in a field V, the line containing the two points lies in the plane V. "..." The intersection of two fields forms a line. "..." The line contains at least two points and the field contains at least three points. "... there were many more ... but in essence that's the axiom ...
Q: Okay ... good ... you still remember ... what about non-Euclide Geometry?
Nil.04: ... the fundamental difference from geometry was originated from the axiom of parallel lines ... For Euclide geometry ... the axiom was "through one point outside the line g there was a line parallel to g" ... whereas for Non-Euclide geometry there were two groups ... First Lobachevsky Geometry and the other Riemman Geometry ... The basic statement of the two geometries was "through the point P outside the line g, there were two lines that were parallel to g" ... and the other "there were no parallel lines" ... the last was the basic statement for Riemman Geometry ...
Q: ... Okay ... next how do you complete the task about ... "At Lobachevsky Geometry, in Saccheri rectangles, the length of the upper segment length was longer than the lower segment"
...
Nil. 05: Yes ... I have done this on the assignment sheet ... (see Figure 2) ...
Q: Okay ... please explain ...
Nil.06: ... I illustrate the image of Saccheri $A_1B_1B_2A_2$ ... next to the ray A_1A_2 take point A3 with $(A_1A_2A_3)$, from A_3 pull the line $A_1A_2 \perp A_3B_3$ and $A_2B_2 \simeq A_3B_3$, obtained by the Saccheri rectangle $A^2B^2B^3A^3$, this process continues until $A_{n-1}B_{n-1}B_nA_n$, applies also $B_1B_2 \simeq B_2B_3 \simeq \cdots \simeq B_{n-1}B_n$ ...
Q: ... okay ...
Nil.07: With many angles summation $B_1B_2 + B_2B_3 + \ldots + B_{n-1}B_n \geq B_1B_n$… $(n-1) B_1B_2 \geq B_1B_n$ $\Rightarrow A_1A_n \leq A_1B_1 + (n-1)B_1B_2 + B_nA_n$…$A_1A_n \leq (n-1) B_1B_2 + 2A_1B_1$…. This applies to $n = 2$ ... . Then suppose $A_1A_2 > B_1B_2$, …. $A_1A_2- B_1B_2 > 0$… suppose that $A_1A_2-B_1B_2=k$, then $2A_1B_2=l$…... applies $A_1A_n = (n-1) A_1A_2$ ... then applies $(n-1)A_1A_2 \leq (n-1)B_1B_2+2A_1B_1 \Rightarrow (n-1) (A_1A_2 - B_1B_2) \leq 2A_1B_1$ consequently $(n-1) k \leq l$ ... this was contrary to the axiom of archemides ... it means that the presuppositions were wrong and must be $A_1A_2 \leq B_1B_2$… so it was proven…

Genetic decomposition of interview footage with **Nil** was consistent with the results of work on paper and pencil (see Figure 2 and Figure 3).

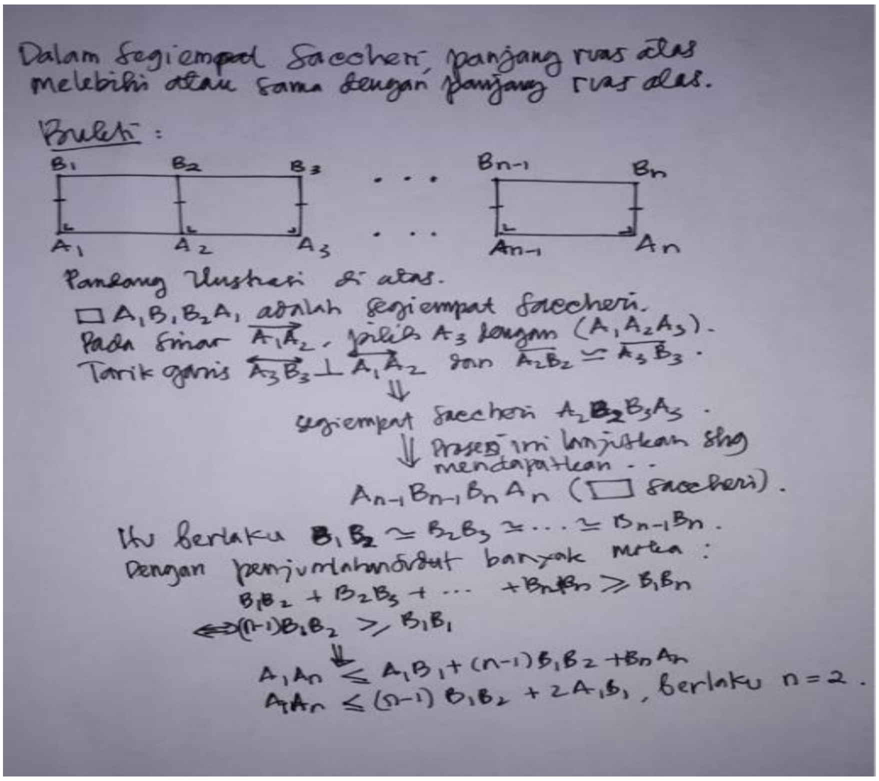

Figure 2. Nil proves Saccheri quadrilateral properties Part 1

Nil has a very complete understanding. He was able to connect precisely and logically every object that was needed. That was also a conceptual understanding. The **Nil** cognitive process reaches a mature scheme. He called back the schema in long-term memory and was processed through the ability of procedural understanding well. That was an achievement of metacognitive understanding. As such it was an understanding of cognition in general, as well as awareness and knowledge of one's own kongnis. **Nil** has strategic knowledge in the form of strategies for thinking, and problem solving. His abilities also appear in the form of contextual and conditional knowledge. Finally he has self-knowledge related to the strengths and weaknesses of self about cognition and learning [12]. Students with relational responses were able to identify and use the underlying conceptual structure. It was continued by extended abstract students who were able to build a general structure and show extensions outside the context given by the original[11].

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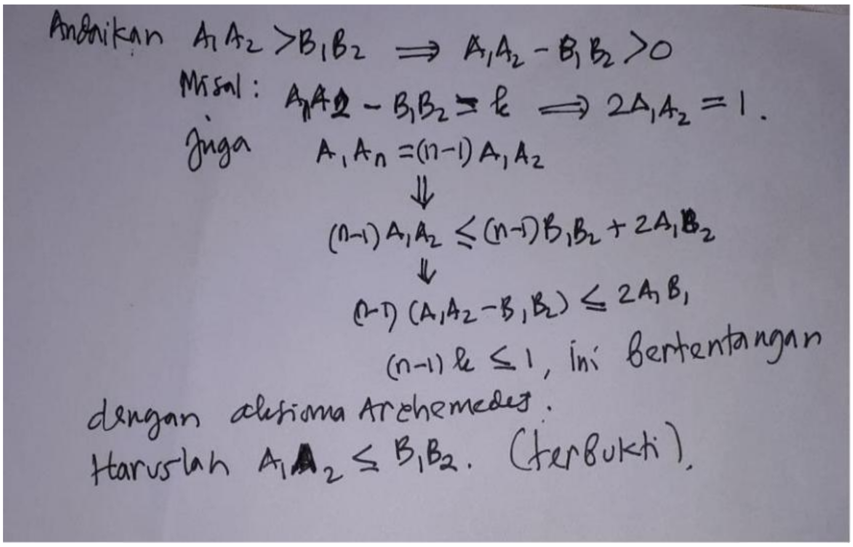
Andaikan $A_1A_2 > B_1B_2 \Rightarrow A_1A_2 - B_1B_2 > 0$
Misal: $A_1A_2 - B_1B_2 = k \Rightarrow 2A_1A_2 = 1$.
Juga $A_1A_n = (n-1)A_1A_2$
$\Downarrow$
$(n-1)A_1A_2 \le (n-1)B_1B_2 + 2A_1B_2$
$\Downarrow$
$(n-1)(A_1A_2 - B_1B_2) \le 2A_1B_1$
$(n-1)k \le 1$, ini bertentangan
dengan aksioma Archemedes.
Haruslah $A_1A_2 \le B_1B_2$. (terbukti).

Figure 3. Nil proves Saccheri quadrilateral properties Part 2

Footage 2:
Q: Next, try to explain the proof of the statement that "From the point P outside the line g, then there were not so many lines parallel to the line g" ...?
Nil.08: ... see the illustration in the picture (see Figure 4) that I have made ... (see Figure 3) ... It will be proven $\angle$ XPQ $\simeq$ $\angle$ YPQ ... to prove ... suppose $\angle$ YPQ> $\angle$ XPQ then there was a ray $\overrightarrow{PM}$ cut g in N and $\angle$ YPQ $\simeq$ $\angle$ MPQ, take the point K with (NQK) such $\overline{NQ} = \overline{KQ}$ then according to the axiom kekongruenan then $\triangle$ QNP $\triangle$ $\triangle$ QKP, it was then obtained $\angle$MPQ$\simeq$$\angle$KPQ$\simeq$$\angle$XPQ.
Q: Good ... continue ...
Nil.09: from my argument, it must be $\overline{PX}$ and $\overline{PK}$ coincide. However, this was impossible because $\overline{PX}$ does not cut g, so $\angle$ YPQ $\not>$ $\angle$ XPQ. In the same way it can be shown $\angle$ YPQ $\not<$$\angle$ XPQ to be correct $\angle$ XPQ $\simeq$ $\angle$ YPQ ...
Q: Okay ... next ...
Nil. 10: ... I will show that $\angle$ YPQ and $\angle$ XPQ were sharp angles ... This was also through contradiction ... Suppose that $\angle$ XPQ $\not<$$\angle$ YPQ right angle must have XPY in line, even though this was not; Suppose that $\angle$ XPQ and $\angle$ YPQ were blunt angles or more than 90°, then the line $\overleftrightarrow{TS}$ must be perpendicular to PQ in P loaded by d ($\angle$ XPY), this was not possible because $\overleftrightarrow{TS}$ was parallel to g.
Q: What was your conclusion?
Nil.11: it was that $\angle$ XPQ and $\angle$ YPQ were not blunt angles ... so the truth was that $\angle$ XPQ and $\angle$ YPQ were acute angles ... and consequently there were infinitely many lines through P that were parallel to g ... this was the end of the proof (see Figure 4).

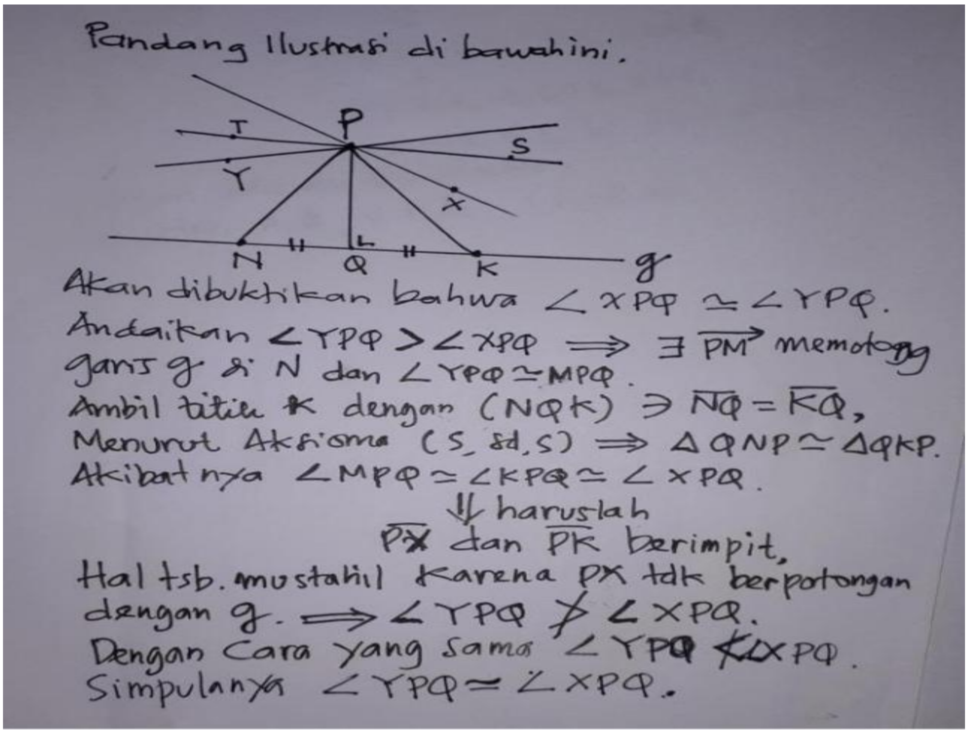

Figure 4. Nil proves the parallel nature of Lobachevsky's Geometry

Based on Footage 2, the **Nil** shows the ability of understanding factual, conceptual, procedural and metacognitive were very good. He was able to give more than one interpretation of an argument. It was an ability to associate integration between interpretations so as to form a new idea. He was able to carry out problem solving activities, provide an explanation of the solution correctly. The **Nil** justifies these solutions to construct new structures (i.e. from Euclide Geometry to Non-Euclide). He was able to demonstrate multidimensional thinking, and can connect with items outside the existing ones so that new ideas were formed. Based on genetic decomposition analysis, it can be concluded that the quality of student responses refers to the SOLO Taxonomy between abstract levels of relational and extended abstraction, therefore students were included in the abstract level classification. The results of other studies show that at the extended abstract level, which was the highest, students can generalize structures beyond what was given, can see structures from many different perspectives, and transfer ideas to new areas. They may have the competence to generalize, hypothesize, criticize, theorize, and so on [13]. The character of students who were at the abstract level were able to use all the statements given to solve the problem, can explain the relationship of statements given with arguments in solving problems, able to explain the usefulness of each statement used to solve the problem, as a result of proven statements, can explain statements that were compiled as a result of existing statements by using good arguments and drawing conclusions that have been made on paper and pencil, but have not been able to make evidence, and he tries to make new statements more than the original statement refers to existing statements, but failed to prove the truth [14]. Thus, at the extended abstract level, students were able to understand the deductive axiom system, and build a new system. Like, students were able to prove that through point T which was not located at line g, there were not so many lines parallel to g. He was able to compare the deductive structure of Lobachevsky's Geometry with Euclid's Geometry. Finally, extended abstract students were able to present several elements and pass interdependence between one another, so that it became an integrated entity. He generalizes to new structures.

Therefore, we need to convey that in order for students to reach the highest level it was necessary to implement a learning approach that was close to students' thoughts and culture. It was an ethnomathematics approach oriented to realistic mathematics (see [14][15][16][17]).

4. Conclusion

In this research we get students who have abilities in the extended abstract category. Students in the extended abstract provide several possible correct conclusions. Abstract principles were used to interpret concrete facts and appropriate responses that were separate from the context. This was done consistently so that it builds new structures.

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