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The Valuation of Deposit Insurance with Risk Using Fourier Transform

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Abstract. Deposit insurance is a tool to stabilize a banking system. In deposit insurance, the insured risk is the risk of the bank's failure to fulfill its obligations to the customer because of its revoked license. The application of a flat rate deposit insurance system will lead to moral hazard problems, and cause a banking crisis as the banks get involved to risky activities. To anticipate this, it is necessary to design a deposit insurance premium which calculating the different risk levels of each bank. One method that can be used is Fourier transforms. The purpose of this study is to obtain a deposit insurance with risk premium model from the Black-Scholes Model by using Fourier transform. Based on the simulation results, the research found the value of volatility and interest rate is directly proportional to the value of deposit insurance premiums, as well as the value of debt obligations. The high value of deposit insurance premiums is also influenced by the value of dividends, however the value of insurance premiums has changed insignificantly prior to *n* period, therefore the value of deposit insurance premiums from zero to *n* period has changed insignificantly.

1. Introduction

In 1980 until 2007 inflation occurred in the economic sector. This provoked the community to get over and overcome the inflation, one of solution is by investing. The main purpose of investment is to have better life in the long run. In other words, they want to have a profit. One of the investments chosen by the community is a bank investment. A bank is chosen based on risk of loss, company reputation, and liquidity. Investment security, company image and easy disbursement are important for an account holder.

Indonesia established a deposit insurance system to maintain stability in the banking sector after the abolition of *Blanket Guarantee* in 2005 [1]. The choice of deposit insurance is better than the implicit protection scheme which is based on four things; first, a deposit insurance system that functions well will produce something faster, smoother and more predictable when the bank is bankrupt. Second, the deposit insurance system provides better protection for small depositors. Third, providing a way to overcome costs through premium. Finally, the deposit insurance system will be more effective to overcome a *Bank Run* [2].

The application of a deposit insurance system with a flat rate will cause *Moral Hazard* problems. The deposit insurance system can also cause a banking crisis as the banks get involved in risky activities [3]. However, most deposit insurance, including in Indonesia, applies it [1]. Economists

support the existence of deposit insurance that includes risk as it is considered more fair and efficient than flat premiums [4].

Design determination of deposit insurance can be based on theoretical aspects (option theory) and based on empirical aspects (rating camel). Based on empirical aspects, it refers to the determination made by FDIC. However, FDIC determines premiums on the basis of CAMEL Ratings which the results tend to be underpriced and unfair. Determination of the rate (basis point) is lack of an actual risk calculation [5]. Merton's research was determining deposit insurance premiums using an option theory [6]. However, the absence of future risks consideration on determining premiums. Ronn and Verma anticipated the risks by calculating the decline in the value assets and if the assets value are lower than deposits [4].

Fourier transformation is widely used to obtain the solution of various problems. Jiang used Fourier transformation method to apply a price approach at Asian option [7]. Fourier transform can be used as a tool to simplify partial differential equations in the deposit insurance. In this study, it is expected that a formula will be obtained to evaluate the value of deposit insurance premiums including the risk of Black-Scholes Model by applying Fourier transform.

2. Deposit insurance with risk

Deposit insurance is one of tools to stabilize banking systems [2]. Deposit insurance is a customer protection tool to keep the customers' trust and to ensure a finance stability in a banking system [8]. In deposit insurance, the insured risk is the bank failure in fulfilling its obligations to customers because the license was revoked. The revocation is due to a bank financial health problem and a supervisor tolerance level on the bank condition.

Merton [6] examined " an isomorphic correspondence between loan guarantees and common stock put options, and then to use the well-developed theory of option pricing to derive the formula". Putri [9] described the similarities between the deposit insurance and option mechanism. The stock as the underlying asset of option contract is similar to the bank's asset. The bank and guarantor of an insure deposit are considered as the holder and the writer of deposit insurance.

The mathematical model of deposit insurance proposed by Merton [6] based on a Black-Scholes model and risk adjusted deposit insurance by Ronn and Verma [4]. The dynamic of bank's asset value V is assumed to follow a geometric Brownian motion

$$dV = \mu V \, dt + \sigma V \, dz \tag{1}$$

Where μ is the expected rate return, σ is the volatility and Z is the standart Wiener process.

The risk factor in this deposit insurance is adding dividends. The effect of dividends on an equity is recorded based on the type of issued dividends. When a company issues dividends to its shareholders, the value of the dividends is deducted from their retained earnings. Even if the dividends are issued as additional shares, the value of the shares is deducted. However, a cash dividends result in a reduction in retained earnings directly, stock dividends resulting in the transfer of funds from retained earnings to paid-in capital. While cash dividends reduce an equity, the stock dividends only rearrange the allocation of equity fund. In other words, the value of asset V from a bank will decrease by $V\delta$ so that for n periods the asset value is equal to $V(1-\delta)^n$.

A portfolio will consist of a deposit insurance G,V is bank's assets and risk factor $(1-\delta)^n$, dividend per dollar of value of the assets δ and $\Delta = \frac{\partial G}{\partial V}$ unit of bank's asset V. so we have the portfolio as the following,

$$\Pi = G - \Delta V (1 - \delta)^n \tag{2}$$

If the deposit insurance value $G(V, t)$ depends only on V and t, then by using Ito's lemma based on Eq(1) and Eq(2), the infinitesimal change of the portofolio value implies that

$$d\Pi = \frac{\partial G}{\partial t} + \frac{1}{2}\sigma^2 (V(1-\delta)^n)^2 \frac{\partial^2 G}{\partial V^2} \tag{3}$$

Return from Π invested into risky assets can be seen as a growth $r \Pi \, dt$ at time dt, with interest rate r and substituting Eq (3), it follows that

$$r \Pi \, dt = \left(\frac{\partial G}{\partial t} + \frac{1}{2} \sigma^2 \, (V \, (1 - \delta)^n \,)^2 \frac{\partial^2 G}{\partial V^2} \right) dt \tag{4}$$

and Eq (4) implies,

$$\frac{\partial G}{\partial t} + \frac{1}{2} \sigma^2 \, (V \, (1 - \delta)^n \,)^2 \frac{\partial^2 G}{\partial V^2} + \frac{\partial G}{\partial V} \, r \, V - r \, G = 0 \tag{5}$$

Eq (5) is called the Black-Scholes model of the deposit insurance with risk.

In the case of asset value V is less than total debt B, assuming that all pre-insurance debt for seniority is similar, the deposit holders will be entitled to the value of their future deposits, or less than total debt.[4]. In other word, they will obtain

$$\min \left\{ F_v \, (B_1) \, , \frac{V_T B_1}{B_1 + B_2} \right\}$$

Thus, the maturity value of the deposit insurance is given by

$$\max \left\{ 0, F_v \, (B_1) - \frac{V_T B_1}{B_1 + B_2} \right\}$$

Where

F_v = denotes the future value operator
V_T = is the terminal value of the bank's assets
B_1 = the face value of the insured deposits
B_2 = the face value of all debt liabilities other than the insured deposits
B = $B_1 + B_2$ = face value of total debt liabilities

3. Solution using Fourier transform

In this section we discuss the solution of BlackScholes partial differential equations Eq (5) to find the solution of deposit insurance in BlackScholes model including the risk.

To simplify the calculations and guarantee that eq (5) is integral able and fulfill Fourier domain by following transform;

$$x = \ln \left(\frac{V_T B_1 / B}{F_v \, B_1} \right)$$

$$\tau = \frac{\sigma^2}{2} \, (T - t)$$

$$Q \, (x \, , \tau) = \frac{e^{2x} G \, (V, t)}{F_v \, B_1}$$

With $= \frac{2r}{\sigma^2}$, then Eq (5) become:

$$-\frac{\partial Q}{\partial \tau} + k \left(\frac{\partial Q}{\partial x} + 2Q \right) + \left(\frac{\partial^2 Q}{\partial x^2} + 3 \frac{\partial Q}{\partial x} + 2Q \right) - k \, Q = 0 \tag{6}$$

In order to fulfill Fourier transformation requirements, the condition of the risk adjusted to deposit insurance is

$$\lim_{|x| \to \infty} Q \, (x, \tau) = 0$$

and

$$\lim_{|x| \to \infty} Q' \, (x, \tau) = 0$$

Fourier transform in each equation in Eq(6). So, we result the solution as the following

$$f(\omega,\tau) = e^{(k+2)\tau} f(\omega,0) e^{(-i\omega(k+3)-\omega^2)\tau} \tag{7}$$

After we know that the solution in Fourier space, then we find invers Fourier transform. Previously the characteristic function of X will be seen.

If $X \sim N(\mu,\sigma^2)$ then the characteristic function of X [10] is

$$\chi(\omega) = e^{i\omega\mu - \frac{1}{2}\sigma^2\omega^2}$$

So, the characteristic function of Eq(7) is:

$$\chi(\omega) = e^{(-i\omega(k+3)-\omega^2)\tau}$$

Which mean $-(k+3)\tau$ and varians 2τ. By convolution property in Fourier transform, we get

$$\mathcal{F}\{Q(\omega,\tau)\} = e^{(k+2)\tau} \mathcal{F}\left\{Q(\omega,\tau) * \frac{1}{2\sqrt{\pi\tau}} e^{-\frac{(\omega+(k+3)\tau)^2}{4\tau}}\right\}$$

$$\mathcal{F}^{-1}\{\mathcal{F}Q\{(\omega,\tau)\}\} = e^{(k+2)\tau} \mathcal{F}^{-1}\left\{\mathcal{F}\left\{Q(\omega,\tau) * \frac{1}{2\sqrt{\pi\tau}} e^{-\frac{(\omega+(k+3)\tau)^2}{4\tau}}\right\}\right\}$$

Then

$$Q(\omega,\tau) = e^{(k+2)\tau}\left(Q(\omega,\tau) * \frac{1}{2\sqrt{\pi\tau}} e^{-\frac{(\omega+(k+3)\tau)^2}{4\tau}}\right) \tag{8}$$

Considering $W(\omega,\tau) = \frac{1}{2\sqrt{\pi\tau}} e^{-\frac{(\omega+(k+3)\tau)^2}{4\tau}}$, then Eq (8) can be rewritten as the following

$$Q(\omega,\tau) = e^{(k+2)\tau}\left(Q(\omega,0) * W(\omega,\tau)\right)$$

$$Q(\omega,\tau) = \frac{e^{(k+2)\tau}}{2\sqrt{\pi\tau}} \int_{-\infty}^{\infty} Q(\omega,0) e^{-\frac{(\omega-x+(k+3)\tau)^2}{4\tau}} d\omega \tag{9}$$

Then substituting the initial condition of deposit insurance with risk $Q(\omega,0) = (e^{-2\omega} - e^{-\omega})^+$ to Eq (9). $(e^{-2\omega} - e^{-\omega})^+$, that will have negative value if and only if $\omega > 0$ then we get the invers Fourier transform is:

$$Q(\omega,\tau) = \frac{e^{(k+2)\tau}}{2\sqrt{\pi\tau}} \int_{-\infty}^{0} e^{-\frac{8\omega\tau+(\omega-x-(k+3)\tau)^2}{4\tau}} dw - \frac{e^{(k+2)\tau}}{2\sqrt{\pi\tau}} \int_{-\infty}^{0} e^{\omega} e^{-\frac{8\omega\tau+(\omega-x-(k+3)\tau)^2}{4\tau}} dw \tag{10}$$

To simplify calculation Eq (10)

$$A_1 = \frac{e^{(k+2)\tau}}{2\sqrt{\pi\tau}} \int_{-\infty}^{0} e^{-\frac{8\omega\tau+(\omega-x-(k+3)\tau)^2}{4\tau}} dw$$

$$A_2 = \frac{e^{(k+2)\tau}}{2\sqrt{\pi\tau}} \int_{-\infty}^{0} e^{\omega} e^{-\frac{8\omega\tau+(\omega-x-(k+3)\tau)^2}{4\tau}} dw$$

To obtain solution of A_1 and A_2 presented with cumulative density function of a standart normal distribution

$$A_1 = e^{-k\tau-2x}\,\Phi\left(\frac{-x-(k-1)\tau}{\sqrt{2\tau}}\right) \tag{11}$$

and

$$A_2 = e^{-x}\,\Phi\left(\frac{-x-(k+1)\tau}{\sqrt{2\tau}}\right) \tag{12}$$

Solution of Eq(9) after invers is

$$Q(x,\tau) = e^{-k\tau-2x}\,\Phi\left(\frac{-x-(k-1)\tau}{\sqrt{2\tau}}\right) - e^{-x}\,\Phi\left(\frac{-x-(k+1)\tau}{\sqrt{2\tau}}\right)$$

Than by reconsidering the initial G form, we will get

$$G(V,t) = B_1\,\Phi(z_2) - \frac{B_1}{B}\,V\,(1-\delta)^n\,\Phi(z_2) \tag{13}$$

$$z_1 = \frac{\ln\left(\frac{e^{rT}B}{V(1-\delta)^n}\right) - \left(r-\frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$$

$$z_2 = z_1 + \sigma\sqrt{T-t}$$

where $\Phi(\cdot)$ is comulative density functions of normal distribution. Eq (13) is an analytical solution for deposit insurance with risk. Using Fourier transformation, this solution is similar to Ronn's paper.

4. Simulation

Based on Eq (13) in which it's an analytical solution of a deposit insurance including the risk that can be found by numerical simulation using MATLAB. By this simulation, we can see some figures representing premium of deposit insurance that must paid by the bank.

If asset of bank X (trillion in IDR) is $V = 100$, an interest rate $r = 15\%$ and dividend $\delta = 0.00001$, we obtain premium of deposit insurance with risk as the following
For B1= 60 (trillion in IDR)

Tabel 1. Deposit Insurance with Risk $B_1 = 60$

| B_1	B_2	G				
		$\sigma^2 = 0.004$	$\sigma^2 = 0.0045$	$\sigma^2 = 0.005$	$\sigma^2 = 0.0055$	$\sigma^2 = 0.006$
60	60	12.4396	12.4397	12.4400	12.4404	12.4411
60	50	8.1288	8.1359	8.1446	8.1549	8.1666
60	40	3.3793	3.4460	3.5110	3.5746	3.6363
60	30	0.4099	0.4757	0.5403	0.6037	0.6658

For 60 (trillion in IDR) there are 3 conditions; face value of total debt liabilities is greater than the bank assets $B > V$ when the face value of all debt liabilities other than B_2 insured deposits are 60 and 50 (trillion in IDR). The face value of total debt liabilities equals to bank $B = V$ assets when the face value of all other insured deposits B_2 debt liabilities is 40 (trillion in IDR), and face value of total debt liabilities smaller than bank assets $B < V$ when the face value of all debt liabilities other than the insured deposits B_2 debt liabilities is 30 (trillion in IDR). The greater the face value of all debt liabilities other than the insured deposits B_2 causes the face value of total debt liabilities B grows bigger, so the value of the deposit insurance premium that must be paid is bigger. Likewise with volatility σ^2, the greater the volatility σ^2 on the face value of total debt liabilities B, the greater the value of deposit insurance premiums.
B1 = 50 (trillion in IDR)

Table 2. Deposit Insurance with Risk $B_1 = 50$

B_1	B_2	G				
		$\sigma^2 = 0.004$	$\sigma^2 = 0.0045$	$\sigma^2 = 0.005$	$\sigma^2 = 0.0055$	$\sigma^2 = 0.006$
50	70	10.3663	10.3664	10.3667	10.3670	10.3676
50	60	6.7740	6.7799	6.7872	6.7957	6.8055
50	50	2.8161	2.8717	2.9258	2.9787	3.0303
50	40	0.3416	0.3964	0.4503	0.5031	0.5549

When $B_1 = 50$ equals to $B_1 = 60$. In this case, there are three conditions; the face value of total debt liabilities is greater than the assets of bank $B > V$ when the face value of all debt liabilities other than the insured deposits B_2 of 70 and 60 (trillion in IDR). The face value of total debt liabilities equals to the assets of bank $B = V$ when the face value of all other than the insured deposits B_2 debt liabilities (trillion in IDR), and the face value of total debt liabilities is smaller than assets bank $B < V$ when the face value of all debt liabilities other than the insured deposits B_2 (trillion in IDR). The greater the face value of all other debt liabilities than the insured deposits B_2 causes the face value of the total debt liabilities B grows bigger, so the value of the deposit insurance premium that must be paid is bigger. Likewise with the volatility value, the greater the value of volatility σ^2 on the face value of the same total debt liabilities B causes the bigger the value of deposit insurance premiums.
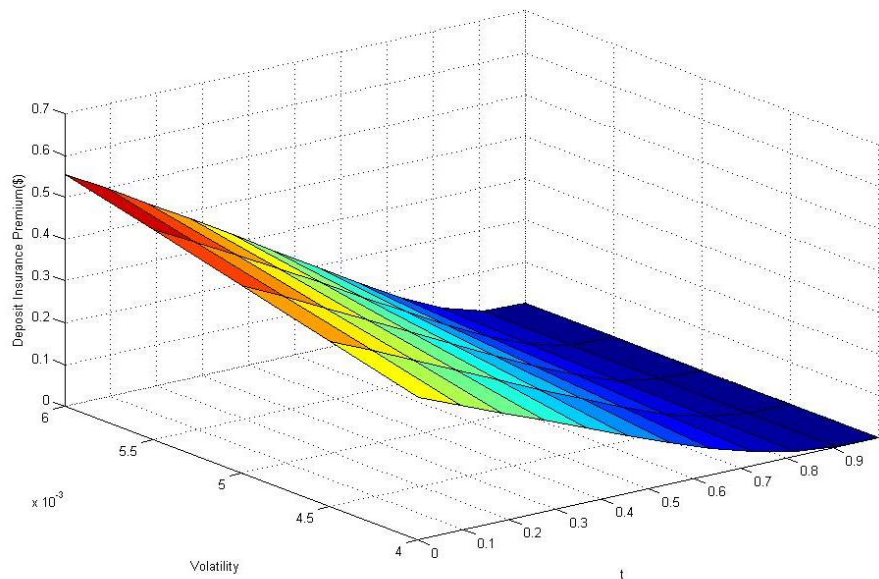
Figure 1. Premium Deposit Insurance with Risk $B_1 = 50$ and $B_2 = 50$

Visualizing the value of deposit insurance premiums including the risks when B1 = 50 and B2 = 50 shown in Figure (1). A debt value of a company influenced by three things, they are: risk rate of return, provisions and limit agreements, and probability that the company can't fulfill the contract partially or fully[11]. These three things are the basis increasing value of deposit insurance premiums. To recognize the effect of interest rates r and volatility on premium values is presented in Figure (2)
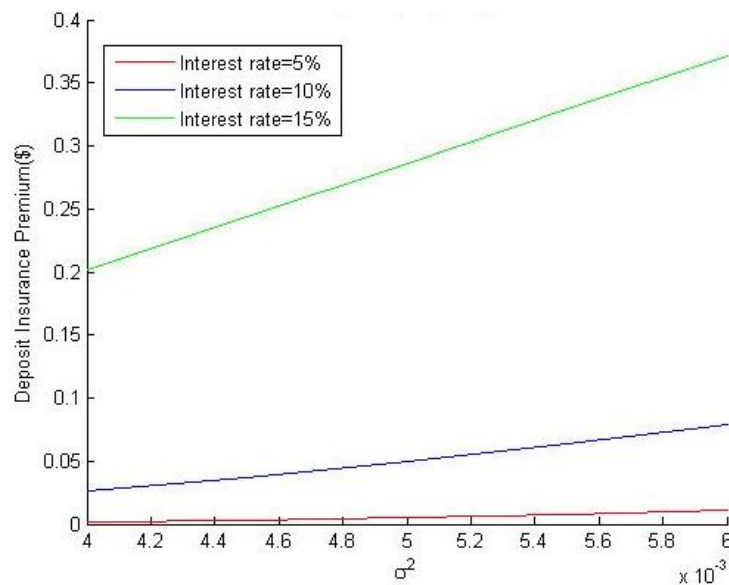
Figure 2. The Effect of Interest Rate and Volatility on Deposit Insurance Premium

Based on figure (2) the interest rate 5%, the increasing of insurance premiums is seen insignificant. For interest rate 10%, the increasing is faster than interest rate 5%. For interest rate 15% causes the increasing of insurance premiums more significant. So, it can be said that by taking the same volatility value, if the interest rate is greater, then the increase in premium value will be sharper. The greater volatility and interest rate cause greater premiums that must be paid off.

Different volatility values show different results. The lowest deposit insurance premium value G is taken at $\sigma^2 = 0.004$ and the highest deposit insurance premium value G is taken at $\sigma^2 = 0.006$. This means that the greater the volatility that is determined, the greater the premium that must be paid.

The effect of dividends on the value of deposit insurance premiums that includes risks shown in Figure (3).
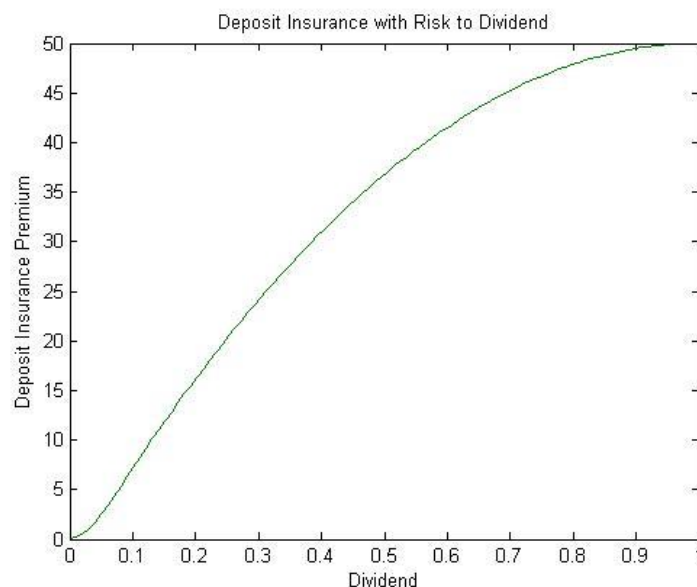

Figure 3. The Effect of Dividend on The Value Deposit Insurance Premium

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Based on Figure (3) the value of deposit insurance premiums is higher as the value of distributed dividends. The dividend distribution affects the bank assets. The greater distributed dividend, the greater the assets reduction.
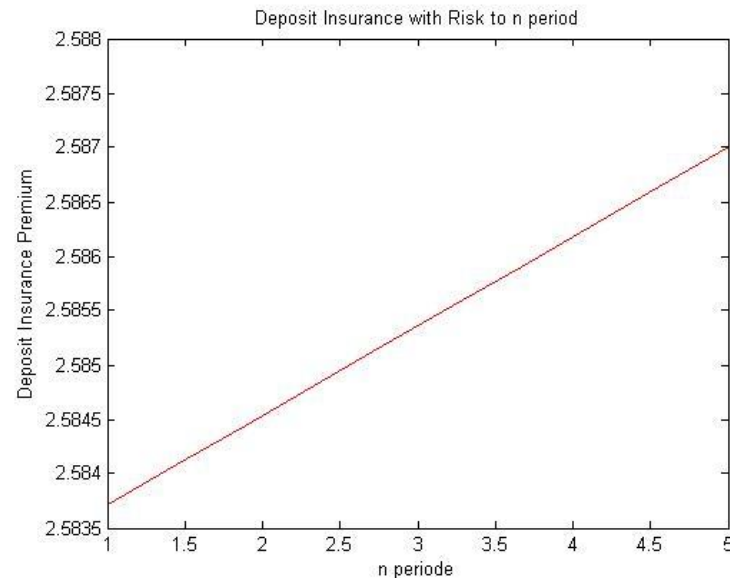

Figure 4. The Effect of n on The Value deposit Insurance Premium

Figure (4) shows the effect on the value of deposit insurance premiums. In this simulation, the value of asset V undergoes small changes up to *n*-th period, therefore the value of deposit insurance premiums from zero period to *n*-th period is nearly unchanged.

5. Conclusion

Based on the results and discussion presented in the previous chapter, the following points can be summarized:

1. The model of deposit insurance with risk premium in the form of an equation in second order partial ferential which is influenced by variable *V* (asset value), *t* (time), and *G* (premium value). The influential parameters are σ^2 (volatility), *r* (interest rate), δ (dividend) and *n* (time period).

2. The solution of deposit insurance premium including the risk is an analytical solution. The value of deposit insurance premium *G* is influenced by the amount of nominal value of the insured deposit B_1 minus amount of asset value *V*. The insured debt value B_1 is influenced by the normal CDF distribution of z_2 which is a combination of z_1 and volatility that is affected by maturity. z_1 is influenced by interest rate *r*, volatility σ^2, asset value *V*, total debt value *B*, dividend δ and maturity date T.

3. Based on the simulation, it can be concluded that:
   - The greater volatility and interest rate, causes greater deposit insurance premium that must be paid.
   - The more debt obligations, the more deposit insurance premiums which must be paid. The value of deposit insurance premiums is higher as the distributed value dividends.
   - The value of deposit insurance premiums has changed marginally up to the *nth* period. Therefore, the value of deposit insurance premiums is nearly seen from zero to period *n* with no changes.

Acknowledgments

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