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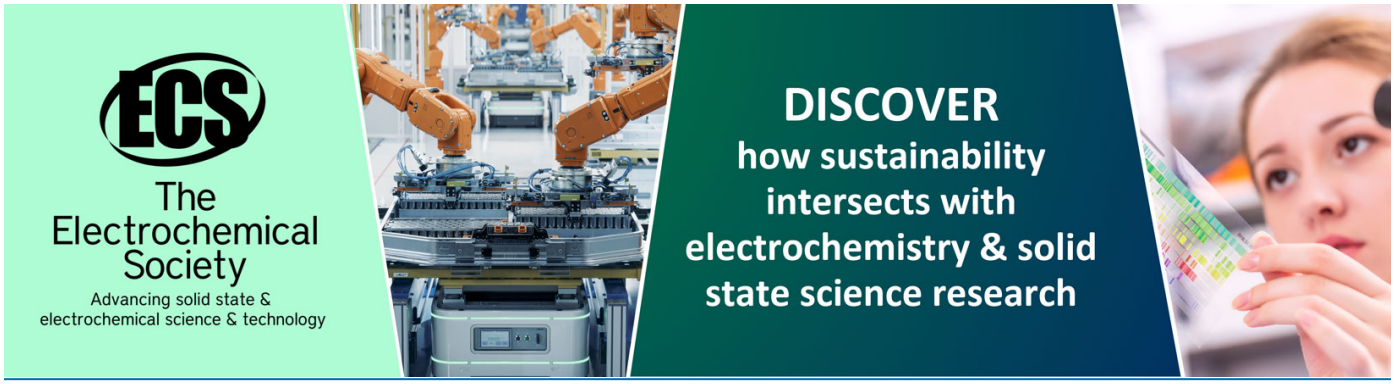

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PID controller for a differential drive robot balancing system

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Abstract. Controlling a differential drive robot balancing system is a challenging task since it is highly unstable, non-linear and under-actuated system. This paper proposes a PID controller algorithm to balance a differential drive robot at upright position. The system model derived using Lagrangian method is discussed. The resulting mathematical model of the proposed closed-loop system was simulated. To verify the system performances, the real time experiments have been conducted. An Arduino Uno and an MPU6050 were used as the main controller and the main sensor, respectively. The PID parameters were manually tuned until the desired performance was achieved. From the simulation and the real time experiment results, the proposed method has demonstrated its capability to stabilize robot at upright position.

1. Introduction

The world of robots has helped a lot of human life in various fields [1]. Very rapid development in Robot technology so that many tasks ranging from simple things to very complicated things can be done by robots so that the results of the process are more efficient, faster, more precise and economical [2]. The existence of robots does not only fill the work of human work in daily life, robots are also applied in industrial fields that can produce increased productivity and cut labor costs [3]. This resulted in a significant increase in profits for current industry players [4]. Two wheeled self-balancing mobile robot is a special type of wheeled mobile robot [5]. Signal processing and control techniques are the main factors in which robot performance and stability rely on [6]. The main focus of this research project is to develop a two-wheeled equilibrium robot that will be controlled by a PID-controller-based microcontroller to improve its durability in terms of performance stability [7]. Two-wheeled equilibrium robot is a type of mobile robot [8]. To achieve equilibrium requires signal processing and control techniques and is a major factor in robot performance and stability [9]. In recent years, many researchers conducted research on wheel equilibrium robots because they have characteristics in terms of non-linearity, instability, having many variables, and strong coupling [10]. The control system commonly used by experts includes using the PID control system [11]. This control system consists of a Proportional, Integral and Derivative (PID) control system [12]. This control system is a controller to accelerate the stability of the system with the duration of time as quickly as possible with minimal oscillation [10]. Independent control systems and equilibrium are the main focus in designing systems with the aim of: (1) Achieving effective stability; (2) increasing response speed; (3) Responding to errors; and (4) Prevent excessive oscillation, fluctuations and robot vibrations. Very effective PID

control is used in closed control systems so that the output of the system can be evaluated whether it is in accordance with the set points we expect or not. In this study it was intended to design equilibrium two-wheeled robots. The control technique uses an Arduino microcontroller. While the input signal is controlled using an accelerometer sensor and gyroscope. The accelerometer is used to detect slope and the gyroscope is used to detect the angular velocity of the robot body. The scope of this research is focused on robot assembly and kinematic mathematical modeling including designing and running an Arduino-based PID Controller on Robots. This paper is organized as follows: Section 2 presents mathematical and simulation modeling. Section 3 PID controller. Section 4 presents result and discussion. Section 5 or the Final section is a conclusion.

2. Mathematical and simulation modelling

The robot consists of main body and wheels. The free body diagram of the robot is presented in Figure 1.

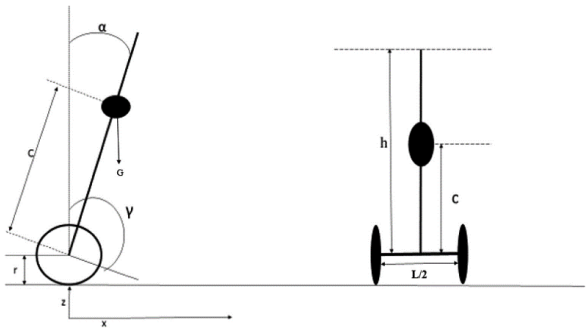


Figure 1. Free-body diagram of the robot.

To model the robot, the following assumptions are made: (1) The robot mechanical structure is rigid, (2) The mass (m) and the radius (r) of the robot wheels are the same, (3) The link length of each wheel to the centre of the robot mass is the same ($l_r = l_l = l/2$), (4) Slip between the wheel and the ground surface is not considered, (5) Internal losses are neglected and (6) Inductance and frictions on the armature are neglected. The robot is considered to possess three degree freedom, consisting of yaw angel (y), tilt angle (α), and transitional motion (x). The two Lagrangian equations are as follows:

$$L = T - V \tag{1}$$

$$\frac{\vartheta}{\vartheta t}\left(\frac{\vartheta L}{\vartheta \dot{q}}\right) - \frac{\vartheta L}{\vartheta q} = F \tag{2}$$

where L is the Lagrangian, the kinetic energy is denoted by T, V denotes the potential energy, F is the forced function, q_x is the generalized coordinate. This robot has been controlled by two inputs torques applied to the motors produced by voltage.

$$T = T_C^L + T_C^R + T_W^L + T_W^R \tag{3}$$

where T_C^L is the main body kinetic energy with respect to linear displacement, T_C^R is the main body kinetic energy with respect to angular displacement, T_W^L is the wheel kinetic energy due with respect to angular displacement, T_W^R wheel kinetic energy with respect to linear displacement.

$$T_C^R = \frac{1}{2}\left[I_x\dot{\alpha}^2 + I_y\dot{\gamma}_y^2 sin\ \alpha^2 + I_z\gamma^2 \cos\alpha^2\right] \tag{4}$$

$$T_W^R = \frac{1}{2}Mr^2\left[\dot{\alpha}_r^2 + \dot{\alpha}_l^2\right] + \frac{1}{2}I\left[\dot{\alpha}_r^2 + \dot{\alpha}_l^2\right] \tag{5}$$

and,

$$\alpha_r = x + L\gamma, \alpha_1 = x - L_\gamma \tag{6}$$

$$T_W^R = \left(M + \frac{I}{r^2}\right)(\dot{x}^2 + L^2\gamma^2) \tag{7}$$

$$V = M_c g d \cos\alpha + M_c g r \tag{8}$$

From Eq. (1), the Lagrangian equation is as follows:

$$\begin{aligned} L = &\left[M + 2M_w + \frac{2I}{r^2}\right]\dot{x} - \left[Md^2 + \frac{I_x}{r^2}\right]\ddot{\alpha} \\ &+ \left[\left(M + \frac{I}{r^2}\right)L^2 + \frac{1}{2}\left(I_z \cos\alpha^2 + I_y \sin\alpha^2 + \mathrm{M_c d}\ \sin\alpha^2\right)\right]\gamma^2 \\ &+ \mathrm{M_c d}\ \cos\alpha\,\dot{x}\dot{\alpha} - [\mathrm{M_c}\ g d\cos\alpha + \mathrm{M_c}\ g r] \end{aligned} \tag{9}$$

For *x*-coordinate, we have the following equations:

$$\left(\frac{\vartheta L}{\vartheta \dot{x}}\right) = \left[\mathrm{M_c} + 2M + \frac{2I}{r^2}\right]\dot{x} + \mathrm{M_c} d\dot{\alpha}\cos\alpha \tag{10}$$

$$\frac{d}{dt}\left(\frac{\vartheta L}{\vartheta \dot{x}}\right) = \left[M_c + 2M + \frac{2I}{r^2}\right]\ddot{x} - M_c d\dot{\alpha}^2 \sin\alpha + M_c d\ddot{\alpha}\cos\alpha \tag{11}$$

$$\frac{\vartheta L}{\vartheta x} = 0 \tag{12}$$

$$\left[M_c + 2M + \frac{2I}{r^2}\right]\ddot{x} - M_c d\dot{\alpha}^2 \sin\alpha + M_c d\ddot{\alpha}\cos\alpha = \frac{\tau_r + \tau_l}{r} \tag{13}$$

$$\ddot{x} = \frac{\left[\frac{\tau_r + \tau_l}{\tau} - M_c d\ddot{\alpha}\cos\alpha + M_c d\dot{\alpha}^2 \sin\alpha\right]}{\left[M_c + 2M + \frac{2I}{r^2}\right]} \tag{14}$$

For α-coordinate, we have the following equations:

$$[M_c d^2 + I_x]\ddot{\alpha} + M_c d\ddot{x}\cos\alpha - \left[M_c d^2 + I_y - I_z\right]\dot{\gamma}^2 - M_c g d\ \sin\alpha = -[\tau_r + \tau_1] \tag{15}$$

From Eqs. 11 and 12, for the $\ddot{\alpha}$can be the subject of the formula as :

$$\begin{aligned} \ddot{\alpha} = &\left[\left(M_c + 2M + \frac{2l}{r^2}\right) + M_c d\cos\alpha\right][\tau_r + \tau_1]/\left\{\left[M_c + 2M + \frac{2l}{r^2}\right][M_c d^2 + I_x]\right. \\ &\left. - M_c^2 d^2\cos\alpha\right\} \\ &- M_c^2 d^2 \alpha^2 \cos\alpha\ \sin\alpha\ /\left\{\left[M_c + 2M + \frac{2l}{r^2}\right][M_c d^2 + I_x] - M_c^2 d^2\cos\alpha\right\} \\ &+ \left[M_c d^2 + I_y - I_z\right]/\left\{\left[M_c + 2M + \frac{2l}{r^2}\right][M_c d^2 + I_x] - M_c^2 d^2\cos\alpha\right\} \\ &- M_c g d\ \sin\alpha\ \left(M_c + 2M + \frac{2l}{r^2}\right)/\left\{\left[M_c + 2M + \frac{2l}{r^2}\right][M_c d^2 + I_x] - M_c^2 d^2\right. \end{aligned} \tag{16}$$

Simplify to get:

$$\ddot{\alpha}=\frac{\left[M_c d^2+I_y-I_z\right][M_c r^2+2Mr^2+2I]cos\ \alpha\ sin\ \alpha}{[M_c rd\ \sin\alpha]^2+([M_c+2M]r^2+2I)I_{\square}+2M_c d^2[Mr^2+I]}\cdot\dot{\gamma}^2 - \frac{M_c^2 d^2 r^2 cos\ \alpha\ sin\ \alpha}{[M_c rd\ \sin\alpha]^2+([M_c+2M]r^2+2I)I_x+2M_c d^2[Mr^2+I]}\cdot\dot{\alpha}^2 + \frac{[M_c r^2+2Mr^2+2I]M_c gd\ sin\ \alpha]}{[M_c rd\ \sin\alpha]^2+([M_c+2M]r^2+2I)I_x+2M_c d^2[Mr^2+I]} - \frac{[M_c r^2+2Mr^2+2I]M_c dr\ cos\ \alpha]}{[M_c rd\sin\alpha]^2+([M_c+2M]r^2+2I)I_x+2M_c d^2[Mr^2+I]}[\tau_r+\tau_l] \quad (17)$$

For *x* coordinate, the Lagrangian from Eq. (9) is as follows:

$$\ddot{\alpha}=\frac{\left[\frac{\tau_r+\tau_l}{r}+M_c d\dot{\alpha}^2\sin\alpha-\left[M_c+2M+\frac{2I}{r^2}\right]\right]\ddot{x}}{M_c d\cos\alpha} \quad (18)$$

By substituting Eqs. (16) in (12), then it gives Eq. (19) as follow:

$$[M_c d^2+I_x]\frac{\left[\frac{\tau_r+\tau_l}{r}+M_c d\dot{\alpha}^2\sin\alpha-\left[M_c+2M+\frac{2I}{r^2}\right]\right]\ddot{x}}{M_c d\cos\alpha}+M_c d\cos\alpha\,\ddot{x}-[M_c d^2+I_y - I_z]\cos\ \alpha\sin\ \dot{\gamma}^2-M_c gd\sin\ \alpha=-(\tau_r+\tau_l) \quad (19)$$

Collecting terms with $\ddot{x}$, and making it the subject of the following formula:

$$\ddot{x}=\frac{M_c d\cos\alpha\left[M_c d^2+I_y-I_z\right]cos\ \alpha\ sin\ \alpha}{[M_c d^2+I_x][M_c r^2+2Mr^2+2I]-[M_c dr\cos\alpha]^2}\cdot\dot{\gamma}^2 - \frac{M_c^2 d^2 gr^2 cos\ \alpha\ sin\ \alpha}{[M_c d^2+I_x][M_c r^2+2Mr^2+2I]-[M_c dr\cos\alpha]^2} + \frac{r^2\,[M_c d^2+I_z][M_c d\ sin\ \alpha}{[M_c d^2+I_x][M_c r^2+2Mr^2+2I]-[M_c dr\cos\alpha]^2}\dot{\alpha}^2 + \frac{r^2\,[M_c d^2+I_X+M_c dr\ cos\ \alpha]}{[M_c d^2+I_x][M_c r^2+2Mr^2+2I]-[M_c dr\cos\alpha]^2}(\tau_r+\tau_l) \quad (20)$$

For γ-coordinate: The Lagrangian is given in Eq. (21) as follow:

$$\left[2\left(M+\frac{I}{r^2}\right)L^2+I_y\ sin\ \alpha^2+I_z(cos\ \alpha)^2+M_c d^2\ sin\ \alpha\right]\ddot{\gamma}+2\left[\left[M_c d^2+I_y-I_z\right]cos\ \alpha\ sin\ \alpha\right]\dot{\gamma}\dot{\alpha} = \frac{L}{r}(\tau_r+\tau_l) \quad (21)$$

Simplified further to make $\ddot{y}$ the subject of the following formula:

$$\ddot{\gamma}=\frac{L}{2(r\left[M+\frac{I}{r^2}\right]L^2+I_y\ sin\ \alpha^2+I_z(cos\ \alpha)^2+M_c d^2\ sin\ \alpha}(\tau_r+\tau_l) - 2\frac{2\left[\left[M_c d^2+I_y-I_z\right]cos\ \alpha\ sin\ \alpha\right]\dot{\gamma}\dot{\alpha}}{2(r\left[M+\frac{I}{r^2}\right]L^2+I_y\ sin\ \alpha^2+I_z(co\square\ \alpha)^2+M_c d^2\ sin\ \alpha} \quad (22)$$

To linearize the non-linear model, it is assumed that the robot conditions are stabilized at the zero tilt angle. For α=0, which implies that sin α=α, cos α=1, $\dot{y}$=0, and $\dot{\alpha}$=0 (24). Therefore, Eqs. (17), (20), and (22) become:

$$\ddot{x} = \frac{M_c^2 d^2 g r^2}{[M_c d^2 + I_x][M_c r^2 + 2Mr^2 + 2I] - [M_c dr]^2}\alpha + \frac{r\,[M_c d^2 + I_X + M_c dr]}{[M_c d^2 + I_x][M_c r^2 + 2Mr^2 + 2I] - [M_c dr]^2}(\tau_r + \tau_l) \quad (23)$$

$$\ddot{\alpha} = \frac{[M_c r^2 + 2Mr^2 2I]M_c gd}{[(M_c 2M)r^2 + 2I]I_x + 2M_c d^2(Mr^2 + I)}\alpha - \frac{[M_c r^2 + 2I] + M_c dr}{[(M_c 2M)r^2 + 2I]I_x + 2M_c d^2(Mr^2 + I)}(\tau_r + \tau_l) \quad (24)$$

$$\ddot{\gamma} = \frac{L}{2(r\left[M + \frac{I}{r^2}\right]L^2 + I_z}(\tau_r - \tau_l) \quad (25)$$

From Eqs. (23), (24), and (25) after substitution of robot parameters, Eqs. (26), (27), and (28) are obtained:

$$\ddot{x} = 0.188\alpha + 3.247(\tau_r + \tau_l) \quad (26)$$

$$\ddot{\alpha} = 5.1\alpha - 70(\tau_r + \tau_l) \quad (27)$$

$$\ddot{\gamma} = 12.85(\tau_r + \tau_l) \quad (28)$$

Where

$$\begin{bmatrix}\dot{x}\\ \dot{\alpha}\\ \dot{\gamma}\\ \ddot{x}\\ \ddot{\alpha}\\ \ddot{\gamma}\end{bmatrix} = \begin{bmatrix}0 & 0 & 0\,1\,0\,0\\ 0 & 0 & 0\,0\,1\,0\\ 0 & 0 & 0\,0\,0\,1\\ 0 & 0.188 & 0\,0\,0\,0\\ 0 & 5.1 & 0\,0\,0\,0\\ 0 & 0 & 0\,0\,0\,0\end{bmatrix}\begin{bmatrix}x\\ \alpha\\ \gamma\\ \dot{x}\\ \dot{\alpha}\\ \dot{\gamma}\end{bmatrix} + \begin{bmatrix}0 & 0\\ 0 & 0\\ 0 & 0\\ 3.247 & 3.247\\ -70 & -70\\ 12.85 & -12.85\end{bmatrix}\begin{bmatrix}\tau_r\\ \tau_l\end{bmatrix} \quad (29)$$

These Eqs. (24), (25), and (26) are transformed into state-space form based on the TWSB robot parameters in table 1.

Table 1. Robot parameters.

Parameter	Symbol	Quantity	Unit
Height of the chassis	h	0.08	m
Width of the chassis	w	0.147	m
Distance between wheels	L	0.082	m
Diameter of wheel	d	0.083	m
Mass of the chassis	Mc	0.305	kg
Mass of wheel	M	0.051	kg
Centre of mass	C	0.04	m
Acceleration due to gravity	g	9.81	m/s
Moment of inertia of chassis wrt. x-axis	lx	0.07124E-3	kgm2
Moment of inertia of chassis wrt. z-axis	lz	0.725E-3	kgm2
Moment of inertia of the wheel	l	0.044E-3	kgm2

3. PID controller

This section presents PID controller methodology for robot development. The main controlling system of the mobile robot adopts PID control. The mobile robot uses sensor feedback data as PID control variable to calculate an output response to do correction and follow the predefined trajectory. The equation of PID controller is as follow [7]:

$$u(t) = P + I + D = K_p e(t) + K_1 \int e\,(t)dt + K_d \frac{d}{dt} e(t) \tag{30}$$

where *P* is the proportional term in the discrete form is written as:

$$P = K_p(e_t - e_{t-1}) \tag{31}$$

The *I* is an integrate term that accounts the total error history.

$$I = K_i \sum_{t=0}^{\infty} e_t \tag{32}$$

The *D* stands for derivative and

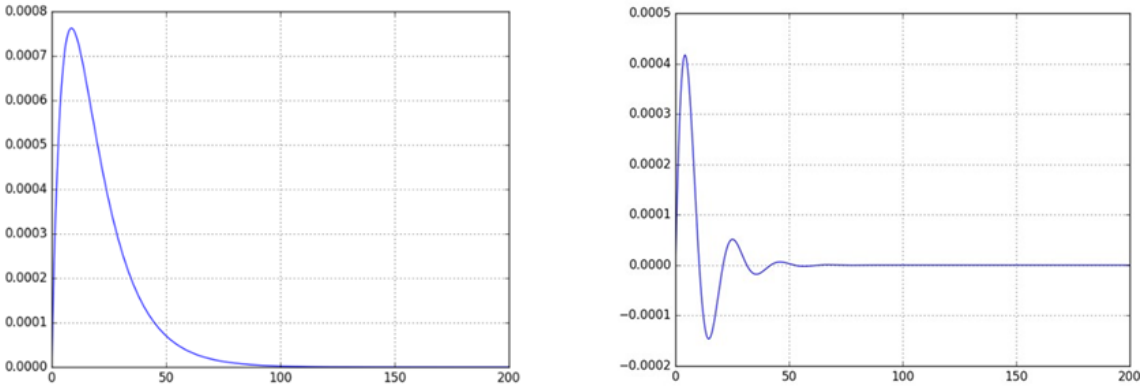
$$D = K_d \left(\frac{e_t - e_{t-1}}{\Delta t}\right), \tag{33}$$

where Δt is the time sampling.
where K_p, K_i *and* K_d denote the coefficients of the proportional, integral and derivative terms, respectively.

4. Result and discussion

4.1. Result

When a robot does not use a PID controller (without a controller) only uses PWM, the robot is difficult to achieve balance. This can be seen in Figure 2 (a). When a robot using a PID controller with parameter values modified Kp = 5000, Ki = 1000 and Kd =500, the results can be seen in figures 2 (b).



(a) (b)

Figure 2. (a) Testing Robot without a controller, (b) Testing Robot with a controller.

4.2. Discussion

Testing is done by changing the parameters of the *Ki*, K_p and K_d. From figures 2 can be seen that, when we use PID controller, the occurrence of oscillation even though it is small, the maximum steady state time is achieved.

5. Conclusion

After testing the data and analyzing the test, it can be concluded that the robot is successfully balanced with the most optimal performance by PID controller with the value is Kp: 5000 Ki: 1000 Kd: 500 The smaller the robot's deviation means the more stable the robot is.

Acknowledgments

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