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Transversely modulated wave packet

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Abstract. The wave packet consisting of two harmonic plane waves with the same frequencies, but with different wave vectors is considered. The dispersion relation of a packet is structurally similar to the dispersion relation of a relativistic particle with a nonzero rest mass. The possibilities of controlling the group velocity of a quasi-monochromatic wave packet by varying the angle between the wave vectors of its constituent waves and of creating a onedimensional spatial structure in the region of wave packet propagation are discussed. The interaction of two transversely modulated wave packets is considered.

1. Introduction

The widespread practical use of wave packets in microwave technology, as well as in the technique of the optical range, in particular in the technique of ultrashort laser pulses [1-9], necessitates a detailed study of specific wave packets [10-12]. Noteworthy is a packet consisting of two quasimonochromatic plane waves propagating at an angle to each other. Despite the obvious simplicity of such a packet, its properties, in particular, the dependence of its group velocity on the angle between the components, have not been studied in detail.

2. Wave packet design

Let's consider two harmonic plane waves propagating at an angle ϑ to each other

$$a_1 = A\cos(\mathbf{K}_1 \mathbf{r} \cdot \omega t), \tag{1}$$

$$a_2 = A\cos(\mathbf{K}_2 \mathbf{r} \cdot \omega t). \tag{2}$$

Here, A is the amplitude, ω is the cyclic frequency, \mathbf{K}_1 and \mathbf{K}_2 are wave vectors, moreover, $\mathbf{K}_1 = \mathbf{k}$ +q, $K_2 = k-q$. The vector q is directed perpendicular to the vector k (figure 1).

Waves (1) and (2) satisfy the linear wave equation. Linear combinations of solutions also satisfy it, including

$$a=a_{1}+a_{2}=A\{\cos[(\mathbf{k}+\mathbf{q})\mathbf{r}-\omega t]+\cos[(\mathbf{k}-\mathbf{q})\mathbf{r}-\omega t]\}=$$

=2Acos $\frac{\mathbf{k}+\mathbf{q}-\mathbf{k}+\mathbf{q}}{2}\cos\frac{2\mathbf{k}\mathbf{r}-2\omega t}{2}=2Acosq\mathbf{r}\cdot\cos(\mathbf{k}\mathbf{r}-\omega t)=a_{\mathbf{q}}a_{\mathbf{k}}.$ (3)

Here

$$a_q=2\cos qr,$$
 (4)

$$a_{\mathbf{k}} = A\cos(\mathbf{kr} \cdot \omega t). \tag{5}$$



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For simplicity, we assume that (1-3) describe waves, with a1, a2, and a varying in directions perpendicular to the plane of the figure.

Note that in (1-3) $\omega \neq c\mathbf{k}$, but

$$\omega = c(\mathbf{q}^2 + \mathbf{k}^2)^{1/2}.$$
 (6)

This relation may be considered as the dispersion relation for the wave packet (3).



Figure 1. The directions of the wave vectors of the considered waves.

3. Wave packet group velocity control

As is well known [1,4,5], the dispersion relation determines the basic properties of a wave packet, including its group velocity. Let's pay attention to the fact that in the case under consideration the dispersion relation (6) is structurally similar to the dispersion relation for a relativistic particle (e.g., [13-15]), having a rest mass.

Thus, the superposition of waves (1) and (2) leads to the formation of a transversely modulated wave packet (3) with a group velocity

$$\mathbf{v} = \frac{\partial \omega}{\partial \mathbf{k}} = \mathbf{c} \frac{\mathbf{k}}{\sqrt{(\mathbf{q}^2 + \mathbf{k}^2)}} = \mathbf{c} \cdot \cos \frac{\vartheta}{2}.$$
 (7)

For the practical use of specific wave packets it is necessary to control their group velocity [10-12,16-22].

It can be seen from formula (7) that the group velocity of the considered wave packet can be controlled by varying the angle ϑ between the wave vectors of its constituent waves.

The considered idealized monochromatic wave packet has only the transverse modulation described by the factor (4). As is well known, the monochromaticity of the waves, including the monochromaticity of the considered wave packet, is only a convenient idealization. Real waves have a variation in frequencies, that is, are quasi-monochromatic. This leads to the presence of longitudinal modulation of wave packets [1-9].

The nonmonochromaticity of the considered wave packet can be created artificially in accordance with a change in the transmitted signal. It is of interest to investigate experimentally the possibility of controlling the group velocity of a packet by changing the angle between the wave vectors of its constituent waves within the coherence time.

4. The possibility of creating a one-dimensional spatial structure in the region of wave packet propagation

The presence of a time-independent factor a_q in expression (3) allows us to represent the considered wave packet in the form

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$$a=B\cos(\mathbf{kr}\cdot\boldsymbol{\omega}t),\tag{8}$$

where

$$B=2A\cos qr=2A\cos ql. \tag{9}$$

Here

$$\mathbf{q} = \left| \mathbf{q} \right| = \mathbf{k} \cdot \mathbf{t} \mathbf{g} \frac{\vartheta}{2},\tag{10}$$

l is the distance from the considered point to the plane of symmetry of the wave packet.

The quasi-amplitude B is periodic in the direction perpendicular to the direction of propagation of the wave packet with a spatial period

$$b = \frac{2\pi}{q} = \frac{2\pi}{k \cdot tg\frac{\vartheta}{2}}.$$
 (11)

It can be seen from formula (11) that the spatial structure arising in the propagation region of the wave packet depends both on the modulus of the vector \mathbf{k} (on the frequency of the interacting waves), and on the angle ϑ .

5. The interaction of two transversely modulated wave packets

In the case when a spatial structure (one-dimensional crystal) with a spatial period b is created in the propagation region of one high intense wave packet, then another low intense wave packet with parameters \mathbf{k} , \mathbf{q} , ϑ will experience the influence of this structure. When considering such an interaction, this spatial periodicity must be taken into account. Let the second wave packet be described by expressions (3, 8). Substituting them in the wave equation

$$\Delta a = \frac{1}{c^2} \frac{\partial^2 a}{\partial t^2},\tag{12}$$

we find that the function a_q satisfies the Helmholtz equation

$$\Delta a_{\mathbf{q}} + q^2 a_{\mathbf{q}} = 0 \tag{13}$$

with periodic boundary conditions

$$a_{\mathbf{q}}(\mathbf{l}+\mathbf{b}) = a_{\mathbf{q}}(\mathbf{l}) \tag{14}$$

or

$$\cos[q(l+b)] = \cos q \tag{15}$$

That is, the allowed values of the transverse wave vector of the second wave are discrete in this case

$$q = \pm \frac{2\pi}{b}n,$$
(16)

where n are integers. Thus, the first wave packet for the second wave packet creates a diffraction grating, the parameters of which are conveniently controlled not only by changing the frequency of the first packet, but also by changing the angle ϑ of the first packet.

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