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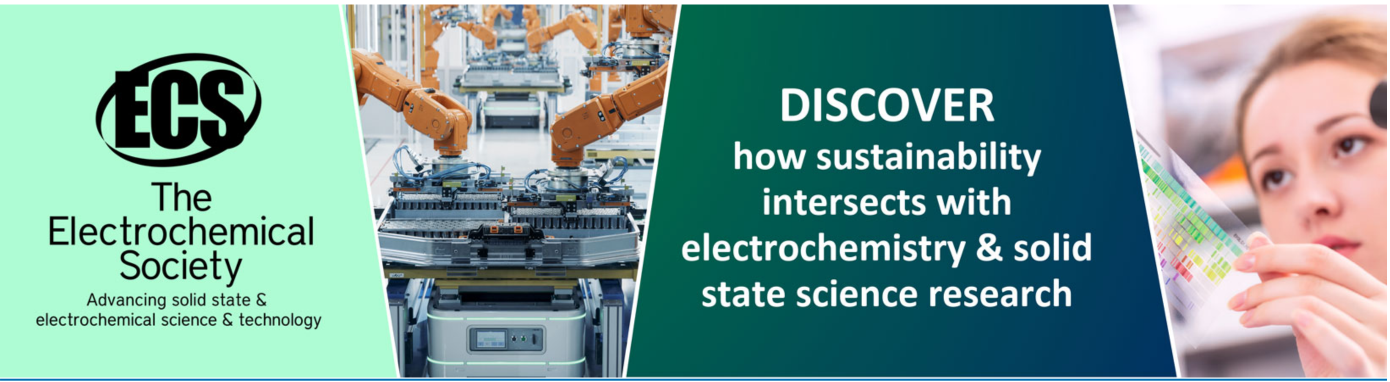

Forming of three-dimensional optical fields consistent with the superposition of scalar spherical harmonics

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Abstract. Spherical functions are the angular part of the family of orthogonal solutions of the Laplace equation written in spherical coordinates. They are widely used to study physical phenomena in spatial domains bounded by spherical surfaces and in solving physical problems with spherical symmetry. In this paper, the superposition equation of spherical harmonics satisfying the Helmholtz equation was obtained. Modelling and visualization of three-dimensional fields, coordinated with separate spherical harmonics and their superpositions, was carried out.

1. Introduction

Due to the decrease in the size of optical devices, much attention has recently been paid to the description of non-paraxial propagation of light fields [1-12] and the development of algorithms for modeling such propagation [13-26].

The nonparaxial scalar model based on Rayleigh-Sommerfeld theory [27] allows to obtain results at very close distances from the aperture [28, 29]. Note that the use of a scalar wave model in the near diffraction zone is valid only for one of the transverse components of the electric field. Moreover, with the increase in the numerical aperture, the role of the longitudinal component of the electric field becomes very important, its contribution may exceed the contribution of transverse components [11]. However, there are known situations [30-32] when the substance or device is sensitive only to the transverse or longitudinal components of the electric field. Thus, scalar field calculations become relevant not only for individual components, but also for the whole picture.

Note that the vector version of Rayleigh-Sommerfeld integrals, as well as the method of plane wave decomposition, have a representation of various components of the electromagnetic field through close expressions, which allows the use of parallel calculation algorithms and high-performance computing facilities [33, 34].

Laser beams with screw phase features attract much attention of researchers [35-45]. This is due to their special properties, including the presence of orbital angular momentum, which is used in optical manipulation for rotation of micro-objects captured by the beam [46, 47], for compaction of information transfer channels [48-54], as well as for structuring the surface of materials [55-60].

As a rule, the propagation of such beams is considered in a cylindrical coordinate system [24-26, 39-42, 61, 62]. However, the shape of objects and optical elements in some applications involve the use of a spherical coordinate system. In both cases, the wave function decomposition of the corresponding systems is used.

In this paper I consider the optical fields, which are a superposition of scalar spherical wave functions. The construction and visualization of such fields is the first step to modeling the propagation of optical fields based on spherical harmonics decomposition.

2. Theoretical part

The Helmholtz equation in spherical coordinates has the following form:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi(r,\theta,\varphi)}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi(r,\theta,\varphi)}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi(r,\theta,\varphi)}{\partial\varphi^2}+k^2\psi(r,\theta,\varphi)=0. \quad (1)$$

Consider the solution in the form:

$$\psi(r,\theta,\varphi)=R(r)\cdot\Theta(\theta)\cdot\Phi(\varphi). \quad (2)$$

After substitution (2) in (1) we obtain:

$$\frac{\Theta(\theta)\cdot\Phi(\varphi)}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial R(r)}{\partial r}\right)+\frac{R(r)\cdot\Phi(\varphi)}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Theta(\theta)}{\partial\theta}\right)+$$
$$+\frac{R(r)\cdot\Theta(\theta)}{r^2\sin^2\theta}\frac{\partial^2\Phi(\varphi)}{\partial\varphi^2}+k^2R(r)\cdot\Theta(\theta)\cdot\Phi(\varphi)=0. \quad (3)$$

Multiply (3) on $\frac{r^2\sin^2\theta}{R(r)\cdot\Theta(\theta)\cdot\Phi(\varphi)}$, get:

$$\frac{\sin^2\theta}{R(r)}\frac{\partial}{\partial r}\left(r^2\frac{\partial R(r)}{\partial r}\right)+\frac{\sin\theta}{\Theta(\theta)}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Theta(\theta)}{\partial\theta}\right)+\frac{1}{\Phi(\varphi)}\frac{\partial^2\Phi(\varphi)}{\partial\varphi^2}+k^2r^2\sin^2\theta=0. \quad (4)$$

Since only the third term depends on φ, let:

$$\frac{1}{\Phi(\varphi)}\frac{\partial^2\Phi(\varphi)}{\partial\varphi^2}=-m^2, \quad (5)$$

then

$$\Phi(\varphi)=\exp(im\varphi). \quad (6)$$

After substituting (5) for (4) and dividing by $\sin^2\theta$, we obtain:

$$\frac{1}{R(r)}\frac{\partial}{\partial r}\left(r^2\frac{\partial R(r)}{\partial r}\right)+\frac{1}{\Theta(\theta)\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Theta(\theta)}{\partial\theta}\right)-\frac{m^2}{\sin^2\theta}+k^2r^2=0. \quad (7)$$

Next, denoting $x=\cos\theta$, $dx=-\sin\theta d\theta$, and $\sin^2\theta=1-x^2$, we get:

$$\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Theta(\theta)}{\partial\theta}\right)-\frac{m^2}{\sin^2\theta}\Theta(\theta)=\frac{1}{\sin\theta}\left(\cos\theta\frac{\partial\Theta(\theta)}{\partial\theta}+\sin\theta\frac{\partial^2\Theta(\theta)}{\partial\theta^2}\right)-\frac{m^2}{\sin^2\theta}\Theta(\theta)=$$
$$=\frac{\partial^2\Theta(\theta)}{\partial\theta^2}+\frac{\cos\theta}{\sin\theta}\frac{\partial\Theta(\theta)}{\partial\theta}-\frac{m^2}{\sin^2\theta}\Theta(\theta)=\sin^2\theta\frac{\partial^2y(x)}{\partial x^2}-2\cos\theta\frac{\partial y(x)}{\partial x}-\frac{m^2}{\sin^2\theta}y(x)=$$
$$=\left(1-x^2\right)\frac{\partial^2y(x)}{\partial x^2}-2x\frac{\partial y(x)}{\partial x}-\frac{m^2}{\left(1-x^2\right)}y(x).$$

Given that Legendre functions $y(x)=P_n^m(x)$ satisfy the equation:

$$\left(1-x^2\right)\frac{d^2y(x)}{dx^2}-2x\frac{dy(x)}{dx}+\left[n(n+1)-\frac{m^2}{\left(1-x^2\right)}\right]y(x)=0, \quad (8)$$

for $\Theta(\theta)=P_n^m(\cos\theta)$ we get:

$$\frac{1}{\Theta(\theta)\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Theta(\theta)}{\partial\theta}\right)-\frac{m^2}{\sin^2\theta}=-n(n+1) \quad (9)$$

Then instead of (7) you can write:

$$\frac{1}{R(r)}\frac{\partial}{\partial r}\left(r^2\frac{\partial R(r)}{\partial r}\right)-n(n+1)+k^2r^2=0. \tag{10}$$

The solution to this equation

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial R(r)}{\partial r}\right)+\left[k^2r^2-n(n+1)\right]R(r)=0,$$

are spherical Bessel functions, in particular:

$$R(r)=j_n\left(kr\right). \tag{11}$$

Thus, the Helmholtz equation (1) is satisfied by the fields representing the superposition of spherical harmonics:

$$W(r,\theta,\varphi)=\sum_{n,m}c_{nm}\,j_n\left(kr\right)Y_{nm}(\theta,\varphi), \tag{12}$$

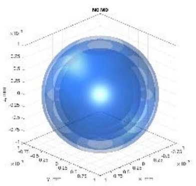
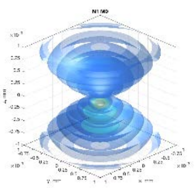
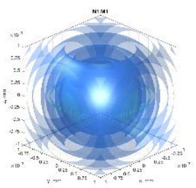
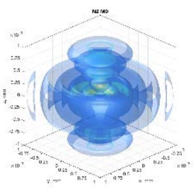
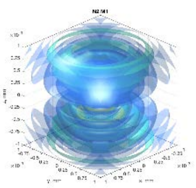
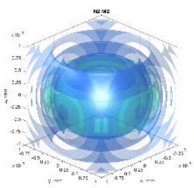
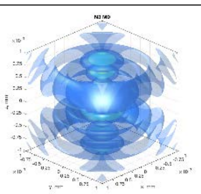
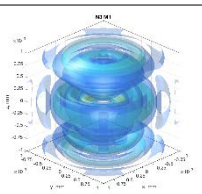
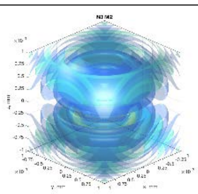
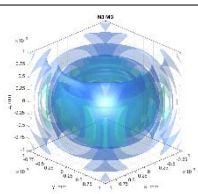
where

$$Y_{nm}(\theta,\varphi)=\left(-1\right)^m\sqrt{\frac{2n+1}{4\pi}\frac{\left(n-|m|\right)!}{\left(n+|m|\right)!}}P_n^m\left(\cos\theta\right)\exp\left(im\varphi\right),\ n\geq|m|. \tag{13}$$

3. Modelling results

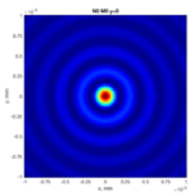
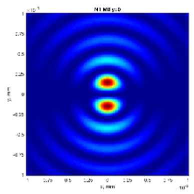
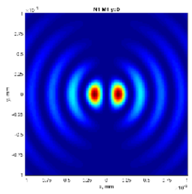
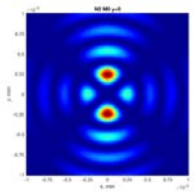
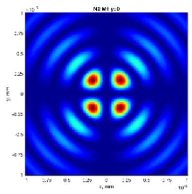
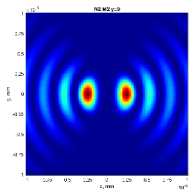
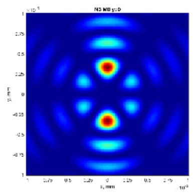
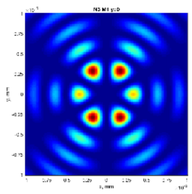
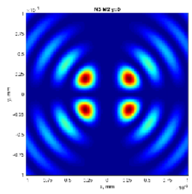
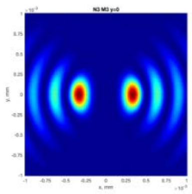
Table 1 presents the results of the formation of three-dimensional optical fields consistent with individual spherical harmonics with fixed indices *n* and *m*. Calculations were performed for $\lambda=0{,}5\ \mu m$, in the range of coordinates $x,y,z\in\left[-2\lambda,2\lambda\right]$.

Table 1. Individual three – dimensional optical fields.

		m			
		0	1	2	3
n	0				
	1				
	2				
	3				

The resulting fields have an axial (z-axis) symmetry, so for a better representation of the fields in table 2 their cross sections are presented, which shows the dependence of the structure of the optical field on the orders of the function coefficients.

Table 2. Cross sections of three-dimensional optical fields, for y=0.

		m			
		0	1	2	3
n	0				
	1				
	2				
	3				

The obtained cross sections indicate that the distribution of the magnitudes in optical fields occurs according to a certain principle depending on n and m.

In the cross section at $m=0$, the space is "separated" by n planes, forming $2n$ energy "petals". As m increases, the number of "separable" planes decreases and becomes equal to $n–m+1$, since at $m>0$ there is a constant vertical separation of the whole picture, increasing proportionally to m.

In a three-dimensional field at $m=0$, a complex of toroidal and two cone-shaped structures is formed in an amount equal to $n+1$. With increasing m, the number of structures decreases and becomes equal to $n–m$. Cone-shaped structures located at the poles at $m=0$ are modified and transformed into toroidal structures in the cases of increasing m due to the appearance of an energy gap increasing according to m.

Table 3 presents the three-dimensional fields agreed by superpositions for all possible m for a certain n and it cross section for axes.

Table 3 and table 4 illustrate the three-dimensional optical fields consistent with the solution proposed in the theoretical part. The resulting three-dimensional fields are described by superpositions with all possible coefficients m for a particular order n (table 3). The results shown in table 3 and table 4 show the effect of higher order n fields.

Table 3 and table 4 show similar results for different superpositions, which indicates a strong influence of components of fields with high order n.

The results in table 5 show that superpositions provide a wide range of different optical fields with varying degrees of complexity of structures. From simple as an hourglass to complex, such as some twisted "drops".

Table 3. Three – dimensional optical fields and their cross sections for fixed order *n*.

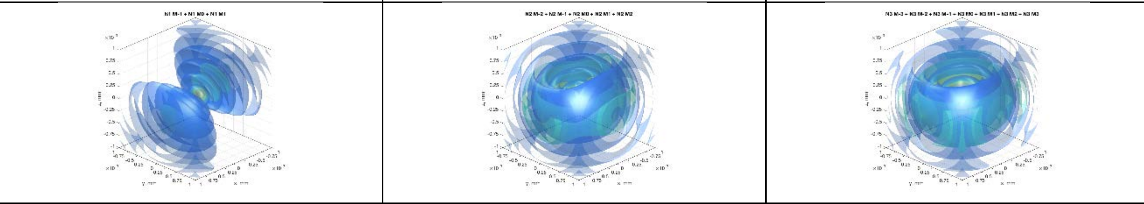
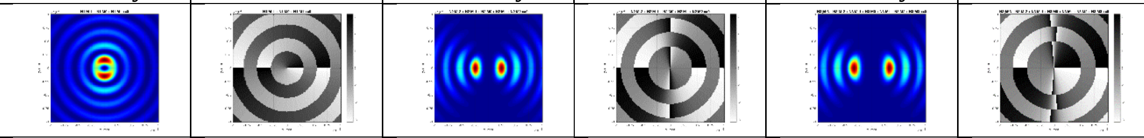
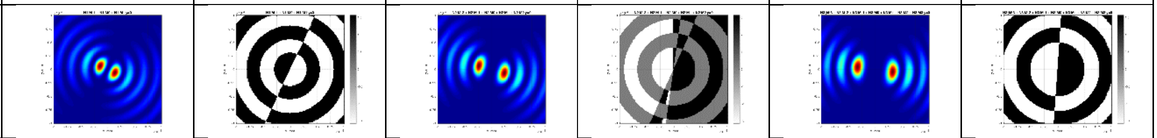
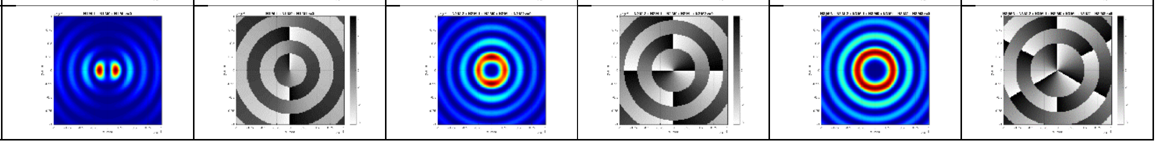
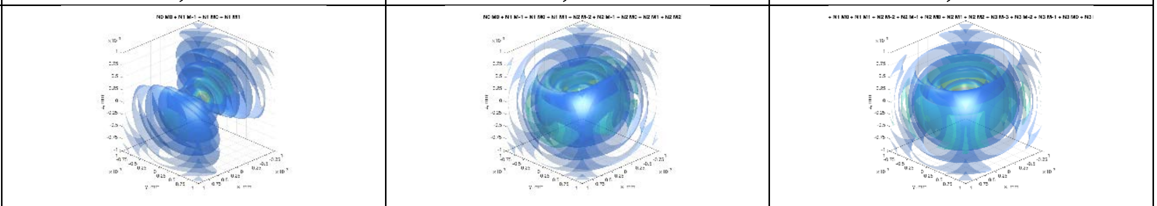
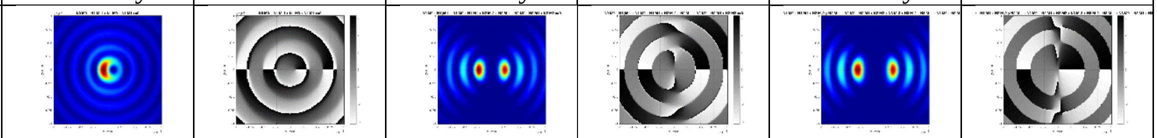
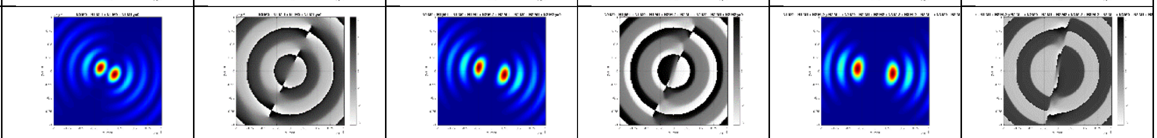
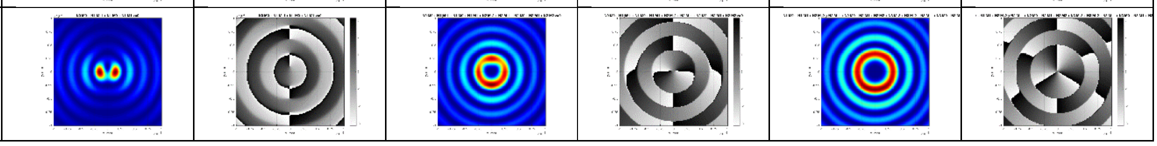
	n=1, m=-1:1		*n=2, m=-2:2*		*n=3, m=-3:3*	
3D						
	Intensity	Phase	Intensity	Phase	Intensity	Phase
Plane ZY						
Plane XZ						
Plane XY						

Table 4. Three – dimensional optical fields and their cross sections for all order *n*.

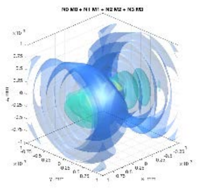
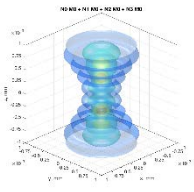
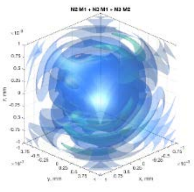
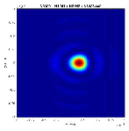
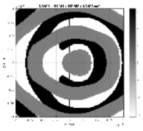
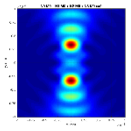
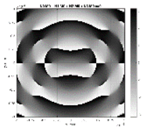
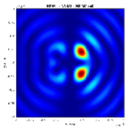
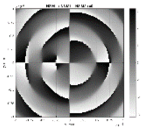
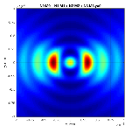
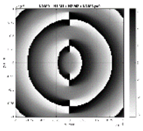
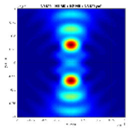
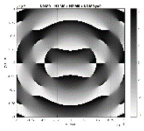
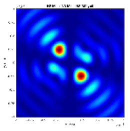
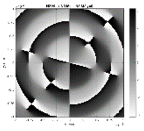
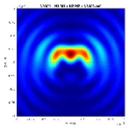
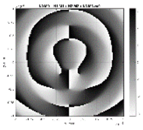
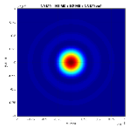
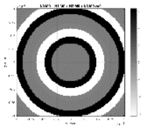
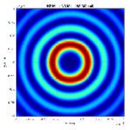
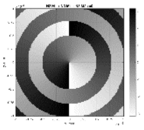
	n=0:1, m=-1:1		*n=0:2, m=-2:2*		*n=0:3, m=-3:3*	
3D						
	Intensity	Phase	Intensity	Phase	Intensity	Phase
Plane ZY						
Plane XZ						
Plane XY						

4. Conclusion

In this work, the solution of the Helmholtz equation in the form of superpositions of scalar spherical harmonics was obtained, a software tool that implements a mathematical model of this solution was developed, 25 solutions were generated and visualized in the form of three-dimensional fields with different coefficients, the analysis of the results was carried out.

The results showed the dependence of the structure of a single spherical harmonic on its orders n and m, as well as the weight effect on the overall picture of higher orders in superpositions.

Table 5. Complex structures of three – dimensional optical fields and their cross sections.

	n=0:3, m=n		*n=0:3, m=0*		*n=2,3,3, m=1,1,2*	
3D						
	Intensity	Phase	Intensity	Phase	Intensity	Phase
Plane ZY						
Plane XZ						
Plane XY						

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