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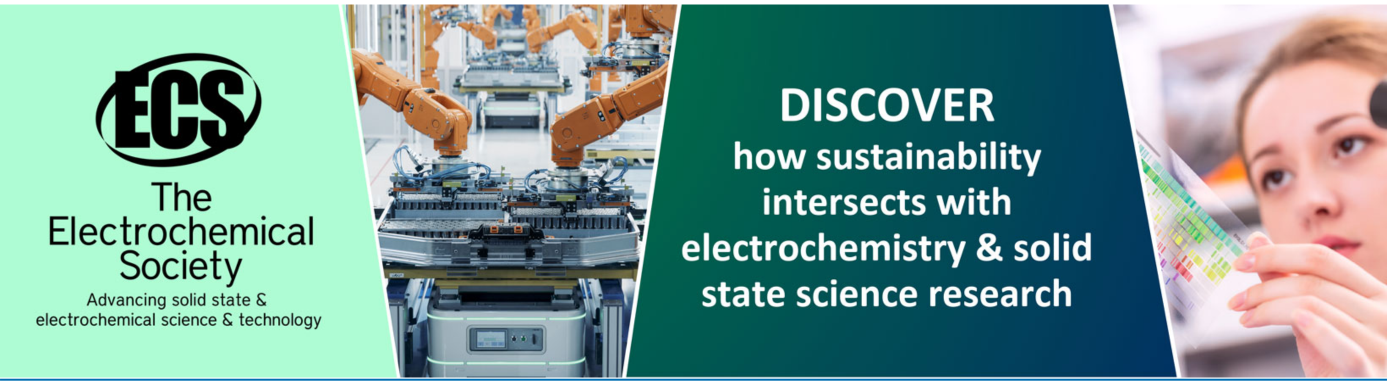

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The cognitive process of extended trans students in understanding the real number system

Dewi Herawaty, Wahyu Widada, Iran Sairan, Fizi Herdian, Khathibul U Z Nugroho and Abdurrobbil Falaq Dwi Anggoro

Universitas Bengkulu, Jl. W. R. Supratman, Kandang Limun, Muara Bangka Hulu, Bengkulu 38371

E-mail: dherawaty@unib.ac.id

Abstract. There were seven levels of student schema development in learning real analysis. Extended trans was the highest cognitive level. The purpose of this study was to describe the cognitive process of students in understanding the real number system. This was the initial part of development research. We want to know the highest initial ability students have about real number systems. The subjects of this study were 12 students of Real Analysis at the Bachelor of Mathematics Education Program. Students were interviewed based on assignments. Data were analyzed using fixed comparison techniques. The results of this study were that students can build linkages between actions, processes, objects, and other schemes (performing retrieval of the previous schema) by using and selecting procedures or operations in real number systems. He was able to apply the concept of the properties of bounded set to problem solving so that a mature scheme was formed. The scheme can be used to solve related problems. Also, can build new structures based on mature schemes that they already have. The conclusion of this study was that students can interiorize, encapsulate and thematize schemes that mature into a deductive structure.

1. Introduction

The cognitive process of students can be analyzed through the development of the scheme. According to Piaget & Garcia the development of the scheme was a process that was physical, and always changing. Knowledge grows based on certain mechanisms and includes three levels (intra level, inter level, and trans level), which occur in a fixed sequence and were called triads [1]. Triad level was a leveling theory about the development of one's scheme in accordance with the genetic decomposition they have. Genetic decomposition was a structured collection of cognitive processes carried out by someone in solving a mathematical problem by basing on the activities of action, processes, objects and schemes (APOS) [2][3][4]. APOS was a theory of how the possibility of learning a mathematical concept or principle can be used as an elaboration tool about mental construction of actions, process objects and schemes [5]. It was an activity that occurs in an information processing system.

Information processing system was a system of cognitive processes related to processing, storing, and recalling knowledge from the brain [6]. Therefore, any information that enters (through a sensory register) will be processed and will be stored in the long-term memory [7]. Students were active information processors, so they were able to represent each information according to the level of knowledge they have. It was a knowledge representation structure stored in memory [6]. Knowledge comes from action, and most cognitive development depends on how far the child actively manipulates and actively interacts with his environment [8]. Cognitive development was not an

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accumulation of separate pieces of information. It was the construction of a mental framework by students to understand their environment. Students have the freedom to build their own understanding [9]. Like lectures in real analysis, it requires high level cognitive processes. Students were required to think axiomatic deductively. It was studying very rigorous analysis and proof of [10].

Real analysis was one of the difficult subjects for most students. That resulted in them experiencing a concept error. Widgets & derivatives were not misinterpreting, not understanding points, difficulty using second derivatives and extreme points. Also, don't understand the conceptually about horizontal asymptotes [11]. Our experience in lectures, students have difficulties in proving theorems. Students also have difficulty arranging conjectures. That was what has happened so far in the lectures on Real Analysis that we have developed.

Students make conjectures, then proving them to be true statements was a very high level of cognitive processes [12]. That requires precision in mathematical thinking. Students must have a good previous scheme, and be able to process it properly. In real analysis learning, students were subjects of learning who must make use of schemes in their long-term memory logically. Information that enters long-term memory cannot be lost, it may only be temporarily forgotten. Also, it only loses the ability to rediscover information stored in memory [6]. Schemes stored in memory and their cognitive processes can be analyzed through genetic decomposition [13][14][15]. It was a collection of mental that can be classified into 3 levels, namely intra, inter, and trans [8].

In its development, Widada formulated the theory of schema development in five levels, namely intra, semi-inter, inter, semi-trans, and trans levels [5]. The theory continued with research in 2009, which produced six levels, namely pre-intra, intra, semi-inter, inter, semi-trans, and trans. Finally, there were 7 levels of scheme development. That was to produce the seventh level, namely the extended trans level [16]. It was a system in cognitive processes. According to Widada & Herawaty [11], it was a system of actions, processes, objects and other schemes related to a concept was a coordination carried out by individuals as a manifestation of their understanding of the concept. This system becomes coherent, that is, an individual will have meaning (either explicit or implicit) in the formal definition. He can solve problems about what was related to the mathematical concept. Thus, we were interested in discussing the cognitive processes of extended trans students in understanding real number systems.

2. Methods

This was the initial part of development research. It was the stage need assessment. The main activity was the task-based interview with the research subject. We want to know the highest initial ability students have about real number systems. The subject was chosen with a snowball technique. The subjects of this study were 12 students of Real Analysis at the Bachelor of Mathematics Education Program. The chosen subject was directly carried out the process of collecting data through task-based interviews. The main instrument in this study was the interviewer (in this case the researcher himself) and guided by another instrument in the form of a task sheet about the problems of the properties of real numbers. The problem given was: Analyze the following statement, "Let $S \neq \emptyset$, $S \subset \mathbf{R}$. $r \in \mathbf{R}$ **was** lower bound of S. r = inf S if and only if $\forall \varepsilon > 0$, $\exists s_\varepsilon \in S \ni r + \varepsilon > s_\varepsilon$. was that statement true? If it's true, then prove it, and if it's wrong, then give the counter example". Data were analyzed qualitatively by analyzing genetic decomposition, and Constant Comparative Technique.

3. Results and Discussion

The researcher conducted interviews with the subject of research based on assignments. The task was stated in a worksheet about the bounded sets. It was as follows: our subject asks to analyze a statement, Let $S \neq \emptyset$, $S \subset \mathbf{R}$. $r \in \mathbf{R}$ **was** lower bound of S. r = inf S if and only if $\forall \varepsilon > 0$, $\exists s_\varepsilon \in S \ni r + \varepsilon > s_\varepsilon$. We ask students to determine the truth value. They were asked to prove or look for a counter example. It depends on the value of its existence.

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Of the 12 students we interviewed in depth, we analyzed qualitatively. Next we do a constant comparison analysis. We will present one student (**Bik**.) Who has very good character. These characters were high cognitive processes. He was able to process a collection of statements in the form of actions, processes, objects and schemes carefully [2].

Bik provides solutions starting from paper and pencil activities. He based it into a logical activity in the form of interiorization, encapsulation and thematization of mathematical objects [5]. For more details, we present the results of paper and pencil and excerpts of interviews between researchers and **Bik**. See Figure 1 (solution from **Bik**.).

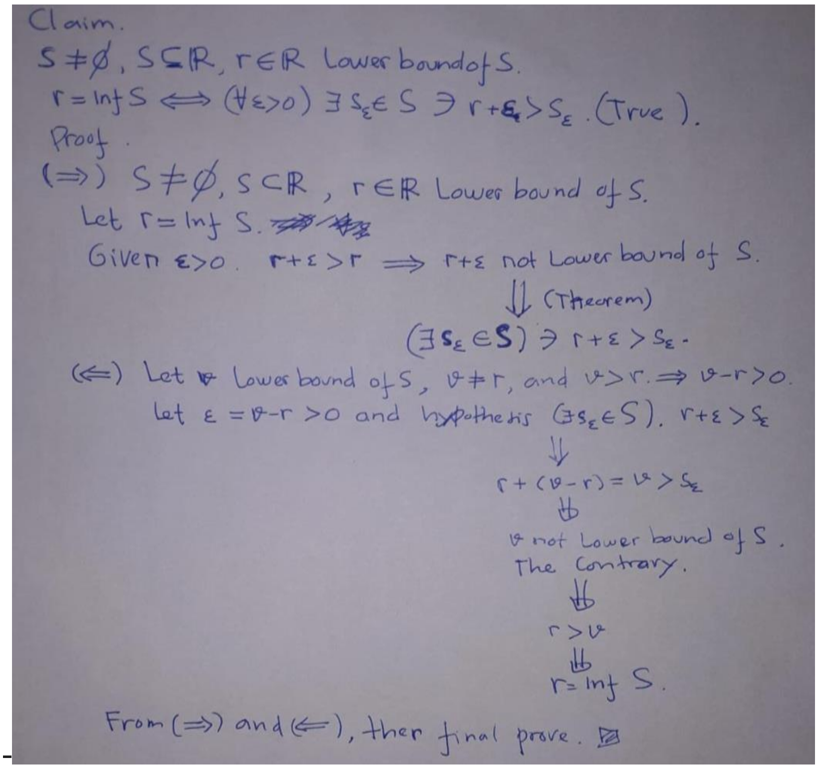
Claim.
$S \neq \emptyset$, $S \subseteq \mathbb{R}$, $r \in \mathbb{R}$ Lower bound of S.
$r = \inf S \Longleftrightarrow (\forall \varepsilon > 0)\ \exists s_\varepsilon \in S \ni r + \varepsilon > s_\varepsilon$. (True).
Proof.
$(\Rightarrow)$ $S \neq \emptyset$, $S \subset \mathbb{R}$, $r \in \mathbb{R}$ Lower bound of S.
Let $r = \inf S$.
Given $\varepsilon > 0$. $r + \varepsilon > r \Longrightarrow r + \varepsilon$ not Lower bound of S.
$\Downarrow$ (Theorem)
$(\exists s_\varepsilon \in S) \ni r + \varepsilon > s_\varepsilon$.
$(\Leftarrow)$ Let v Lower bound of S, $v \neq r$, and $v > r$. $\Rightarrow v - r > 0$.
Let $\varepsilon = v - r > 0$ and hypothesis $(\exists s_\varepsilon \in S)$, $r + \varepsilon > s_\varepsilon$
$\Downarrow$
$r + (v - r) = v > s_\varepsilon$
$\Downarrow$
v not Lower bound of S.
The Contrary.
$\Downarrow$
$r > v$
$\Downarrow$
$r = \inf S$.
From $(\Rightarrow)$ and $(\Leftarrow)$, then final prove. ☒

Figure 1. Solution of Bik

We conducted interviews in depth based on the tasks completed by **Bik**. That was we do it casually so as not to seem to interrogate. We believe that **Bik** expresses his cognitive process well, real and truly. That was a guarantee that the interview was valid. The interview footage between researchers (= Q) and research subjects (= **Bik**.) was as follows.

Q: What can you explain from the answers you have made.

Bik.01: ... based on my analysis, I claim that the statement was true ... then I tried to prove it.

Q: Try to explain the proof that you have compiled?

Bik.02: Because the statement was in the form of implications, then I prove two directions, namely the direction to the right and left. First, suppose that r a real number was infimum of S ... given ε was any positive real number.

Q: Why does ε have to be any real number?

Bik.03: Because to ensure that the statement applies to every positive real number ...

Q: Continue!!! ...

Bik.04: .. continue to be r + ε> r, ... which means that r + ε was not the lower bound of S ..., then there was s_ε at S so that r + ε> s_ε ...... proof to the right was complete ...

Q: Good ... what next?
Bik.05: ... I can explain proof of direction to the left, ... suppose v was the lower bound of S, with $v \neq r$. suppose $v > r$, mala $v-r > 0$ ... Let $\varepsilon = v-r > 0$ ... according to the hypothesis there was $s_\varepsilon \epsilon S$ so that $r+\varepsilon > s_\varepsilon$. Therefore, $r + (v-r) = v > s_\varepsilon$ ..., means v was not the lower limit of S. It was a contradiction ... it must be $r > v$. Thus $r = \inf S$. The proof to the right was also complete.
Q: What can you conclude?
Bik.06: ... because the evidence to the right and left was complete ... meaning that the claim was proven correct.
Q: What was the continuation of the true value statement?
Bik.07: ... I can make a new statement ... namely "For example, the set of parts of the set of real numbers was not empty, and S has a lower limit, then S has an infinite."
Q: were you sure this statement was true ...?
Bik.08: I ... I'm sure this was true ... because it must be trying to prove it ...

Interviews between researchers (Q) and research subjects (**Bik**) give meaning that the cognitive process of **Bik** was achieving the extended trans [17]. In the description of the interview, **Bik** began by interiorizing the actions into a logical process. The process was to analyze the problem, and produce a claim that the statement was true (**Bik**.01). He tried to do a further process to prove it. He tried these processes by encapsulating them into an object about the technique of verifying the statement of implications carried out with two implications (**Bik**.02).

The objects that he revealed, **Bik** thematized from various previous schemes that were called from his long-term memory. He was able to finish half the evidence correctly (**Bik**.04). It was a mature scheme that he shows and he wakes up in his cognitive process. Therefore, **Bik** reaches a level that exceeds the level of trans [17][14].

Bik shows his cognitive abilities well. He was doing cognitive processes by analyzing, recalling schemes and connecting them into a mature new scheme. He was able to prove the implications left to right (**Bik**.05). **Bik** was able to synthesize that proof to the right and left was complete, and this was proof of complete claim (**Bik**.06). It also shows that **Bik** was at a higher level than trans level [18][19].

The cognitive process carried out by **Bik** makes sure that he really exceeds the trans level. **Bik** was able to compile a new statement as one of the logical consequences of the claim that he has proven. The new statement he believes was true (**Bik**.07). He also competed to prove it. This gives the meaning that **Bik** was at the extended trans level [20].

Based on cognitive processes carried out by **Bik**. We conclude that **Bik** was at the highest level, and this was called the extended trans level [17][20]. Thus, this study gives meaning that students can build links between actions, processes, objects, and other schemes. They can perform retrieval from the previous schema by using and selecting procedures in real number systems. Students were able to apply the properties to problem solving so that mature themes were formed. It was a scheme that can be used to solve related problems. Also, can build new statements based on the mature scheme. Students can complicate, summarize and make mature schemes into deductive structures. Those were students in the extended trans level category.

4. Conclusion

We were conclude that this research was that the cognitive processes of students can be analyzed through mental activities that express through action-process-objects and schemes. That was in the form of genetic decomposition. Students were able to achieve the highest cognitive process, namely the extended trans level. He was able to associate mental objects in the information processing system precisely. He can make a new scheme that was mature. Finally, we concluded that the students reached the extended trans level.

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