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Student Teachers' Construction of Mathematical Theorem of Set and Cardinality

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Abstract. Constructivists suggest that teachers should guide and facilitate their students in reinventing and or reconstructing mathematics concepts as in reconstructing a theorem. A theorem in mathematics holds a vital role as a fundamental aspect in building the mathematics itself. Hence, students must learn and acquire this ability. Furthermore, by reconstructing a theorem, students not only learn about the theorem but also learn about problem-solving since the theorem that should be reconstructed can be presented as a problem. In which, this skill is needed and promoted in the 21st century. Hence, the teacher has to be able to reconstruct and prove their construction of theorem. However, many studies were focused on investigating students' mathematical proof ability, not on the ability to reconstruct a theorem. This study is aimed to investigate mathematics undergraduate students' ability to reconstruct and prove a theorem. A problem to make a general statement regarding the numbers of all possibility of the cardinality of A   B, with A, B is set, and n(A) is a and n(B) is b, is given to 60 undergraduate students who are majoring mathematics education. The statement then is analyzed and categorized into 0 to 5 based on the developed framework. The result shows that most students' answers (29 answers) are categorized as an incomplete or irrelevant theorem (level 0), while only seven answers can be categorized as level 5 (well structured). As for level 2, 3, and 4, there are 13, 8, 3 answers representatively. Also, there is no student answer, which is categorized as level 1. The mistakes occur because students do not fully understand the mathematics notion. Then, the theorem made is different from the one that is asked, the given condition mentioned by the students is not general enough (does not represent all cases); the conclusion made is false. However, no conclusion is unrelated to the condition.

1. Introduction

Constructivism, the idea that students have to construct their knowledge with the help of their teacher, has still been an ideal view in teaching [1]. In teaching mathematics, based on this view, teachers should guide and facilitate their students in reinventing and or reconstructing mathematics concepts as in reconstructing a theorem [2]. This constructivism means that students have to do the thinking process while the teacher acts as a facilitator and give a scaffolding and guidance in the learning process.

A theorem in mathematics holds an important role as a fundamental aspect in building the mathematics itself. Besides, to construct another theorem, it is a necessity for the construction to be based on or in synch with the other theorem [3]. Hence, students need to have and learn this ability. Furthermore, by reconstructing a theorem, students not only learn about the theorem but also learn about problem-solving since the theorem that should be reconstructed can be presented as a problem [4]. In which, this skill is needed and promoted in the 21st century [5].

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The 21st-century learning requires the learner to study in the learning activity actively [5]. The core of this century is skills of high order things, communication, creativity and innovation, problem-solving, and confidence [6]. Mathematical problem solving is a students' process in solving a mathematical problem as a product of creative thinking. Then problem posing is a constructing problem in mathematical terms based on some given information to find a solution [4]. So, problem posing is a counterpart of problem-solving in mathematics problem-posing gives other benefits, such as to increase problem-solving ability, making students active, and enriching fundamental concepts [7]. So, the task of submission a problem should be given in Mathematics Education students to understand the information in axiom or theorem.

Since the teacher should be able to guide and facilitate their students in reconstructing a mathematical theorem, the teacher has to be able to reconstruct and prove their construction of theorem. Siswono [8] finds out that mathematics students teachers find difficulties in constructing a theorem from the given axioms. The results of the study claim that the unsophisticated students' theorem constructions are caused by students' mistakes of a mathematical concept, mistake on the proof, and mistake related to the mathematics sentence [8]. However, the analysis of the theorem structure is missing, This study aims to explore mathematics education undergraduate students' ability to construct a theorem, focus on the structure of the theorem. So that we can find out more data on the mathematics teacher students' theorem construction and its proof.

2. Theorem Construction and Proof Scheme

A theorem in mathematics is defined as a statement that needs to be justified and be proven [9]. As for the formulation, a theorem can be written as a conditional sentence. In other words, it can be presented as *"if ..... then ....."* statement. The *"if"* statement is called premises or condition, and the *"then ..."* statement is called a conclusion of the given premises.

Derived from the definition of theorem and the fact that all theorem can be written as conditional sentences consist of condition and conclusion, a classification level of students' theorem construction is produced. The theorem construction is classified into six levels, start from level 0, which associated with an irrelevant theorem or incomplete theorem to level 5 for sophisticated theorem (Table 1).

Table 1. The descriptor for coding of students' theorem construction

Level	Characteristics	Description
0	Irrelevant or incomplete theorem	Do not know how to construct theorem (leave a blank page) Construct irrelevant theorem with the task
1	Incomplete condition and unrelated conclusion	The condition is incomplete, just using one or more condition, but do not cover all possible condition Relevant to the task Conclude with no relation to the written condition. Unable to relate the conclusion and condition
2	Incomplete condition and wrong conclusion	Relevant to the task The conditions are incomplete The conclusion is related to the condition but wrong
3	Incomplete condition and right conclusion	Relevant to the task The conditions are incomplete The conclusion is related to the condition and right conclusion

4	Complete condition and wrong conclusion	Relevant to the task The conditions are complete, cover all possible cases The conclusion is related to the condition but wrong
5	Well-structured (Complete condition and right conclusion)	Relevant to the task The conditions are complete The conclusion is related to the condition and right

3. Method

This research aims to know how is mathematics education undergraduate students' ability to construct a theorem focus on the structure of the theorem. Sixty mathematics student teachers are given a problem to construct a theorem based on the condition given.

Problem:
Given $n(A) = a$ and $n(B) = b$ with A and B are a set. Construct a general statement regarding the numbers of all possibility of $n(A \cup B)$.

To construct the theorem, students need to define the condition of their general statement since the condition given in the problem is still too general. In other words, students need to think of the condition of the sets, whether they are different or same, dependent or independent. The conjectured condition of the general statement should not to particular and have to cover all the possible cases. The general statement could not only made for a particular case, for instance, only for condition $A = B$. As for the conclusion, students need to understand the difference between the numbers of all possibility of $n(A \cup B)$ and the numbers of all possibility of $(A \cup B)$ thoroughly. Next, the statement obtained needs to be proven. The constructed theorem made by the students then is analyzed and categorized based on the descriptor for coding of students' theorem construction (Table 1).

4. Result and Discussion

In general, the level of students' theorem construction can be divided into four categories, such as failed to understand the problem or construct irrelevant theorem (level 0), construct unrelated conclusion using incomplete condition (level 1), construct wrong theorem with related conclusion using different level in showing the conditions (level 2, 4), and construct the right theorem that have related conclusion using different level in showing the conditions (level 3,5). Table 2 shows the frequency distribution of students' theorem construction on the task.

Table 2. The distribution of students' theorem construction

Level of theorem construction	Frequency	%
0	29	48.33%
1	0	0.00%
2	13	21.67%
3	8	13.33%
4	3	00.05%
5	7	11.67%

Table 3 shows that most students, there are 29 students, are categorized in level 0. This finding means that most of the students failed to construct the relevant theorem. As for level 2 to 4, there are 13, 8, and 3 answers that are categorized in this level, respectively, while no one made an unrelated conclusion. The main cause of the level 0 answers is that the students construct a theorem that is irrelevant to the given task. The factors are that students do not fully understand the mathematics symbol. Hence they mistook the numbers of all possibility of $n(A \cup B)$ as the numbers of all possibility of $(A \cup B)$. A sample of the students' results can be seen in figure 1.

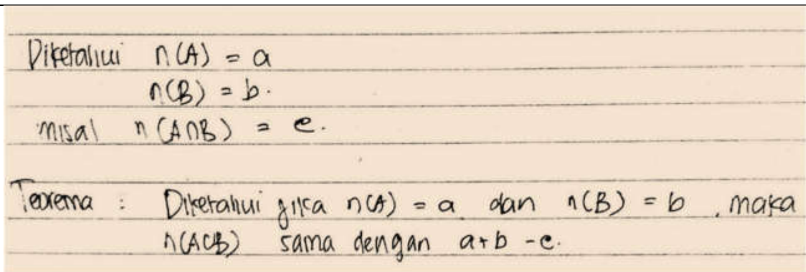
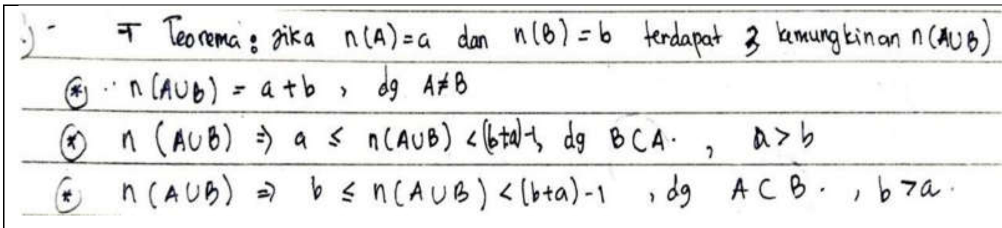


Figure 1. Students' responses which categorized in level 0

From Figure 1, we can see that the students did not understand that the researcher asked the problem. The purpose of the problem is to construct a theorem related to the number of possibilities. However, in this case, the student answered by constructing a theorem related to the irrelevant set. So, the students did not answer the question. Their carelessness or misconceptions cause it in the concept of probability of cardinality. Students conclude that the probability of cardinality has the same meaning as the cardinality of the set chapter.

In other cases, there are no students that the answer was categorized in level 1 that describe there are no related between condition and the conclusion. Furthermore, 13 students answer in level 2. It indicates that students show incorrect theorem because of the incomplete condition, but the conclusion inferred is related to the conditional statement, as in figure 2.



"Theorem: If $n(A) = a$ and $n(B) = b$ there are three possibilities of $n(A \cup B)$

- $n(A \cup B) = a + b; wit \quad A \neq B$
- $n(A \cup B)\ is\ a \leq n(A \cup B) < (b + a) \quad 1; wit \quad B \subset A, a > b$
- $n(A \cup B)\ is\ b \leq n(A \cup B) < (b + a) \quad 1; wit \quad A \subset B, b > a$"

Figure 2. Students' response which categorized in level 2

The example of the incomplete condition of a theorem is shown in Figure 2. The condition should cover all possibility of a relation between $A, B, a,$ and b. However, some students only consider part of it; for instance, is they only wrote the condition for $A \neq B$ with A and B are independent set. Students do not write the condition for $A \neq B$ with A and B are dependent. Also, as for the other condition, they neglect the fact that there is a possibility for $A = B$. Even though students realize that it is possible for $B \subset A\ so\ a > b$ and vice versa, but they do not realize that if $B \subset A\ and\ A \subset B$ then $A = B$ result on $a = b$. This condition is not yet covered in the theorem.

Regardless of the given incomplete condition, students failed to make the right conclusion. For the case that $B \subset A$ or $A \subset B$, the cardinality of $(A \cup B)$ should be the same as the number of elements of the more significant set. While the students answer that $n(A \cup B)$ wit $B \subset A$ is $a \leq n(A \cup B) < (b + a)$, so the conclusion is incorrect.

As for level 3, eight students can make a relation to the conclusion and construct the correct theorem, yet the condition is incomplete.

* Teorema untuk menentukan banyak kemungkinan $n(A \cup B)$.
Jika, $n(A) = a$
$n(B) = b$
maka, banyak kemungkinan $n(A \cup B)$ adalah bilangan terkecil dari a, b. ditambah 1
Contoh : $n(A) = a$
$n(B) = b$
misal $a < b$
maka banyak kemungkinan $n(A \cup B) = a + 1$

The theorem of determining the number of possibility of $n(A \cup B)$
If, $n(A) = a; n(B) = b$
Then, the numbers of $n(A \cup B)$ is the smallest of a, b plus 1.
For example: $n(A) = a; n(B) = b$
Assume $a < b$ then the number of possibility of $n(A \cup B) = a + 1$

Figure 3. Students' response which categorized in level 3

From figure 3, we can see that students can construct the correct theorem for the cardinality of $A \cup B$, is $a + 1$. However, the condition is $a < b$. It indicates that the theorem was applicable for $a = b$, so the condition cannot be applied generally or incomplete.

Table 1 also shows that there are only three students can show the complete condition, which is related to the conclusion, but the theorem was incorrect, as shown in the figure below.

b) $n(A) = a$
$n(B) = b$
Teorema :
Banyaknya kemungkinan $n(A \cup B)$ adalah 3, yaitu :
1. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
saat $0 < n(A \cap B) < n(A)$, $n(A) < n(B)$
2. $n(A \cup B) = n(A) + n(B)$
saat $n(A \cap B) = 0$
3. $n(A \cup B) = n(A)$ atau $n(A \cup B) = n(B)$

Figure 4. Students' responses

Figure 4 shows that students at level 4 mention all the conditions of the theorem. However, the students' answers, which are categorized as level 4, indicate the wrong conclusion. Students have the right thinking process, yet for a condition in which $0 < n(A \cap B) < n(A)$ with $n(A) < n(B)$ (case 1 figure 4), students do not consider all the possibilities of $n(A \cap B)$ itself. Instead, students see the condition as one whole possibility. This situation leads to the wrong conclusion.

Using the complete condition related to the conclusion, seven students can construct the theorem correctly. The students in this situation can be categorized in level 5 (highest level) of the constructing mathematical theorem. The students' artifact for level 5, shown in figure 5.

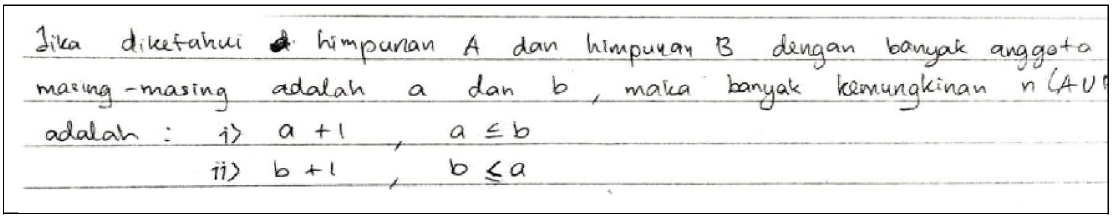


Figure 5. Students' responses which categorized level 5

At the highest level, students can construct the theorem correctly based on a complete condition which related to the conclusion. They mention all the conditions using notation $a \leq b$ and vice versa, which means the theorem will be applied generally. There is no misconception between the concept of cardinality and cardinality of possibility, and the conclusion also obtained mathematically right.

5. Conclusion

The result shows that most students failed to construct the theorem. Even though some of the constructed theorems provide the right conclusion, but if the condition is not cover all cases, the theorem not yet to be called well-structured and vice versa. The same category is also applied for the students who able to construct a well-structured theorem, yet the theorem is irrelevant to the task. There are only a few students (7 out of 60) who can construct a well-structured theorem. This finding suggests that the teacher or lecturer still need to improve student's ability in theorem construction, especially in writing or listing all cases for conditional sentences as well as understanding mathematics symbol. While regarding whether or not the constructed theorem can be proven as true still needs further investigation.

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