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The application of fuzzy analytic hierarchy process (FAHP) approach to solve multi-criteria decision making (MCDM) problems

N F Mahad[1*], NMohamed Yusof[1] and N F Ismail[2]

[1]Universiti Teknologi MARA Cawangan Negeri Sembilan, Kampus Seremban, Persiaran Seremban Tiga/1, Seremban 3, 70300 Seremban, Negeri Sembilan, Malaysia
[2]Universiti Teknologi MARA Cawangan Negeri Sembilan, Kampus Kuala Pilah, Parit Tinggi, 72000 Kuala Pilah, Negeri Sembilan, Malaysia

*faradilah315@uitm.edu.my

Abstract. This paper focuses on the application of Fuzzy Analytic Hierarchy Process (FAHP) approach to solve multi-criteria decision making (MCDM) problems. MCDM is a process which involves a decision maker or a group of decision makers to evaluate and to choose the best alternatives based on the criteria decided by the decision maker(s). A real-life empirical example about supplier selection is used to implement the FAHP method. The objectives of the study are: (1) to implement FAHP approach with different linguistic scales to solve MCDM problems; and (2) to compare the relative weights of each alternative with respect to the criterion that was computed using different linguistic scales. There are three sets of scales denoted as S1, S2 and S3 used in this paper. Four criteria which are delivery, price, service and payment terms and three alternatives which are Supplier A, Supplier B and Supplier C has been considered in this study. The first objective is achieved since FAHP can be used to solve the MCDM problems. Meanwhile, for second objective, the Coefficient of Variations (CV) has been used to do the comparison. The findings revealed that scale S2 is the most preferable linguistic scale for this case study.

1. Introduction

Multi-criteria decision making (MCDM) is about making choices, supporting and understanding them, in the presence of multiple and conflicting criteria [1]. MCDM can be defined as a process of evaluating a set of alternatives or options and selecting the best alternatives with respect to the related criteria. MCDM is divided into two types which are multi-objective decision making (MODM) and multi-attribute decision making (MADM). MODM is a problem with an infinite number of possible values for the decision arguments and hence for the objective functions [2]. MODM is naturally associated with mathematical programming when dealing with optimization problems. Meanwhile, MADM methods is used to solve problems with discrete decision spaces and a predetermined or a limited number of alternatives [3]. Contrast with MODM techniques, MADM heavily involves human participation and judgments [3].

The MCDM technique deals with complex problems by breaking them into more manageable pieces to allow data and judgements to be brought to bear on the pieces [2]. Thus, the MCDM problems can be structured as a hierarchy involves a goal, a set of criteria or sub-criteria (if any) and a set of alternatives. One of the most powerful MCDM methods that can be used to solve problem with conflicting and multiple criteria is Analytic Hierarchy Process (AHP). The advantages of AHP are the relative ease in which it can handles multiple criteria, it is an easily understandable method, and it can effectively handle both qualitative and quantitative data [4]. However, AHP is often criticized for its inability to handle the uncertainty of the decision maker's perception [5]. Since AHP method are unable to handle the imprecision and vagueness of human thinking, FAHP is suggested by many researchers to solve MCDM problems including selection problems. Fuzzy sets or fuzzy numbers which incorporates the vagueness of human thinking is used when one needs to deal with the uncertain judgments to express criteria importance over another [6].

In this paper, the application of FAHP approach with different sets of linguistic scales is employed to solve the MCDM problems. Thus, the objectives of the study are: (1) to implement FAHP approach with different linguistic scales to solve MCDM problems; and (2) to compare the relative weights of each alternative with respect to criterion that was computed using different linguistic scales. A real-life empirical example about supplier selection is used to demonstrate the application of FAHP. There are 6 sections that is included in this study which are (1) Introduction, (2) Literature Review, (3) Data Analysis, (4) Result and Discussion, (5) Conclusion and Recommendation and (6) Acknowledgement.

2. Literature Review

AHP method is often criticized for the use of unequal scales and the inability to adequately handle the uncertainties and accuracy that exists in the process of pair-wise comparison[7]. The traditional AHP is also unable to accurately reflect the human thinking style [8] and it is problematic to use the right value to express the expert's view in performing a comparison of alternatives [9]. In situations with incomplete information, crisp numbers are incapable to describe alternatives with different criteria precisely.

Thus, FAHP is used to overcome the problems. FAHP is a combination of fuzzy set theory and AHP hierarchical analysis to make a single decision method and it is the best tool to deal with qualitative evaluation [10]. Zadeh proposed fuzzy set based on a degree of membership to evaluate the alternatives appropriately[11]. A fuzzy set is a class of objects with a continuum of grades of membership characterized by a membership function, which assign to each object a membership degree between zero and one[12] to mathematically represent uncertainties and vagueness to generate decision. Fuzzy set theory providing a more widely information rather than classic sets theory since it has a capability to represent vague data and reflecting real world [13].

The decision maker usually tends to choose interval judgment where it is more convincing than a fixed value judgment. FAHP method has been used for determination of weight of the criteria given by decision makers and then ranking of the method will be evaluated by traditional AHP method [12]. In the FAHP procedure, fuzzy numbers are used to do pairwise comparison in the judgment matrix. Fuzzy numbers become more meaningful to quantify a subjective measurement that usually fuzzy and imprecise into a range of exact value[14].

Fuzzy numbers can be represented using triangular fuzzy numbers (TFNs) or trapezoidal fuzzy numbers (TrFNs) to express decision makers' judgment due to their simplicity in modelling and easy interpretation. TFNs and TrFNs are two restrict fuzzy sets with convexity and normalization, and have been widely used to modeling fuzzy data [15]. However, this study focused on TFNs since it is adequate to represent the level of fuzzy linguistic variables. A TFN is denoted simply as (l,m,u) and the parameters l,m and u represent the smallest possible value, the mid-point value and the largest value that describe a fuzzy event [16]. TFNs are used because it has a good ability to ensure integrality of decision information [15]. Some studies used five-point, six-point or seven-point linguistic scales which were converted into the TFNs.

3. Data Analysis

In this section there are seven steps involved to calculate the priority weights of each criterion and alternative by using FAHP. Step 1 – 7 below explain the methodology proposed by [17].

Step 1. Construct a hierarchy structure for the MCDM problems.

Step 2. Construct the pairwise comparison for each criteria (attribute) and alternative with preference scale from Table 1. Three sets of linguistic scales from[18], [8] and [19] denoted as S1, S2 and S3 respectively were used in this study.

Step 3. Calculate the value of Fuzzy synthetic extent with respect to the object using:

$$S_i = \sum_{j=1}^{m} M_{g_i}^{j} \otimes \left[\sum_{i=1}^{n} \sum_{j=1}^{m} M_{g_i}^{j} \right]^{-1} \tag{1}$$

Table 1. Triangular Fuzzy Importance Scale

Linguistic Scale	S1 [18]		Linguistic Scale	S2 [8]		Linguistic Scale	S3 [19]	
	TFNs Scale	TFNs Reciprocal Scale		TFNs Scale	TFNs Reciprocal Scale		TFNs Scale	TFNs Reciprocal Scale
Just Equal (E)	$(1,1,1)$	$(1,1,1)$	Equal Importance (EI)	$(1,1,1)$	$(1,1,1)$	Equally Importance	$(1,1,3)$	$\left(\frac{1}{3},1,1\right)$
Weakly more important (WMI)	$\left(1,\frac{3}{2},2\right)$	$\left(\frac{1}{2},\frac{2}{3},1\right)$	Moderate Importance (MI)	$\left(\frac{1}{2},1,\frac{3}{2}\right)$	$\left(\frac{2}{3},1,2\right)$	Weakly Importance	$(1,3,5)$	$\left(\frac{1}{5},\frac{1}{3},1\right)$
Strongly more important (SMI)	$\left(\frac{3}{2},2,\frac{5}{2}\right)$	$\left(\frac{2}{5},\frac{1}{2},\frac{2}{3}\right)$	Strong Importance (SI)	$\left(1,\frac{3}{2},2\right)$	$\left(\frac{1}{2},\frac{2}{3},1\right)$	Essentially Importance	$(3,5,7)$	$\left(\frac{1}{7},\frac{1}{5},\frac{1}{3}\right)$
Very strongly more important (VSMI)	$\left(2,\frac{5}{2},3\right)$	$\left(\frac{1}{3},\frac{2}{5},\frac{1}{2}\right)$	Very Strong Importance (VSI)	$\left(\frac{3}{2},2,\frac{5}{2}\right)$	$\left(\frac{2}{5},\frac{1}{2},\frac{2}{3}\right)$	Very Strong Importance	$(5,7,9)$	$\left(\frac{1}{9},\frac{1}{7},\frac{1}{5}\right)$
Absolutely more important (AMI)	$\left(\frac{5}{2},3,\frac{7}{2}\right)$	$\left(\frac{2}{7},\frac{1}{3},\frac{2}{5}\right)$	Demonstrated Importance (DI)	$\left(2,\frac{5}{2},3\right)$	$\left(\frac{1}{3},\frac{2}{5},\frac{1}{2}\right)$	Absolutely Importance	$(7,9,9)$	$\left(\frac{1}{9},\frac{1}{9},\frac{1}{7}\right)$

To obtain $\sum_{j=1}^{m} M_{g_i}^{j}$, perform the fuzzy addition operation of *m* extent analysis values for a matrix such that:

$$\sum_{j=1}^{m} M_{g_i}^{j} = \left(\sum_{j=1}^{m} l_j, \sum_{j=1}^{m} m_j, \sum_{j=1}^{m} u_j \right) \tag{2}$$

and to obtain $\left[\sum_{i=1}^{n} \sum_{j=1}^{m} M_{g_i}^{j} \right]^{-1}$, perform the fuzzy addition operation of $\sum_{j=1}^{m} M_{g_i}^{j} \left(j = 1,2,...,n \right)$ values such that:

$$\left[\sum_{i=1}^{n} \sum_{j=1}^{m} M_{g_i}^{j} \right]^{-1} = \left(\frac{1}{\sum_{i=1}^{n} u_i}, \frac{1}{\sum_{i=1}^{n} m_i}, \frac{1}{\sum_{i=1}^{n} l_i} \right) \tag{3}$$

Step 4. Obtain the degree of possibility of each criterion and alternatives. The degree of possibility of $M_2 = (l_2, m_2, u_2) \geq M_1 = (l_1, m_1, u_1)$ is:

$$V(M_2 \geq M_1) = \sup_{y \geq x}\left[\min\left(\mu_{M_1}(x), \mu_{M_2}(y)\right)\right] \tag{4}$$

equivalently expressed as:

$$V(M_2 \geq M_1) = hgt(M_1 \cap M_2) = \mu_{M_2}(d) = \begin{cases} 1, & \text{if } m_2 \geq m_1 \\ 0, & \text{if } l_1 \geq u_2 \\ \dfrac{l_1 - u_2}{(m_2 - u_2) - (m_1 - l_1)}, & \text{otherwise} \end{cases} \tag{5}$$

where *d* is the ordinate of the highest intersection point *D* between μ_{M_1} and μ_{M_2}. To compare M_1 and M_2, we need both of $V(M_1 \geq M_2)$ and $V(M_2 \geq M_1)$.

Step 5. Obtain the minimum degree of possibility for each criterion and alternatives. The degree of possibility for a convex fuzzy number to be greater than *k* convex fuzzy $M_i = (i = 1, 2, ..., k)$ is:

$$V(M \geq M_1, M_2, ..., M_k) = V[(M \geq M_1) \text{ and } (M \geq M_2) \text{ and} \dots \text{and } (M \geq M_k)] \\ = \min V(M \geq M_i), i = 1,2,3,...,k \tag{6}$$

Assume that $d'(A_i) = \min V(S_i \geq S_k)$ (7)

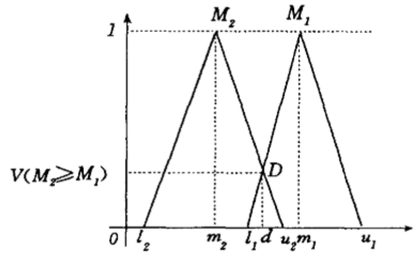

Figure 1. The intersection between M_1 and M_2

For *k* = 1,2,3,…,*n*; The weight vector is given by:

$$W' = (d'(A_1), d'(A_2), ..., d'(A_n))^T \tag{8}$$

where $A_i (i = 1, 2, ..., n)$ are *n* elements.

Step 6. Normalization of matrix comparison and obtain weight vector. The normalized weight vector is defined as:

$$W = (d(A_1), d(A_2), ..., d(A_n))^T \tag{9}$$

where *W* is a non-fuzzy number.

Step 7. Compute the relative weights and ranking the alternatives based on relative weight performance of alternatives. The best alternatives indicate the strongest weights.

$$w_i = \sum A_i K_{ij} \tag{10}$$

where,

w_i = Overall relative rating for factors *i*

A_i = Average normalized weight for factors*i*

K_{ij} = Average normalized rating for alternatives *j* with respect to factors *i*

3.1. Implementation of FAHP

A real-life empirical example about supplier selection is used as the case study. There are four criteria preferred by the decision makers which are delivery (C_1), price (C_2), service (C_3) and payment terms (C_4). Table 2 below shows the pairwise comparison of criterion for respondent 1 using scale S1 from table 1.

Table 2. Pairwise comparison of criterion

C_n	C_1	C_2	C_3	C_4
C_1	$(1,1,1)$	$(1,1,1)$	$\left(\frac{2}{7},\frac{1}{3},\frac{2}{5}\right)$	$\left(\frac{1}{2},\frac{2}{3},1\right)$
C_2	$(1,1,1)$	$(1,1,1)$	$\left(\frac{1}{3},\frac{2}{5},\frac{1}{2}\right)$	$\left(\frac{2}{5},\frac{1}{2},\frac{2}{3}\right)$
C_3	$\left(\frac{5}{2},3,\frac{7}{2}\right)$	$\left(2,\frac{5}{2},3\right)$	$(1,1,1)$	$\left(1,\frac{3}{2},2\right)$
C_4	$\left(1,\frac{3}{2},2\right)$	$\left(\frac{3}{2},2,\frac{5}{2}\right)$	$\left(\frac{1}{2},\frac{2}{3},1\right)$	$(1,1,1)$

The value of fuzzy synthetic extent with respect to the goal is calculated using equation (1-3).

$$S_{C_1}=(2.7857,3,3.4)\otimes\left(\frac{1}{22.5667},\frac{1}{19.0667},\frac{1}{16.0190}\right)=(0.1234,0.1573,0.2122)$$

$$S_{C_2}=(2.7333,2.9,3.1667)\otimes\left(\frac{1}{22.5667},\frac{1}{19.0667},\frac{1}{16.0190}\right)=(0.1211,0.1521,0.1977)$$

$$S_{C_3}=(6.5,8,9.5)\otimes\left(\frac{1}{22.5667},\frac{1}{19.0667},\frac{1}{16.0190}\right)=(0.288,0.4196,0.593)$$

$$S_{C_4}=(4,5.1667,6.5)\otimes\left(\frac{1}{22.5667},\frac{1}{19.0667},\frac{1}{16.0190}\right)=(0.1773,0.2710,0.4058)$$

Next the degree of possibility of $M_2=(l_2,m_2,u_2)\geq M_1=(l_1,m_1,u_1)$ is calculated using equation (4-5).

$V(S_{C_1}\geq S_{C_2})=1$, $V(S_{C_1}\geq S_{C_3})=0$, $V(S_{C_1}\geq S_{C_4})=0.2354$,
$V(S_{C_2}\geq S_{C_1})=0.9340$, $V(S_{C_2}\geq S_{C_3})=0$, $V(S_{C_2}\geq S_{C_4})=0.1466$,
$V(S_{C_3}\geq S_{C_1})=1$, $V(S_{C_3}\geq S_{C_2})=1$, $V(S_{C_3}\geq S_{C_4})=1$,
$V(S_{C_4}\geq S_{C_1})=1$, $V(S_{C_4}\geq S_{C_2})=1$, $V(S_{C_4}\geq S_{C_3})=0.4420.$

Then, the minimum degree of possibility is calculated using equation (6-8).

$d'(S_{C_1})=\min(1,0,0.2354)=0$,
$d'(S_{C_2})=\min(0.9340,0,0.1466)=0$,
$d'(S_{C_3})=\min(1,1,1)=1$,
$d'(S_{C_4})=\min(1,1,0.4420)=0.4420$.

From these values, the weight vector is $W' = (0,0,1,0.4420)^T$. The weight vector is then normalized and the weight priorities for each criterion is $W = (0,0,0.6935,0.3065)$. This same process is repeated to compute the overall relative weights for each criterion and alternative.

4. Result and Discussion

The result is explained in three parts which are relative weight for criteria, relative weight for alternatives and comparative study using Coefficient of Variations.

4.1. Relative weight for criteria

Table 3 shows the relative weight for criteria using the linguistic scales from table 1.

Table 3. Relative weight for criteria

Criteria	S1		S2		S3	
	Weight	Rank	Weight	Rank	Weight	Rank
C_1	0.0835	2	0.1915	3	0.1777	2
C_2	0.0796	3	0.1616	4	0.1679	3
C_3	0.7603	1	0.4166	1	0.5700	1
C_4	0.0766	4	0.2303	2	0.0845	4
$\sum$	1		1		1	

The result shows that service criterion has the highest weight for all set of scales used in this paper which are 0.7603, 0.4166 and 0.5700 respectively. Payment terms criterion has the lowest weight for S1 and S3 which are 0.0766 and 0.0845 respectively. However, the least preferred criterion for S2 is price which scored 0.1616.

4.2. Relative weight for alternatives

There are 3 alternatives preferred by the decision makers which are Supplier A (A), Supplier B (B), and Supplier C (C). Table 4 shows the relative weight for alternatives with respect to linguistic scales which is shown in table 1.

Table 4. Relative weight for alternatives

Alternative	S1		S2		S3		Expert
	Weight	Rank	Weight	Rank	Weight	Rank	Rank
A	0.0393	2	0.1417	3	0.0556	3	3
B	0.9314	1	0.6770	1	0.7999	1	1
C	0.0293	3	0.1813	2	0.1444	2	2
$\sum$	1		1		1		

The result shows Supplier B is the most preferred alternative for each scale used in this paper. The final weights are 0.9314, 0.6770 and 0.7999 respectively. Supplier A has the lowest weight for scale S2 and S3 which are 0.1417 and 0.0556 respectively. However, the least preferred alternative for scale S1 is Supplier C which scored 0.0293. The finding shows that the ranking of alternatives using scale S2 and S3 are similar with the expert preference. Thus, the first objective of this paper which is to implement FAHP approach with different linguistic scales to solve MCDM problems is achieved since FAHP can be used obtain the weight of the chosen criteria and alternatives.

4.3. Comparative study using Coefficient of Variations

The relative weight from table 3 and table 4 is compared by using Coefficient of Variations (CV). CV is used to measure the dispersion of data points in a data series around the mean. The CV is used to compare the degree of variation from one data series to another. The formula for CV is the standard deviation divided by the mean and is expressed as a percentage.

Table 5. Coefficient of Variations based on relative weight for criteria

Linguistic Scale	Coefficient of Variations
S1	136.08%
S2	45.84%
S3	86.94%

Table 5 shows the CV for relative weight for criteria for each linguistic scale. The lowest CV is identified for S2 which is 45.84%. This means that the respondents have almost the same preferences between the criterion for S2 since the variations of the data is the smallest among the three linguistic scales. The other CV for S1 and S3 is 136.08% and 86.94% respectively.

The analysis for alternatives is done to measure the variability of the data obtained. Table 6 shows the CV for the relative weight for alternatives for each linguistic scale. Table 6 indicates that the lowest CV is for S2 with 89.49% followed by S1 and S3 with 155.39% and 121.75% respectively. The values of CVs are quite high for the all scales used. Thus, the variability for each linguistic scale are quite varies. In other words, the respondents had different judgement on each linguistic scale in term of alternatives involved.

Table 6. Coefficient of Variations based on relative weight for alternatives

Linguistic Scale	Coefficient of Variations
S1	155.39%
S2	89.49%
S3	121.75%

5. Conclusion and Recommendation

FAHP is one of the MCDM tools that was developed to solve the hierarchical and selection problems. It can be concluded that in this case study, the most preferred linguistic scale is S2 since the value of CV is the smallest. This value indicates the extent of variability in relation to the mean of the judgments of the decision makers.

It is recommended that for future research, the number of groups which consists of different sets of linguistic scales and number of samples should be increased to gain significant finding in statistical analysis.

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