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The ability of the civil engineering students to represent partial derivative symbols as metonymy and metaphor

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Abstract. Symbolic representation is a form of one's ideas interpretation to ease the problem-solving. The representation of symbols is an important thing in mathematical learning because it reflects the definition of a concept. Symbolic representation involves metonymy as the symbol scheme and metaphor as the denotation of the symbol. Metonymy and metaphor are related to each other and are involved in a symbolization system to discover new symbols in mathematics. The Civil Engineering students have to deal with a number of new symbols in Calculus II, such as the symbol of a function of two or more variables and its derivative. This symbol, needless to say, is nothing like the symbol of a function of one variable and its derivative which the students had learned in Calculus I offered in the previous semester. This study aimed to describe the ability of the Civil Engineering students to represent symbols as metonymy and metaphor through partial derivative problem-solving. This study was designed as a case study to detect the difficulties faced by the students in solving partial derivative problems. The students were provided with a worksheet containing partial derivative problems which focused on symbolic representation as metonymy and metaphor. The students' answers were analyzed based on how they denoted the partial derivative, both from the notation and the definition of the notation. Interview and observation were conducted to support worksheet findings. The results showed that the students had a poor ability in representing the partial derivative symbol of f with respect to x and y. The students' analogy was that the derivative symbol can be expressed with the d and prime notations such as in the derivative of functions of one variable. Therefore, this symbol was used in the partial derivative. The students were able to provide a correct solution to the partial derivative problems, but less careful in determining the partial derivative symbol. The error in using the partial derivative symbol was not noticed by the students because they focused more on the problem-solving procedure rather than the symbolization.

Keywords: representation, symbols, partial derivative, metonymy, metaphor.

1. Introduction

Mathematical representation is a form of mathematical ideas interpretation aimed to ease the problem-solving processes [1] [2]. The form can be presented through writing, figures, tables, graphs, and symbols [2] [3] [4]. An appropriate and meaningful use of mathematical representation is an effective approach to solving a problem in mathematics [5]. The form of individuals' mathematical representation demonstrates their mathematical thinking activities [6].

Mathematical representation is closely associated with mathematical symbols [7] [8]. Symbolic mathematical representation is expressed through mathematical statements or algebraic symbols [1].

The use of symbols is a determining factor in mathematical achievement. It reflects the definition of a concept. The process of using symbols in mathematics is related to mathematical achievement [9].

Symbolism involves metonymy as the symbol scheme and metaphor as the denotation of the symbol. Information is presented by special symbols which are in this case known as metonymy. A special form of analogy that often serves as the center of meaning construction is called a metaphor [10]. Metonymy and metaphor are related to each other, especially in the process of understanding similar mathematical symbols in different contexts [11]. Metonymy and metaphor are involved in a symbolization system to discover new symbols in mathematics [10] [11].

Research in the past has examined the involvement of metonymy and metaphor in the partial derivative symbolism. The inclusion of symbols as metonymy is indicated by the difference in the notation for a single function and multiple variables, d and ∂. In a function of one variable, $y = f(x)$, the first derivative is denoted by $\frac{dy}{dx}$. On the other hand, in a function of multiple variables, $z = f(x, y)$, the partial derivative of z with respect to x can be written as $\frac{\partial z}{\partial x}$. For instance, a function of multiple variables is known as $z = 5x + 3y$. The partial derivative of z with respect to x can be expressed as $\frac{\partial z}{\partial x} = 5$ instead of $\frac{dz}{dx} = 5$. As a matter of fact, there is no significant difference in the procedures to operate the derivatives of functions of multiple variables. However, based on the partial derivative concept, a partial derivative of a function of multiple variables is expressed as its partial derivative with respect to one of those independent variables. In the above example, $\frac{\partial z}{\partial x}$ is presented as a metaphor, the partial derivative of z with respect to x, holding the other variable constant. If $z = 5x + 3y$, while holding y constant and $dy = 0$, the derivative can be written as $\partial z = 5 \, \partial x$ or $\frac{\partial z}{\partial x} = 5$ [11].

Partial derivative is one of the mathematical concepts learned by the Civil Engineering students in the Calculus class. This concept is offered to the students who have studied a function of two or more variables. There are a lot of new symbols that have not yet been recognized by the students. The partial derivative symbol has a different denotation compared to the derivative symbol of functions of one variable. For instance, the partial derivative of f with respect to x is denoted as $f_x(x, y)$ while the partial derivative of f with respect to y can be expressed as $f_y(x, y)$. Enhancing the ability to operate these new symbols, either as metonymy or metaphor, should become an important part of mathematical learning at the college so that the students can understand the correct and proper use of the symbols.

Studies on mathematical representation need to be conducted to the Civil Engineering students. An analysis of the students' mathematical representation can help detect their difficulties in studying mathematics [5]. Research has reported issues faced by the Civil Engineering students in figuring out the derivatives of algebraic functions. These problems include the students' inability to understand the procedure of algebraic function derivatives, inability to apply mathematical operations, and inability to interpret the problems [12]. Therefore, an in-depth analysis of the Civil Engineering students' ability to represent the partial derivative symbol as metonymy and metaphor needs to be conducted.

2. Research Method

This study was designed as a case study to detect the difficulties faced by the college students in understanding Partial Derivative and to provide insights for the lecturer on the improvement of learning the correct use of mathematical symbols. The subjects of this study were the Civil Engineering students who were enrolled in the Calculus II class. This study was carried out in six stages: (1) identified the problem, (2) gathered information through library study, (3) determined the objective of the study, (4) collected the data, (5) analyzed and interpreted the data, and (6) reported and evaluated the results. The data of this study were obtained through worksheet assignment, interview, and observation. The indicators of the subjects' symbolic representation analysis are presented in Table 1.

Table 1. Aspects and Indicators of Symbolic Representation

Aspects of Symbolic Representation	Indicators of Symbolic Representation

Metonymy (the marking of the symbol)	Writes down the symbol of the partial derivative of $f(x,y)$ with respect to x and y
	Provides an example of a function of multiple variables and determines its partial derivative
Metaphor (the denotation of the symbol)	Explains the result of the derivative symbol categorization
	Determines the partial derivative of the exponential function using the chain rule

Worksheet of partial derivative characterized by symbolic representation was developed to obtain symbol representation as metonymy dan metaphor. The interview and observation were performed to support findings on the subjects' problem-solving. The interview was semi-structured where the questions were adjusted to the condition of the field. The subjects' representation of symbols as metonymy and metaphor in solving partial derivatives was also observed.

The data of this study, thus, were in the form of the analysis of the students' answers on the worksheet, field notes written on the observation sheets, and the subjects' interview recordings. A qualitative analysis by [13] was adopted to examine the data. The steps of the data analysis consisted of 1) preparation and organization of data for analysis, 2) data coding, 3) representation of the findings, 4) interpretation of the findings, and 5) data triangulation. Students' answers on the worksheet were analyzed based on (1) the partial derivative symbolization and (2) the denotation of the partial derivative symbol through problem-solving. The interview recordings were transcribed and presented aligned with the result of the observation.

3. Results and Discussion

3.1. Worksheet Characterized by Symbolic Representation
Worksheet characterized by symbolic representations was developed to obtain data of symbol representation as metonymy and metaphor. The material on the worksheet is Partial Derivatives. The worksheet contains space for writing summaries based on references that have been read by students, partial derivative exercises focused on mathematical symbols, and reflection questions to test student competency.

The summary on this worksheet is filled by resolving the partial derivative problems in general. The problems given include writing symbols of the partial derivative of the function $f(x,y)$ with respect to x and y, writing the difference of the symbol for the function of one variable and function of multiple variables, giving examples of functions of two variables and determine its partial derivative.

The partial derivative exercises that focus on mathematical symbols contain the identification of examples and non-examples of the partial derivative symbols and give their reasons, determine the partial derivative of a function of multiple variables and function values with the correct symbols, and determine partial derivative using the chain rule with the correct symbols, and determine partial derivative using the substitution method with the correct symbols.

The reflection questions contain analysis and evaluation questions to investigate the similarity of the partial derivatives $f(x,y)$ with respect to x and y by using the correct symbols. The second problem is determining the function value of the partial derivative in certain value pairs of x and y with the correct symbols. The third problem is identifying the similarity of partial derivative functions with respect to x and y by using the chain rule.

3.1.1. Representation of the Partial Derivative Symbol as Metonymy
In the first problem, the students were seemingly unable to represent the partial derivative symbol. Various notations were expressed for the partial derivatives. The partial derivative of $f(x,y)$ with respect to x was written as $\frac{\partial f}{\partial x}, \frac{\partial f(x,y)}{\partial x}, f_x(x,y), \frac{\partial z}{\partial x}, z_x$, and $\frac{df}{dx}$. Similarly, the partial derivative of $f(x,y)$ with respect to y was denoted as $\frac{\partial f}{\partial y}, \frac{\partial f(x,y)}{\partial y}, f_y(x,y), \frac{\partial z}{\partial y}, z_y$, and $\frac{df}{dy}$. This indicates that the

students were quite familiar with a variety of partial derivative symbols, but failed to denote the symbol. One of the examples of the students' answers can be seen in Figure 1.
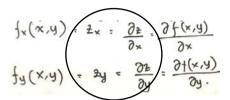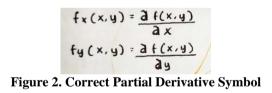

Figure 1. Incorrect Partial Derivative Symbol Using New Variables

In Figure 1, the student added the equals sign (=) in order to emphasize the equal position of the four partial derivatives. It can be interpreted that the four symbols represented the partial derivatives of the function. Unfortunately, an initial assumption that $f(x, y) = z$ did not exist in the student's answer. Instead, the student involved a new notation z to express the partial derivative symbol. The use of $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ could not represent the partial derivative of $f(x, y)$, if there was no assumption that $z = f(x, y)$. Likewise, notations z_x and z_y would not be able to represent the partial derivative of the function. Therefore, either the derivative symbol of $f_x(x, y) = z_x = \frac{\partial z}{\partial x} = \frac{\partial f(x,y)}{\partial x}$ or $f_y(x, y) = z_y = \frac{\partial z}{\partial y} = \frac{\partial f(x,y)}{\partial y}$ was incorrect in such a case. The analysis of symbolic representation as metonymy suggested that the students paid less attention to the importance of making an assumption and using arithmetic operation symbols. If $f(x, y)$, was given, then the partial derivative of $f(x, y)$ with respect to x can be expressed as $f_x(x, y)$ or $\frac{\partial f(x,y)}{\partial x}$. After reading the references related to the topic, the students seemed to understand that $f_x(x, y), \frac{\partial f(x,y)}{\partial x}$, or $\frac{\partial z}{\partial x}$ could be used to denote the partial derivative of the function. However, the significance of the assumption $z = f(x, y)$ was ignored by the students.

Another example of the students' answers which showed their ability to write the derivative symbol correctly is presented in Figure 2.

Figure 2. Correct Partial Derivative Symbol

The student in Figure 2 was able to represent the partial derivative of $f(x, y)$ with respect to x and y. This ability reflected the student's comprehension of the references. In one of the references, the partial derivative symbol was expressed as follows:

"*If $z = f(x, y)$, we employ the following symbols to denote the partial derivative*

$$f_x(x, y) = \frac{\partial z}{\partial x} = \frac{\partial f(x,y)}{\partial x} \text{ and } f_y(x, y) = \frac{\partial z}{\partial y} = \frac{\partial f(x,y)}{\partial y}"$$ [14].

Therefore, the student used notations $f_x(x, y) = \frac{\partial f(x,y)}{\partial x}$ and $f_y(x, y) = \frac{\partial f(x,y)}{\partial y}$ to represent the partial derivative of the function.

The ability to represent symbols as metonymy is highly crucial for college students as it constitutes the form of their mathematical ideas interpretation in solving a problem. In one of the students' answers, we found that the partial derivative was expressed by similar notations to express the derivative of functions of one variable, such as seen in Figure 3.
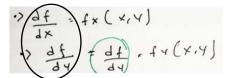
Figure 3. Incorrect Partial Derivative Symbol Using Notation d

The student in Figure 3 wrote $\frac{df}{dx}$ and $\frac{df}{dy}$ to signify the partial derivative. The result of the observation indicated that the students completed the worksheet with the assumption that there was no significant difference between the derivative of functions of one variable and functions of multiple variables. They used notation d as an analogy of the derivative, such as in functions of one variable, so that this notation is also used to denote the partial derivative.

The second problem assigned the students to provide an example of a two-variable function and its partial derivative. The students' answers to this problem were categorized into four types of solution which consisted of: (1) a correct solution and correct symbolic representation, (2) a correct solution but incorrect symbolic representation, (3) an incorrect solution but correct symbolic representation, and (4) an incorrect solution and incorrect symbolic representation.

The incorrect symbolic representation in the first problem had affected the solution offered by the students to the second problem. An example of the correct solution and incorrect symbolic representation is displayed in Figure 4.

Figure 4. A Correct Solution but Incorrect Symbolic Representation

In Figure 4, the student showed a correct example of a two-variable function and its partial derivative. However, the use of notations $\frac{df}{dx}$ and $\frac{df}{dy}$ for the partial derivative of $f(x,y)$ with respect to x and y was considered erroneous. The student did not recognize his error in writing an incorrect partial derivative symbol. The student put more focus on the final result of the partial derivative; thus, ignored the correct symbolic representation of the derivative. Quite contrary, other students showed correct symbolic representation but an incorrect solution due to the inaccuracy in carrying out the partial derivative procedures. Another finding suggested that a student wrote $f'(x,y)$ as a notation to represent the partial derivative to generate the following result (Figure 5).

Figure 5. A Function of Multiple Variables and Its Partial Derivative

This student assumed that functions of multiple variables have the same rules as functions of one variable. The derivative was expressed with the prime symbol and the derivative procedure was carried out based on each variable. Therefore, the student denoted the derivative of x^2 as $2x$ and the derivative of y^2 and $2y$.

3.2. Representation of the Partial Derivative Symbol as Metaphor

The first problem asked the students to explain the classification of five derivative symbols, that are: $g_x(x, y, z), D_x, \frac{\partial k}{\partial a}, f_y(x, y),$ and $\frac{dz}{dx}$. Then, the students were assigned to categorize the symbols into two categories: the symbols that can represent partial derivatives and the symbols that cannot represent partial derivatives. The students also had to explain why they made such categorization. The two major findings were that: (1) the classification of the symbols was correct and the reasons were appropriate, and (2) the classification of the symbols was incorrect and the reasons were inappropriate.

Some students were able to correctly classify $g_x(x, y, z), \frac{\partial k}{\partial a}, f_y(x, y)$ as the partial derivative symbols and $D_x, \frac{dz}{dx}$ as the derivative of functions of one variable. The students' explanations related to the partial derivative symbols contained index x or y such as f_y, using ∂ and containing more than one variables. The students expressed the partial derivative symbol as a partial derivative of one of the variables denoted in the index. The students used ∂ to distinguish partial differentiation of functions of several variables. There were students who provided incorrect differentiation index of the partial derivative. The symbol D_x was wrongly interpreted as the partial derivative of functions of multiple variables with respect to x. This happened because the students noticed index x on notation D_x, so that they made wrong judgement in classifying the derivative symbols.

In the second problem, the students had to determine the partial derivative of the exponential function using the chain rule. Given $f(x, y) = (3xy - 2y)^3$, the students were asked to figure out $\frac{\partial f(x,y)}{\partial x}$ and $\frac{\partial f(x,y)}{\partial y}$. The students wrote $\frac{\partial f(x,y)}{\partial x}$ as the partial derivative with respect to x and $\frac{\partial f(x,y)}{\partial y}$ as the partial derivative with respect to y. The partial derivative of $f(x, y)$ with respect to x was expressed as a partial derivative with respect to x, holding y constant, and vice versa. The students applied the chain rule to the derivative of functions of one variable to determine the partial derivative of the exponential function. However, there was an incorrect procedure shown in the process of simplifying the result of the derivative, such as presented in Figure 6.

Figure 6. Incorrect Procedure in Simplifying the Partial Derivative of the Exponential Function Using the Chain Rule

Figure 6 suggested that there was no significant difference between the chain rule applied for the partial derivative of one-variable exponential functions and the chain rule applied for two-variable exponential functions. The difference only lay on the partial derivative of $3xy - 2y$ with respect to x or y. In Figure 6, the student explained the steps of finding the partial derivative of the exponential function with respect to x as follows: (1) figured out the third derivative and multiplied it with $(3xy - 2y)^{3-1}$, (2) found the differentiation of algebraic function $3xy - 2y$ with respect to x, (3) performed the multiplications at stage 1 and stage 3, (4) simplified the result. Similar steps were applied to determine the partial derivative of the exponential function with respect to y, only the differentiation of algebraic function $3xy - 2y$ was done with respect to y. An error made by the student in simplifying the result of the derivative (marked on Figure 6) was due to his clumsiness in using algebraic operations.

In addition, there was a student who made an assumption to represent the chain rule of a two-variable exponential function, such as shown in Figure 7. The purpose of making the assumption was to ease the process of solving the partial derivative of the two-variable exponential function.

Figure 7. The Partial Derivative of the Exponential Function Using the Chain Rule

In Figure 7, the student represented the use of the chain rule by making an initial assumption, $u = 3xy - 2y$. The error in using symbols as metonymy was reoccurring when the student used the prime notation to express the partial derivative symbol. However, the student reclassified u' by using $u_x(x, y)$ and $u_y(x, y)$. Then, the student wrote $\frac{\partial f(x,y)}{\partial x}$ as the partial derivative of f with respect to x and y using the correct chain rule procedure. The incorrect use of the prime notation or the incorrect use of d was mostly found in the students' answers because the students made an analogy that the derivative of a function could be represented by the prime or d notations.

4. Conclusions

Symbolic representation as metonymy and metaphor is obtained based on the analysis results of the worksheet characterized by symbolic representations. The findings of the study indicate the students' lack of ability to represent derivative symbols as metonymy due to their poor understanding of the significance of writing correct mathematical symbols. The students assumed that the final solution was far more important than the mathematical symbols used in the problem-solving. However, the results showed that the students were able to represent the partial derivative symbol as a metaphor. The students were able to understand that the partial derivative of $f(x, y)$ with respect to x could be denoted through the partial derivative procedure with respect to x, holding y as constant. Likewise, the partial derivative of a function with respect to y can be expressed by holding x as constant. In addition, error in representing partial derivative as metonymy and error in doing algebraic operations were still made by the students.

5. Suggestion

It is highly recommended for the future researchers to consider developing teaching materials that contain topics around mathematical symbolic representation so that the students can understand the correct use of mathematical symbols as metonymy and metaphor.

References

[1]   Astuti E P 2017 Representasi Matematis Mahasiswa Calon Guru dalam Menyelesaikan Masalah Matematika *Jurnal Tadris Matematika* **10** 70-82

[2]   Sabirin M 2014 Representasi dalam Pembelajaran Matematika *Jurnal Pendidikan Matematika* **1** 33-44

[3]   Tajudin N M and Chinnappan M 2016 The Link between Higher Order Thinking Skills, Representation, and Concepts in Enchancing TIMMS Tasks *International Journal of Instruction* **9** 199-214

[4]   Lestari K E and Yudhanegara M R 2017 Analisis Kemampuan Representasi Matematis

Mahasiswa pada Mata Kuliah Geometri Transformasi Berdasarkan Latar Belakang Pendidikan Menengah *Jurnal Matematika Integratif* **13** 29-34

[5] Jitendra A K, Nelson G, Pulles S M, Kiss A J and Houseworth J 2016 Is Mathematical Representation of Problems an Evidence-Based Strategy for Students with Mathematics Difficulties? *Exceptional Children* **83** 8-25

[6] Hutagaol K 2013 Pembelajaran Kontekstual untuk Meningkatkan Kemampuan Representasi Matematis Siswa Sekolah Menengah Pertama *Jurnal Infinity* **2** 85-99, 2013

[7] Stalnaker D and Zanibbi R 2015 Math Expression Retrieval Using an Inverted Index Over Simbols Pairs *Recognition and Retrieval XXII* (San Fransisco: International Society for Optics and Photonics)

[8] Leibovich T and Ansari D 2016 The Simbol-Grounding Problem in Numerical Cognition: A Review of Theory, Evidence, and Outstanding Questions *Canadian Journal of Experimental Psychology* **70** 12-23

[9] Reynvoet B and Sasanguie D 2016 The Simbol Grounding Problem Revisited: A Thorough Evaluation of The ANS Mapping Account and The Proposal of An Alternative Account Based on Simbol-Simbol Associations *Frontiers in Psychology* **7** 1581

[10] Presmeg N C 2013 Reasoning with Metaphors and Metonymies in Mathematics Learning *Mathematical Reasoning* 275-288

[11] Malviya S 2019 Simbol as Metonymy and Metaphor: A Sociological Perspective on Mathematical Simbolism *Science, Technology, & Society* **24** 53-72

[12] Hasanah H 2019 Analisis Kesulitan Mahasiswa dalam Menyelesaikan Soal Turunan Fungsi Aljabar *Jurnal Industri dan Teknologi Terpadu* **2** 76-84

[13] Creswell J 2015 *Riset Pendidikan: Perencanaan, Pelaksanaan, dan Evaluasi Riset Kualitatif dan Kuantitatif* (Yogyakarta: Pustaka Pelajar)

[14] Varberg D, Purcell E and Rigdon S 2007 *Calculus, 9th Edition* (Upper Saddle River, NJ: Prentice Hall)