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Upper bound for the power outputs of linear vibrational power harvesters: translational vs. rotational geometries

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Abstract. Inertial harvesters utilize the motion of a translating or rotating mass to scavenge power from ambient vibrations. For translational harvesters, analytical expressions describing the influence of size on the output power of these harvesters have been derived previously by modeling vibrations as sinusoids or white noise. In this paper, we present a general method to estimate the size-dependent power outputs of both translational and rotational harvesters for vibrations with an arbitrary power spectral density. Using the heights and widths of the peaks in the vibrational power spectral density, we derive simple analytical expressions for the maximum power output of harvesters and the minimum harvester size required to achieve it. To illustrate an application of these estimates, we analyze the case of walking-induced vibrations produced at the hip-pocket and upper-arm locations. We find that rotational designs can outperform translational ones only when the harvester is larger than 26 cm and 8 cm for the hip-pocket and arm locations, respectively.

1. Introduction

The proliferation of electronic devices emphasizes the need to develop sustainable ways of powering them. While batteries are expected to be the preferred power source for most applications, some low-power devices, such as sensor nodes, key fobs, and standalone systems can be integrated with power harvesters if they are sufficiently small and powerful. Applications where battery replacement or recharging is impractical, such as long-term implantable devices, would particularly benefit from compact power harvesters with centimeter-scale or smaller sizes.

Vibration power harvesters convert the translational or rotational kinetic energy from ambient vibrations to electric power [1, 2]. For translational harvesters, the finite size of a harvester restricts the maximum displacement of its inertial mass, thereby limiting the work that can be done by the inertial forces and, therefore, the power output. Mitcheson, et al., derived a closed-form expression for the relationship between size and power output of linear translational harvesters excited by *harmonic vibrations* [1]. In addition, Halvorsen modeled random vibrations and derived analytical expressions for the maximum power outputs of linear translational harvesters driven by *white noise* but without considering the harvester size [3].

For rotational harvesters, the rotation of the inertial mass is driven by vibration-induced torques. In contrast to the translational case, the maximum angular displacement of a rotating inertial mass is, in principle, unlimited; therefore, there are no corresponding limits on the maximum power output of a linear rotational harvester when excited by a purely harmonic (sinusoidal) wave. The lack of a

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corresponding power limit is a potential advantage of rotational harvesters in applications with very

limited sizes, such as MEMS or implantable sensors. However, no real-world vibrations are purely sinusoidal; therefore, the relative benefits of translational vs. rotational harvesters depend on the spectral properties of the vibration source.

Here, we present a method to estimate the power outputs of both translational and rotational harvesters as a function of their sizes, when the exciting vibration has an arbitrary power spectral density (PSD). We obtained analytical expressions for the maximum power output and the minimum harvester size that is needed to reach this maximum power, based on the height and widths of the peaks in the vibration PSD. We demonstrate the usefulness of our approach by deriving practical estimates of the power harvestable from vibrations produced by the human walk.

2. Methods

The steady-state amplitude, x, of a linear vibrational harvester driven at an angular frequency, ω , by a harmonic acceleration, a, is given by

$$x(\omega) = \frac{a}{\omega_0^2 - \omega^2 + i\gamma\omega}.$$
 (1)

Here, ω_0 and γ represent resonance angular frequency and dissipation constant respectively. The resulting relationship between displacement PSD, $\hat{S}_x(\omega)$, and the acceleration PSD, $\hat{S}_a(\omega)$, is given by

$$\hat{S}_{x}(\omega) = \left| x(\omega) \right|^{2} = \frac{\hat{S}_{a}(\omega)}{\left(\omega_{0}^{2} - \omega^{2}\right)^{2} + \left(\gamma\omega\right)^{2}}.$$
(2)

As shown in figure 1(a), the acceleration PSD, $\hat{S}_a(\omega)$, of a typical stochastic vibration source has a global maximum value $\hat{S}_{a,peak}$ at an angular frequency ω_{peak} and a full-width at half maximum \hat{w} . If all damping is due to conversion of mechanical motion to electricity (i.e., no heat-producing dissipation), the specific power output of a damped harvester, P can be written as





Figure 1. (a) Typical power spectral density (PSD) of stochastic vibrations (b) Acquisition of walking-induced vibrations from the hip-pocket and arm locations using a smartphone.

Using standard methods for rational fractions, we obtain the simple analytical expression:

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$$P \le \frac{\pi \tilde{S}_{a,peak}}{2} = \frac{S_{a,peak}}{4},\tag{4}$$

Where $S_{a,peak}$ denotes the peak value of acceleration PSD with the frequency expressed in the units of Hz (rather than angular velocity, as for $\hat{S}_{a,peak}$). The upper bound in equation (4) can be asymptotically reached using a harvester tuned to resonance at ω_{peak} with a high quality factor such that $\hat{w}Q \gg \omega_{peak}$. Therefore, such harvesters will need to be very large to sustain the large displacements of their inertial masses. However, practical applications typically impose a limit on harvester size, restricting the maximum Q that can be used without the proof mass constantly hitting the outer boundaries of the harvester. The standard deviation of displacement for a harvester with $\hat{w}Q \gg \omega_{peak}$ is

$$\sigma_{x} = \left\langle x^{2} \right\rangle^{\frac{1}{2}} = \left[\int_{0}^{\infty} \hat{S}_{x}(\omega) d\omega \right]^{\frac{1}{2}} = \left[\int_{0}^{\infty} \frac{\hat{S}_{a}(\omega)}{(\omega_{0}^{2} - \omega^{2})^{2} + (\gamma \omega)^{2}} d\omega \right]^{\frac{1}{2}} = \left(\frac{\pi Q \hat{S}_{a, peak}}{2\omega_{peak}^{3}} \right)^{\frac{1}{2}}.$$
 (5)

Assuming that the maximum displacement rarely exceeds three standard deviations in each direction, the harvester size, L, is related to Q by

$$L \ge 6\sigma_x = \frac{6}{\omega_{peak}} \left(\frac{\pi Q \hat{S}_{a,peak}}{2\omega_{peak}}\right)^{\frac{1}{2}} = \frac{3}{2\pi f_{peak}} \left(\frac{Q S_{a,peak}}{2\pi f_{peak}}\right)^{\frac{1}{2}}.$$
(6)

If the harvester is tuned to the highest peak in the vibration PSD and Q is high enough that the resonance of the harvester is narrower than the peak in the PSD, the spectrum can be approximated as white noise and the upper bound in equation (4) is achieved asymptotically as $Q \rightarrow \infty$. However, we can use only finite Q in practice, and based on analysis of multiple vibration sources, we have found that $Q = (8\pi^3 \omega_{peak})/9\hat{w}$ is sufficient to nearly achieve this maximum power. As a result, the minimum harvester size, L_{min} needed to achieve the maximum power given by equation (4) can be estimated as

$$L_{\min} = \left(\frac{4\pi^2}{\omega_{peak}}\right) \left(\frac{\hat{S}_{a,peak}}{\hat{w}}\right)^{\frac{1}{2}} = \left(\frac{1}{f_{peak}}\right) \left(\frac{S_{a,peak}}{w}\right)^{\frac{1}{2}}.$$
(7)

The case of rotational harvesters is very different. The angular displacement of its rotary inertial mass is, in principle, not limited by its physical size L. This implies that it is possible to employ rotational harvesters with arbitrarily large values of Q at the peak frequency f_{peak} . Similar to equation (4), the maximum specific power, P_r , of a rotational harvester is then related to its radius of gyration, R_g , and the peak value of its rotational acceleration PSD, $\hat{S}_{\alpha,peak}$ or $S_{\alpha,peak}$, as

$$P_r = \frac{\pi R_g^2 \hat{S}_{\alpha,peak}}{2} = \frac{R_g^2 S_{\alpha,peak}}{4}.$$
(8)

To illustrate the relative advantages of rotational and vibrational harvesters, we applied these formulas to the case of walking-induced vibrations. Figure 1(b) shows the experimental setup we used to record walking-induced vibrations at the hip pocket and upper-arm locations. During a walk that lasted about 11-minutes, the three components of translational and rotational acceleration signals, *a* and α respectively, were recorded using a Nexus 4 smartphone. Acceleration PSDs, S_a and S_α corresponding

to each component of acceleration a and angular acceleration α were obtained using the Welch averaging method. (Recordings from other walks yielded very similar spectra.)

For translational harvesters, the a_y component was chosen for analysis since it had the highest value

of $S_{a, peak}$, while for rotational harvesters, only the α_z component was considered for the same reasons.

The maximum power outputs of translational harvesters were then determined by integrating equation (3) numerically for a wide range of harvester resonance frequencies and quality factors, and then choosing the frequency and quality factor that produced the maximum output for a given harvester size. For rotational harvesters, a ring-type proof mass was considered due to its optimal moment of inertia, and equation (8) was used to calculate its output power as a function of its characteristic size, *i.e.*, the ring diameter.

3. Results and discussion

Figures 2(a) and 2(b) show the translational and rotational acceleration PSDs for the hip pocket and arm respectively. Peak values were found at a frequency of 2 Hz for the a_y component and a frequency of

1 Hz for the α_z component, in agreement with literature-reported values [4, 5]. These are consistent with the fact that one full period of the leg movement corresponds to two foot strikes on the ground. Comparing figures 2(a) and 2(b), we also see that that value of $S_{\alpha,peak}$ is larger at the arm than at the hip pocket. This can be explained by the fact that while walking, arms swing by a larger angle than hips.



Figure 2. Power spectral density (PSD) of translational S_{a_y} (blue) and rotational S_{α_z} (red) acceleration signals recorded at (a) hip pocket and (b) arm locations.

Figures 3(a) and 3(b) show the output powers of translational and rotational harvesters as a function of their sizes for the hip pocket and arm, obtained by numerically optimizing equation (3) by trying out a very wide range of harvester frequencies and quality factors. For translational harvesters, we find good agreement with previously reported trends for optimized harvester architectures [6]. The power outputs of translational harvesters saturate to a maximum value for sizes, $L \gg 1$ m. However, the power outputs of rotational harvesters increase quadratically with harvester size. Comparing these two architectures against each other, the optimal harvester design crosses over from translational to rotational at characteristic harvester sizes of 26 cm and 8 cm for the hip and arm locations, respectively.

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Figure 3. Output powers of translational (blue) and rotational harvesters (red) located at (a) hip pocket and (b) upper arm locations, obtained by numerical optimization of equation (3).

Using equation (4), the maximum power output of the translational harvester is estimated to be $S_{a,peak} / 4 = 57/4 = 14.3$ W/kg for the hip pocket and 72/4 = 18 W/kg for the upper arm. These analytical estimates are in excellent agreement with those obtained by numerically maximizing equation (3), which are ~15 mW/g for the hip pocket and ~19 mW/g for the arm, as illustrated in figures 3(a) and 3(b) respectively. However, these maximum powers are only achieved at harvester sizes given by equation (7), which evaluate to 13 m and 17 m for hip and upper arm locations, again in agreement with the saturation observed numerically in Fig. 3. Thus, the concise analytical equations (4) and (7) have the excellent predictive ability for walking-induced vibrations. In fact, we have also verified the predictive ability of these equations for other vibration sources [7].

4. Conclusion

The method presented in this work allows easy estimation of the maximum harvestable power and the needed harvester size from the power spectral density of recorded vibrations. In the considered example of walking vibrations, translational designs are expected to outperform rotational ones for realistically sized micropower generators. However, the advantage of translational harvested is not a foregone conclusion. For vibration sources with less frequency variation than typical human walking, rotational designs even for very small harvester sizes.

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