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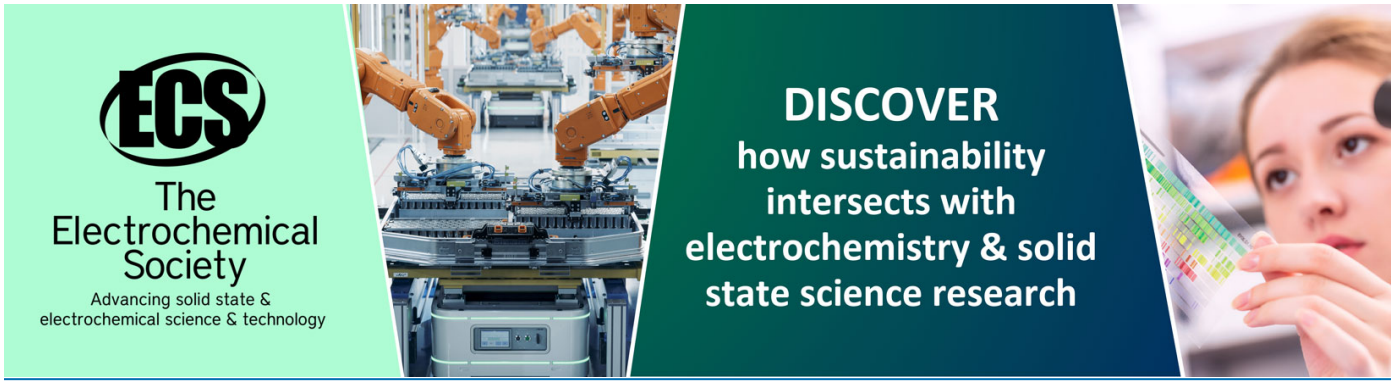

Scaffolding process based on students diagnostic difficulties in proving group problems by using mathematics mapping

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Abstract. This study aims to describe the scaffolding process based on the diagnostic difficulties of students in proving group problems using mathematics mapping. To achieve that goal, an exploratory qualitative study was conducted, with the subject of research being Mathematics Education Students at Universitas PRGI Ronggolawe Tuban. The scaffolding process refers to the level of scaffolding developed by [19], namely explaining, reviewing and restructuring, and developing conceptual thinking. The diagnostic difficulties in proving group problems are reviewed based on the first step of the proof, the path of proof, related concepts, arguments, key expressions, and proof languages. Based on the results of the analysis, it can be concluded that: 1) Difficulties found in the initial stages of proof, and the flow of proof, scaffolding provided is explaining, reviewing, and restructuring; 2) Difficulties in related concepts and proof arguments, the scaffolding provided is explaining, reviewing, restructuring, developing conceptual thinking, and making connections; 3) Difficulties in key expressions, the scaffolding provided is reviewing, restructuring; 4) Difficulties in the language of proof, the scaffolding can be given by using a review.

1. Introduction

Rav in [1] says that proof is the way mathematicians show mathematical tools to solve problems and justify that the proposed solution to the problem is indeed a solution. According to [1], the problem of proof does not only function to show a mathematical statement that is true or false but has a broader meaning, for example, verification, confirmation, systematization, inquiry, communication, and exploration. [2] says that writing mathematical proof will help students understand mathematical objects. Reid in [3] explains that training students to write mathematical proofs will develop their deductive reasoning. Likewise, according to [4], it proves that "complex mathematical activities are with logical, conceptual, social and problem-solving dimensions."

Constructing proof is recognized as an important component in mathematics education. [5] explain that proof is an important part of mathematics because the proof is the daily academic practice of mathematicians. This is also important for mathematics educators because proof involves reasoning, beliefs, and communication and helps meaningful learning. Proof can be used to show students who understand and do mathematics means more than just learning to execute certain procedures. According to [6], proof is an important part of mathematics itself, so it must be discussed with students the proof function in mathematics, other aspects both important and limitations. However, in the classroom, the key role of proof is to convey mathematical understanding, and the importance of the challenge is to find more effective ways of using proof for this purpose.

The important role of proof in mathematics and mathematics education is very clear, but several studies show students tend to have difficulty in constructing proof. The results of the research by [7]

showed that children aged 14-15 years had difficulty in constructing complete evidence-based on deductive reasoning. Previous research has shown that many prospective elementary school teachers face difficulties in understanding the differences of proof, invalid general arguments, and empirical arguments [8]. The proof is a complex mathematical activity, so it becomes important in mathematics learning. Difficulties arise in teaching proof to students in classrooms. As many studies show [8-11], students, in all aspects of proof, have poor understanding and difficulties in construct their proof.

Referring to several opinions on the construction of proof for its importance in mathematics and mathematics education, until the present the problem of proof is still a scourge for students. This can be seen from the results of several studies showing that there is a tendency to construct a proof. For that, a solution needs to be sought to overcome the difficulties of students in learning to prove. In this paper, the authors have an idea to provide a solution to the difficulty of studying the roof through scaffolding.

Scaffolding can be defined as "the process that enables a child to solve a problem, carry out a task, or achieve a goal which would be beyond his unassisted efforts" [12]. They believe that the acquisition of a child's skills is an activity in which relevant skills are ready to be combined and 'bent' into 'higher skills' to fulfill the requirements of new, more complex tasks. This activity can only be successful through a tutor's intervention, which will produce more than just modeling and imitation. The same is explained by [13] that the concept of scaffolding is used to define and explain the role of adults or peers who are better and able to support children's learning and development. [14] also said that scaffolding is a mechanism for observing student processes that are helped to achieve their learning potential.

Why is scaffolding chosen as a solution to overcome difficulties in proving problems? According to [15], the application of scaffolding is useful to enrich students' understanding of mathematical proof. Likewise, [15] outlines several reasons for the success of the scaffolding process: 1) this process can motivate problem-solving procedures for students, 2) the scaffolding method will improve students' current social capacity and relationships, 3) the scaffolding process will increase students 'confidence in problem-solving math problems, and 4) this method of education can show students' mistakes and misunderstandings in the settlement procedure. The scaffolding process according to [16] is to break complex tasks into smaller, more manageable parts that slowly increase cognitive complexity to form a cohesive whole.

To provide scaffolding that is by student difficulties in proving, it is necessary to use mapping. Mapping is done between the difficulties of students in proving the scaffolding given. According to [17], the practice of thematic analysis using mathematics mapping can help anticipate difficulties in the process of planning and curriculum development. Diagnosis by using Mathematics mapping is needed to find where students have difficulties in solving mathematical problems accurately. Based on the abovementioned consideration, the problem is how to describe the scaffolding process based on the diagnostic difficulties of students in proving Group problems using Mapping Mathematics?

2. Research Method

This study aims to describe the scaffolding process based on the diagnostic difficulties of students in proving group problems by using mathematics mapping. To obtain a description of the difficulties that students have in proving Group problems, the first step is giving the subjects a test of proof of the group. Then, to explore students' difficulties in proving the problem of the Group, it is explored by task-based interviews to the research subjects. For this reason, this type of research is qualitative exploration. The research subjects were 34 students of the 2016 Mathematics Education Study Program. Meanwhile, the subjects that would be interviewed were 6 students (BN, AN, LR, EA, PI, CK).

In exploring the construction of proof, it can be determined by six categories, namely the first step of the proof, the flow of proof, related concepts, arguments, key expressions, and proof languages [18]. The scaffolding process refers to the level of scaffolding developed by [19]. Then, on level 2,

there are some steps namely: explaining, reviewing, and restructuring and level 3 includes developing conceptual thinking, making connections, and generating conceptual discourses.

The problem of proving a group contains a non-empty set and an operation. To diagnose student difficulties in proving group problems, some instruments are used in five problems, including: 1) group problems with unmodified (defined) set objects and operations; 2) group problems with unmodified set objects, but modified operations; 3) group problems with set objects modified, but operations are not modified; 4) group problems with set objects and modified operations; and 5) group problems with finite set objects and modified operations.

To determine the validity of the data, it was done by using triangulation method. The data is said to fulfill the validity of the data (valid data) if the data from the answers to the written test are the same as the data from the interview. Furthermore, valid data was analyzed to obtain conclusions from the research results. Furthermore, the data was analyzed qualitatively, carried out by working with data, organizing data, reducing it to manageable units, synthesizing it, finding and finding patterns, discovering what is important and what is learned, and deciding what can be told to others [20]. Referring to the opinion there are six stages of qualitative data analysis carried out, namely: (1) reviewing all data, (2) reducing data, (3) compiling in units, (4) categorizing data, (5) coding, and (6) checking the data

3. Results and Discussion

The scaffolding process used in this study is a form of scaffolding at level two and three of scaffolding proposed by [19]. The intended levels include: Level two, namely: a) explaining, includes interaction showing and telling; b) reviewing provides for the interaction of interpreting students' actions, looking, touching and verbalizing, prompting and probing, parallel modeling, explain and justify; c) restructuring includes the interaction of meaningful contexts, simplifying the problem, rephrasing students' talk, and negotiating meanings. At level three, namely: developing conceptual thinking, making connections and generating conceptual discourse.

The following explanation presents the scaffolding process on each indicator of proof, including the first step of the proof, the flow of evidence, related concepts, arguments, key expressions, and proof language based on difficulties in proving group problems.

3.1 The First Step of Proof

In proving a group, there are 4 (four) axioms that must be proven, including closed nature, associative nature, the existence of identity elements (neutral elements), and inverse properties. Each axiom has a different definition. Closed and associative axioms, the definition begins with "for each ...", the meaning must apply to each element. For the axiom of the existence of elements of identity, the definition begins with "there are/exist ...", meaning that the existence of elements of identity is single. And in the inverse axiom, the definition begins with "for every ..., there are/exists ...", meaning that every element has a single inverse. Different definitions of each axiom affects that each trait has a different initial step of the proof.

In the initial step of the proof, the students tend to experience more difficulties in proving the existence of identity elements and inverse properties compared to closed or associative properties. Student difficulties appear when proving statements (definitions) beginning with " there are/exist..." or "for every ..., there are/exist...". Besides, difficulties in the initial steps of the proof are also caused by obstacles in understanding the problem to be proven.

Scaffolding for the first step of proof, especially proving statements (definitions) beginning with "there are/exist..." or "for each ..., there are/exist...", the first step is to explain the ideas of the initial steps of proof when statement (definition) starts with "there are /exists ..." or "for every ..., there are/exist ..." level 2 scaffolding, which is ***explaining*** (showing and telling). Furthermore, scaffolding ***reviewing*** identifies the aspects that are most related to implicit mathematical ideas or problems that must be solved. In the ***reviewing step***, specifically using interaction through *using prompting and probing questions, parallel modeling and getting students to explain and justify*. Provide instructions through questions that lead and emphasize the direction of student understanding, then provide a

model that can be used for the initial step of the proof. To find out the proof of understanding written, students are asked to explain and justify.

For difficulties in understanding the problem, scaffolding of ***explaining*** (showing and telling) is given by showing the problem to be proven. Scaffolding of ***reviewing*** specifically uses interactions through *using prompting, probing questions and getting students to explain and justify*, to encourage student awareness in understanding problems. Besides, specifically for scaffolding of ***restructuring*** the interaction is *meaningful contexts and simplifying the problem*.

3.2 Flow of Proof.

The system of proof writing is an important aspect, because if the plot of evidence is arranged systematically, the proven traits will be evident. The proof line is related to the sequence of proof. Some student difficulties in the path of proof lie in understanding the problem that will be proven. This is in line with [21] that students still experience difficulties in constructing, understanding and validating proof. Group problems contain a non-empty set as an object and an operation. Student difficulties are found when the given set object is modified, as well as the defined operation. The search results for the subject of difficulty are caused by a lack of understanding of students in defined sets or operations.

Scaffolding difficulties in understanding the problem results in constraints on the flow of proof as well as the difficulty of the initial step of the proof. That is, given scaffolding of ***explaining*** (showing and telling), by showing and telling the problem to be proven, this opens up an idea in the path of proof. Scaffolding of ***reviewing*** specifically uses interactions using prompting and probing questions and getting students to explain and justify, to encourage student awareness in quickly finding the appropriate proof path. Besides, for scaffolding of ***restructuring*** specifically the interaction of meaningful contexts, and simplifying the problem, this helps students easily understand the problem.

For student difficulties in dealing with set objects and defined operations, it is due to lack of understanding defined. For this reason, scaffolding is provided through ***Reviewing***, specifically, the interaction of interpreting students' actions, prompting and probing, parallel modeling, explain and justify. It is necessary to listen to the student's interpretation of what is defined, then directed through the questions, continue to be given a model whether right or wrong, finally students are asked to explain. Besides, given the scaffolding of ***restructuring***, in particular, the interaction of meaningful contexts and simplifying the problem, this helps students easily understand the object or operation defined.

3.3 Related Concepts

Related concepts is very important step because it relates to Mathematical connection skills. That is the ability of students to link previous mathematical concepts, which support in the evidence. In related concepts, in general, the difficulties of students are two: a) initial knowledge possessed; b) connect definitions, theorems, entries, or questions that have been proven before and relate to the evidence faced. The results in proof that it tends to be long-winded (repetitive), not even found. The ability to construct proof according to [22] includes the ability to use methods of proof, definition, lemma, and theorem to show the truth of a statement in mathematics. [23] explains that the main cause of student failure in constructing a proof is lack strategic knowledge of how to choose facts and theorems to be applied.

In the difficulties of initial knowledge of students, scaffolding of ***explaining*** can be provided by showing and telling concepts that have not been understood. Furthermore, to strengthen students, use scaffolding of ***reviewing*** with interpreting students' interactions actions, looking, touching and verbalizing, prompting and probing, parallel modeling, explaining and justifying. For student difficulties in connecting definitions, theorems, entries, or questions that have been proven before and related to the evidence faced, scaffolding of ***restructuring*** is used specifically the interaction of meaningful contexts, simplifying the problem, rephrasing students' talk. Through scaffolding restructuring, students are helped, because definitions, theorems, entries, or previously proven

questions have been simplified, and provided contexts that are meaningful for abstract situations exist. Besides, ***level 3 scaffolding***, namely *developing conceptual thinking and making connections* help students grow the idea of connection with proven problems.

3.4 Proof Argument

The argument in the proof is often expressed in the form of implications or causation. The argument is an important part, so that conclusions that arise and are used for subsequent verification, are not born suddenly. That is, the conclusion arises from logical thought. [24] says that each claim is derived logically from the previous claim and the theorem whose truth has been determined. Students' difficulties are in their use, both the word logic, for example, if... then ... (because ..., then ...), or placement. These results in an inappropriate language of proof. Findings obtained from student activities when proving, among others: a) between hypotheses and unrelated conclusions, b) make conclusions, but there is no supporting hypothesis (sudden conclusion), and c) using hypotheses whose truth is uncertain or not proven.

The argument in the proof has a very important role, because the mindset of proof, the truth can be seen from his argument. Scaffolding used is ***reviewing*** interactions including interpreting students' actions, looking, touching and verbalizing, prompting and probing, parallel modeling, explain and justify. Students can see, say, interpret, explain, and be helped through questions or models that support. Also, students can develop thinking concepts, make connections to previous thoughts that are similar and can be used. For this reason, ***level 3 scaffolding*** is used, namely developing conceptual thinking, and making connections.

3.5 Key Expression

The key expression when showing what you know, what you want to prove, and when using a definition or theorem is still difficult. Besides, the expression in the first step of proof between proof of nature is closed, associative with the nature of the existence of the identity element, and the inverse nature tends to be the same. This result in the final verification not being found or not following what should be indicated. Another difficulty experienced by students in key expressions is when the problem proved requires a related concept in the form of a theorem.

The difficulty of key expressions in showing what is known, what is to be proven, and when using definitions or theorems can use ***reviewing*** scaffolding, especially interaction prompting and probing, parallel modeling, explain and justify. Through scaffolding reviewing, students are helped by their awareness of questions, similar models, finally able to explain. Besides, to better understand the definitions or theorems that will be used using scaffolding of ***restructuring*** specifically the interaction of meaningful contexts and simplifying the problem. Through meaningful contexts and simplification of problems with definitions or theorems, they help students better understand, so the key expressions to prove can also be found.

3.6 Proof Language

Language proof in mathematics (group) is different from other fields of study. Proof of the problem of a group of objects set with one or more operations that are abstract, the way of thinking must be logical. Besides, the language of evidence is allowed to use mathematical symbols (logic). In the language of evidence, researchers examine the use of notations, mathematical terms, and symbols used by students when proving. Also, the mathematical language used when expressing arguments. Based on the results of the study, it was found that most students were still constrained to use notations, mathematical terms, and symbols. According to [25], the ability to identify elements in the set, use language and mathematical notation, use definitions to prove, and the ability to initiate proof are some of the decisive prerequisites in proving.

Scaffolding for constraints uses notation, mathematical terms, and symbols use ***reviewing,*** specifically the interaction of interpreting students' actions, looking, touching and verbalizing, prompting and probing, parallel modeling, explaining and justifying. This step helps students from

starting to see, say, to the student's interpretation of the notation, mathematical terms, and symbols. Then, students are helped through questions, and similar examples until finally they can explain and justify the notation, mathematical terms, and symbols used.

4. Conclusion

The proof is an important aspect that must be developed in learning. However, it is proven that there are still many students who find it difficulties. This study provides an alternative to provide problems in proving, namely by providing scaffolding. Based on the results of the analysis, the authors can describe as follows: Difficulties in the initial steps of proof and the path of proof, the scaffolding that can be given is explaining, reviewing, and restructuring. The difficulty of proving the related concept and proof arguments indicators, the given scaffolding is explaining, reviewing, restructuring, developing conceptual thinking, and making connections. For difficulties in proving key expression indicators, the scaffolding provided is reviewing and restructuring. Meanwhile, difficulties with indicators related to the language of proof, reviewing scaffolding can be given.

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