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Integral equation of a horizontal dipole located near the interface of media

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Integral equation of a horizontal dipole located near the interface of media

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Abstract. In the article the output of integral equations symmetric cylindrical dipole antenna of superconductive material. The antenna is located parallel to the interface between the two media. An example of calculating the current distribution over the antenna depending on the parameters of the medium is given.

1. Problem Statement

We will consider a cylindrical dipole antenna of perfectly conducting material which is located parallel to the interface. The antenna length is $2l$, the radius is a . In the general case, axial and azimuthal currents will be induced under the action of fields reflected from the interface onto the surface of the cylindrical dipole. In the case of a needle dipole ($a << \lambda, a << l$), the contribution of azimuthal current component will be insignificant, as shown in [1].

Under these conditions, the equivalence of cylindrical dipole and band one with the width $d = 4a$ has been proved in [1]. To make it simpler, we will further consider the band dipole located parallel to the interface boundary in the plane XOY of Cartesian $x, y.z$ coordinate system.

The antenna is located in the medium with dielectrical ε permeability and magnetic μ permeability. The second medium has ε_1 and μ_1 parameters. The second medium may also be characterized by σ penetrability or $tg\delta$ dielectric loss tangent. Further, we will use the designations ω – circular frequency, λ – wave length, $k = \dfrac{2\pi}{\lambda}$ - wave number for the medium with the antenna, k_1 – wave number for the second medium, $i = \sqrt{-1}$. The antenna is excited by the $E_z^{(1)}(z)$ primary field in the middle.

2. Integral Equation Deduction

The field induced by the dipole can be written as follows [1]

$$E_x^{(2)} = \frac{1}{i\omega\varepsilon} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{e^{-i\chi_1 x - i\chi_2 y + \beta z}}{\beta} \left[-\chi_1^2 + k^2 \right] f(\chi_1, \chi_2) d\chi_1 d\chi_2 , \tag{1}$$

$$E_y^{(2)} = \frac{1}{i\omega\varepsilon} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{e^{-i\chi_1 x - i\chi_2 y + \beta z}}{\beta} \left[-\chi_1 \chi_2 \right] f(\chi_1, \chi_2) d\chi_1 d\chi_2 , \tag{2}$$

,

$$H_y^{(2)} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-i\chi_1 x - i\chi_2 y + \beta z} f(\chi_1, \chi_2) d\chi_1 d\chi_2 \tag{3}$$

where

$$f(\chi_1, \chi_2) = \frac{1}{8\pi^2}\int_S j_x(x', y') e^{i\chi_1 x' + i\chi_2 y'} dx' dy',$$

$$\beta = \sqrt{\chi_1^2 + \chi_2^2 - k^2}\,.$$

The electro-magnetic field emitted by the dipole at the interface boundary is partly reflected and passes partly into the other medium. We will designate the longitudinal components of the reflected field (E_z^{rl}, H_z^{rl}) and refracted one (E_z^{rr}, H_z^{rr}). Also, we will express the transverse components of these fields via the longitudinal components [2].

Having made all necessary math calculations, we obtained expressions for field components – $E_x^{rl}, E_y^{rl}, H_x^{rl}, H_y^{rl}$, $E_x^{rr}, E_y^{rr}, H_x^{rr} H_y^{rr}$. Unknown A^1, A^2, B^1, B^2 enter the expressions for the components of the reflected and refracted fields. We will use the conditions of continuity of tangential components at the interface:

$$E_x^{(2)} + E_x^{rl}{}_{[z=-h]} = E_x^{rr}{}_{[z=-h]},\ E_y^{(2)} + E_y^{rl}{}_{[z=-h]} = E_y^{rr}{}_{[z=-h]}, \tag{4}$$

$$H_x^{(2)} + H_x^{rl}{}_{[z=-h]} = H_x^{rr}{}_{[z=-h]},\ H_y^{(2)} + H_y^{rl}{}_{[z=-h]} = H_y^{rr}{}_{[z=-h]}. \tag{5}$$

As the result, we obtain integral correlations. Having solved the correlations, we obtain the system of four functional equations. This system can be broken into two independent systems

$$\begin{cases} \dfrac{\omega\varepsilon}{\beta} A^1(\chi_1, \chi_2) - \chi_1 f(\chi_1, \chi_2) = \dfrac{\omega\varepsilon_1}{\beta_1} A^2(\chi_1, \chi_2), \\ \dfrac{-\beta\chi_1 f(\chi_1, \chi_2)}{i\omega\varepsilon} + iA^1(\chi_1, \chi_2) = -iA^2(\chi_1, \chi_2), \end{cases} \tag{6}$$

$$\begin{cases} iB^1(\chi_1, \chi_2) + \chi_2 f(\chi_1, \chi_2) = -iB^2(\chi_1, \chi_2), \\ \dfrac{-k^2\chi_2 f(\chi_1, \chi_2)}{i\omega\varepsilon\beta} + \dfrac{\omega\mu}{\beta} B^1(\chi_1, \chi_2) = \dfrac{\omega\mu}{\beta_1} B^2(\chi_1, \chi_2). \end{cases} \tag{7}$$

Having solved these systems, we found A^1, A^2, B^1, B^2. We will write down the expressions for two components – A^1, B^1:

$$A^1(\chi_1, \chi_2) = \frac{\beta\chi_1 f(\chi_1, \chi_2)}{\omega\varepsilon}\frac{\varepsilon\beta_1 - \varepsilon_1\beta}{\varepsilon\beta_1 + \varepsilon_1\beta} = \frac{\beta\chi_1 f(\chi_1, \chi_2)}{\omega\varepsilon} F_1(\chi_1, \chi_2)\,, \tag{8}$$

$$B^1(\chi_1, \chi_2) = -i\chi_2 f(\chi_1, \chi_2)\frac{\beta_1 - \beta}{\beta_1 + \beta} = -i\chi_2 f(\chi_1, \chi_2) F_2(\chi_1, \chi_2)\,. \tag{9}$$

Having known A^1, A^2, B^1, B^2, we could define the field in the whole space.

Using the deduction methods of one-dimension integral equation, we get the following equation

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$$(AI)(\tau)+(KI)(\tau)+\left(\overset{*}{K}I\right)(\tau)=i\sqrt{\frac{\varepsilon}{\mu}}\frac{\alpha}{k}E^{(1)}(\tau), \tag{10}$$

where

$$(AI)(\tau)=\frac{1}{\pi}\int_0^{+\infty}\chi\int_{-1}^{1}\cos[\chi(\tau-t)]I(t)dtd\chi,$$

$$(KI)(\tau)=\frac{kl\alpha}{2\pi^2}\int_0^{+\infty}\left[(\chi^2-1)I_0\left(ka\sqrt{\chi^2-1}\right)K_0\left(ka\sqrt{\chi^2-1}\right)-\frac{\chi}{2ka}\right]\times$$
$$\times\int_{-1}^{1}\cos[\chi kl(\tau-t)]I(t)dtd\chi,$$

$$\left(\overset{*}{K}I\right)(\tau)=\frac{kl\alpha}{2\pi^2}\int_0^{+\infty}\int_0^{+\infty}\left[\chi_1^2\gamma^2\frac{\gamma_1-\gamma\dfrac{\varepsilon_1}{\varepsilon}}{\gamma_1+\gamma\dfrac{\varepsilon_1}{\varepsilon}}+\chi_2^2\frac{\gamma_1-\gamma}{\gamma_1+\gamma}\right]J_0\left(\frac{\chi_2 kd}{2}\right)\times$$
$$\times\frac{e^{-\gamma kh}}{\gamma\left(\chi_1^2+\chi_2^2\right)}\int_{-1}^{1}\cos\left[\chi_1 l(\tau-t)\right]I(t)dtd\chi_1 d\chi_2,$$
$$\alpha=4\pi(ka)(kl),$$
$$\gamma=\sqrt{\chi_1^2+\chi_2^2-1},$$
$$\gamma_1=\sqrt{\chi_1^2+\chi_2^2-\left(\frac{k_1}{k}\right)^2}.$$

It should be noted that, unlike the integral equations of antennas in free space, the equation (10) contains the integral operators with infinitely differentiated nuclei. The equation (10) comes down to infinite Fredholm systems of the second kind as follows.

$$c_n+\sum_{m=1}^{+\infty}c_m K_{mn}=b_n,\quad 1\le n\le+\infty.$$

3. Calculations Results

In all calculations, the results of which are given below, the primary field was set as $E_z^{(1)}(z)=U_0 f(z)$, where

$$f(z)=\frac{1}{2\Delta}\begin{cases}0,|z|\ge\Delta,\\1,|z|<\Delta\end{cases}.$$

Tension – $U_0=1B$. The parameter Δ/l is denoted by *T*. Input resistance – $Z=R+iX=\dfrac{U_0}{I(0)}$, $I(0)$ – current in the middle of the dipole.

Figure 1 gives the current distribution along the dipole for parameters:
$T=0.05$, $h=0.45\lambda$ $l=0.25\lambda$ $a=0.005\lambda$.

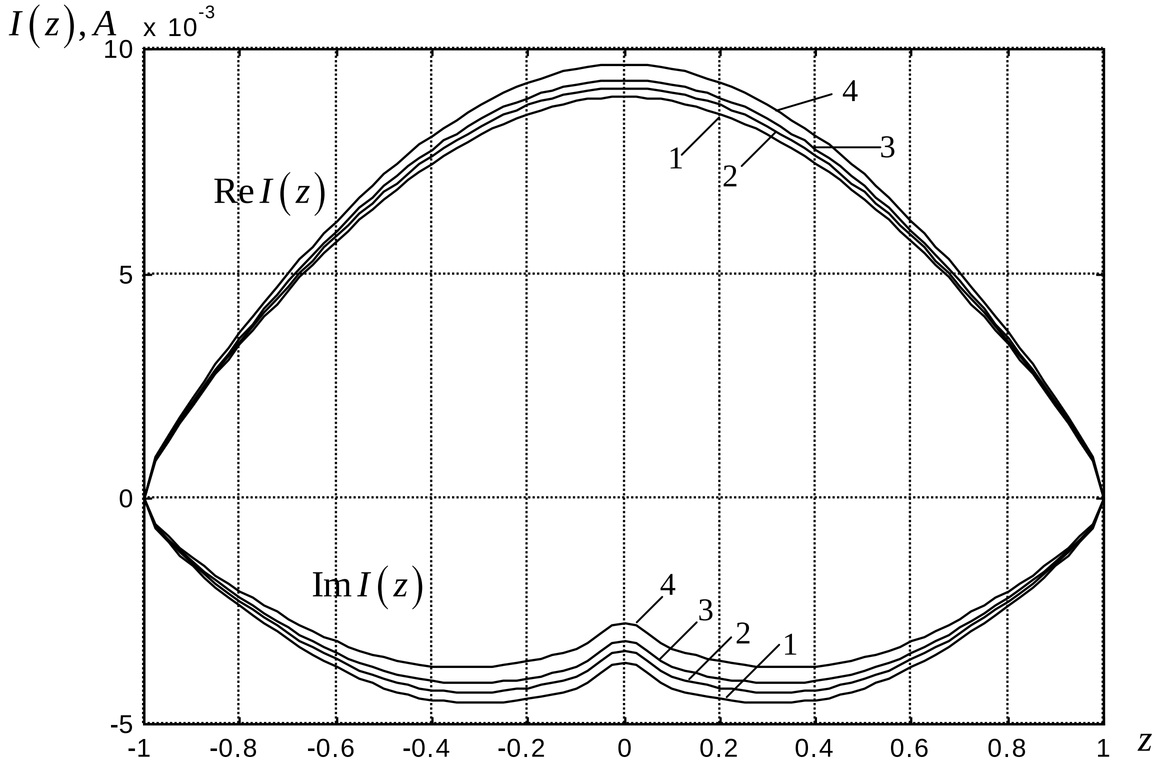

Figure 1. Current distribution along the semi-wave dipole depending on the medium parameters:

1 – $\varepsilon_1 = 5$, $tg\delta = 0.001$; 2 – $\varepsilon_1 = 10$, $tg\delta = 0.01$;

3 – $\varepsilon_1 = 20$, $tg\delta = 0.1$; 4 – $\varepsilon_1 = 80$, $tg\delta = 1$

References

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