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# Neutrino quantum decoherence due to entanglement with magnetic field

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**Abstract.** Neutrino oscillations in a constant magnetic field are considered. The backward influence of neutrinos on the environment is accounted for using the density matrix formalism. The entanglement of neutrinos with the magnetic field can destroy quantum coherent superposition of different neutrino states and thus lead to the suppression of neutrino oscillations. The master equation for the neutrino density matrix accounting for the effects of quantum decoherence due to entanglement with the magnetic field is derived.

#### 1. Introduction

It is well known that an external environment can significantly influence the neutrino oscillations pattern. In studies of a neutrino evolution in a magnetic field the neutrino is usually considered as a closed system (see, for instance, [1]). That means that neutrinos do not have any backward influence on a magnetic field. This influence can be negligible, in principle, but it causes entanglement between neutrinos and the environment. The entanglement can destroy quantum coherent superposition of different neutrino states and thus it can lead to the suppression of neutrino oscillations.

There are three types of quantum decoherence discussing in the literature: 1) dephasing, 2) intenglement with the environment and 3) relevation of "which-path" information [2]. There is another type of decoherence in neutrino physics that is due to the neutrino waves separation which is well studied in the literature and can be referred to the classical decoherence. This last phenomena we do not consider in this paper.

The Lindblad's equation is often used in studies of quantum decoherence of neutrinos (see, for example, [3] and [4]). However, there is a serious unsolved inherent problem of this approach that is manifested by a fact that the decoherence parameter is not fixed by the theory and remains free. This provides an uncertainty or undesired freedom, while the prediction of the theory is compared with an experimental data. The aim of the present note is to fill in the mentioned about gap and to derive the equation for neutrino density matrix accounting for the effects of quantum decoherence due to entanglement with a magnetic field.

#### 2. General formalism

The evolution of a closed system (which is a neutrino plus a magnetic field in our case) is governed by the Liouville equation

$$i\frac{\partial\rho(t)}{\partial t} = \left[H(t), \rho(t)\right],\tag{1}$$

where  $\rho(t)$  is a density matrix that can describe a pure or a mixed state of the system. If we consider the system composed of two subsystems corresponding to neutrinos and the background magnetic field the hamiltonian is composed of three terms:

$$H = H_{\nu} + H_A + H_{int},\tag{2}$$

where  $H_{int}$  stands for neutrinos to magnetic field interactions. If not to be interested in the evolution of the environment (the magnetic field), then its degrees of freedom can be traced out and from (1) we get

$$i\frac{\partial\rho_{\nu}(t)}{\partial t} = \left[H_{\nu}(t), \rho_{\nu}(t)\right] + Tr_A \left(H_{int}\rho - \rho H_{int}\right),\tag{3}$$

where  $\rho_{\nu} = Tr_A \rho$  is a density matrix which describes the evolution of the studied neutrino system. Note that tracing out makes the equation irreversible.

#### 3. Master equation for neutrino

A neutrino can interact with a magnetic field due to the anomalous magnetic moment  $\mu$ . The derivation of the evolution equation below is based on the analogous derivation in the case of particle interaction with a magnetic field due to an electric charge [6]. The interaction hamiltonian (our calculations are performed in the interaction representation) is given by

$$H_{int} = \mu \bar{\psi}(x) \sigma_{\mu\nu} \psi(x) F^{\mu\nu}(x) = j_{\mu\nu} F^{\mu\nu}, \qquad (4)$$

where  $F^{\mu\nu} = \partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu}$  is the electromagnetic field tensor,  $\psi(x)$  is the wave function of the neutrino in the mass basis and

$$\mu\bar{\psi}(x)\sigma_{\mu\nu}\psi(x) = j_{\mu\nu}.$$
(5)

The electromagnetic field potential  $A_{\mu}$  can be expressed in the form  $A_{\mu} = \bar{A}_{\mu} + a_{\mu}$ , where  $\bar{A}_{\mu}$  describes the classical field B and  $a_{\mu}$  describes the electromagnetic field fluctuations. Here below we focus on the entanglement of the neutrino with the magnetic field. Therefore, for simplicity we omit  $\bar{A}_{\mu}$  part in further calculations from here further on.

Equation (3) can be formally solved (integrated). Then, after tracing out the electromagnetic degrees of freedom, we obtain the following equation for the neutrino density matrix in the mass basis:

$$\rho_{\nu}(t_f) = tr_A \left[ Texp\left( \int_{t_i}^{t_f} d^4 x L(x) \right) \rho(t_i) \right], \tag{6}$$

where we introduce the Liouville superoperator  $L(x)\rho_{\nu} = -i[H_{int}, \rho_{\nu}]$ . Using the Wick's theorem [5]:

$$T_A \exp\left[\int_{t_i}^{t_f} d^4 x L(x)\right] = \exp\left[\frac{1}{2} \int_{t_i}^{t_f} d^4 x \int^{t_f} d^4 x' [L(x), L(x')] \theta(t-t')\right] \exp\left[\int_{t_i}^{t_f} d^4 x L(x)\right],$$
(7)

we can eliminate the time-ordering  $T_A$  of the electromagnetic variables and get

$$\rho_{\nu}(t_{f}) = T_{\nu} \left( -\frac{1}{2} \exp\left[ \int_{t_{i}}^{t_{f}} d^{4}x \int_{t_{i}}^{t_{f}} d^{4}x' \Theta(t-t') [F_{\mu\nu}(x), F_{\alpha\beta}(x')] J_{+}^{\mu\nu}(x) J_{+}^{\alpha\beta}(x') - \int_{t_{i}}^{t_{f}} d^{4}x \int_{t_{i}}^{t_{f}} d^{4}x' \Theta(t-t') J_{-}^{\mu\nu}(x) J_{-}^{\alpha\beta}(x') \right] \right) \times tr_{A} \left( \exp\left[ \int_{t_{i}}^{t_{f}} d^{4}x L(x) \right] \right), \quad (8)$$

where  $T_{\nu}$  is time-ordering of the neutrino degrees of freedom and we use the following notations

$$J^{\mu\nu}_{+}(x)\rho_{\nu} = j^{\mu\nu}(x)\rho_{\nu}, \quad J^{\mu\nu}_{-}(x)\rho_{\nu} = \rho_{\nu}j^{\mu\nu}(x).$$
(9)

In equation (8) the averaging over the magnetic field degrees of freedom is included only in the functional

$$W[J_+, J_-] = tr_A\left(\exp\left[\int_{t_i}^{t_f} d^4 x L(x)\right]\right).$$
(10)

To simplify this functional it is convenient to use a cumulant decomposition. Under the initial conditions of weak entanglement of the neutrinos with an external field ( $\rho = \rho_{\nu} \oplus \rho_A$ ) the maximum degree of the cumulant decomposition is two. Moreover, it is possible to show that  $\langle a_{\mu}(x) \rangle = 0$ , thus there is only one term the cumulant decomposition. In the light of the above, functional  $W[J_+, J_-]$  is written in the form

$$tr_A\left(\exp\left[\int_{t_i}^{t_f} d^4x L(x)\right]\right) = \exp\left[\frac{1}{2}\int_{t_i}^{t_f} d^4x \int_{t_i}^{t_f} d^4x' < L(x)L(x') > \right].$$
 (11)

With using (11) equation (8) is modified as

$$\rho_{\nu}(t_f) = T^{\nu} \exp(i\Phi[J_+, J_-])\rho_{\nu}(t_i), \qquad (12)$$

where

$$i\Phi[J_{+}, J_{-}] = \frac{1}{2} \int_{t_{i}}^{t_{f}} d^{4}x \int_{t_{i}}^{t_{f}} d^{4}x' \times \times [-i\partial_{\alpha}\partial_{\mu}D_{F}(x-x')_{\beta\nu}J_{+}^{\alpha\beta}(x)J_{+}^{\mu\nu}(x') + \partial_{\alpha}\partial_{\mu}D_{F}^{*}(x-x')_{\beta\nu}J_{-}^{\alpha\beta}(x)J_{-}^{\mu\nu}(x') + + \partial_{\alpha}\partial_{\mu}D_{-}(x-x')_{\beta\nu}J_{+}^{\alpha\beta}(x)J_{-}^{\mu\nu}(x') + \partial_{\alpha}\partial_{\mu}D_{+}(x-x')_{\beta\nu}J_{-}^{\alpha\beta}(x)J_{+}^{\mu\nu}(x')].$$
(13)

where  $D_F(x - x')_{\mu\nu}$  is the Feynman propagator,  $D_+(x - x')_{\mu\nu}$  and  $D_-(x - x')_{\mu\nu}$  are two-point correlation functions. In the further calculations it is convenient to use the anticommutator function  $D_1(x - x')_{\mu\nu}$ :

$$D_F(x - x')_{\mu\nu} = \langle T[a_\mu(x), a_\nu(x')] \rangle, \tag{14}$$

$$D_{+}(x - x')_{\mu\nu} = \langle T[a_{\mu}(x), a_{\nu}(x')] \rangle, \qquad (15)$$

$$D_{+}(x - x')_{\mu\nu} = \langle T[a_{\nu}(x'), a_{\mu}(x)] \rangle, \qquad (16)$$

$$D_1(x - x')_{\mu\nu} = < T\{a_\mu(x), a_\nu(x')\} > .$$
(17)

In the following we use the Feynman gauge  $D(x - x')_{ij} = \delta_{ij}D(x - x')$  with  $a_{\mu} = (0, -\vec{a})$ . The currents  $j_{ij}$  can be expressed as

$$j_{ij} = 2\mu\bar{\nu}(x)\gamma_0\gamma_i\gamma_j\nu(x) = 2\epsilon_{ijk}S^k,$$
(18)

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where we introduced the spin vector

$$S^{k} = \mu \bar{\nu}(x) \gamma_{0} \Sigma^{k} \nu(x).$$
<sup>(19)</sup>

Finally, we arrive to the following equation for the neutrino density matrix:

$$\frac{d}{dt}\rho_{\nu}(t) = K_0\rho_{\nu}(t) + K_1\rho_{\nu}(t) + K_2\rho_{\nu}(t), \qquad (20)$$

where the superoperators are defined as

$$K_0 \rho_{\nu} = -\frac{1}{i} \left[ H_B, \rho_{\nu} \right]$$
 (21)

$$K_1 \rho_{\nu} = -\frac{1}{2} \int d^3x \int d^3x' \int dx'_0 \vec{\partial}^2 D_1(x - x') [\vec{S}(x), [\vec{S}(x'), \rho_{\nu}]], \qquad (22)$$

$$K_2 \rho_{\nu} = \sum_{i,j} \frac{1}{2} \int d^3x \int d^3x' \int dx'_0 \partial_i \partial_j D_1(x - x') [S^i(x), [S^j(x'), \rho_{\nu}]].$$
(23)

In the final equation (20) the superoperator  $K_0$  corresponds to the hamiltonian arising due to  $\bar{A}_{\mu}$ . The two terms  $K_1$  and  $K_2$  arise due to entanglement with fluctuations of the magnetic field. These terms are real and have a form of the dissipative terms in the Lindblad's equation (see in [7] and [8]).

To explore the influence of transversal field on neutrino evolution it is convenient to rewrite equation (20) in the following form:

$$\frac{d}{dt}\rho_{\nu}(t) = K_{0}\rho_{\nu}(t) + K_{\perp}\rho_{\nu}(t) + K_{\parallel x}\rho_{\nu}(t) + K_{\parallel y}\rho_{\nu}(t) + K_{mixed}\rho_{\nu}(t),$$
(24)

where new superoperators are defined as

$$K_{\perp}\rho_{\nu} = \frac{1}{2} \int d^3x \int d^3x' \int dx'_0 (\partial_1^2 + \partial_2^2) D_1(x - x') [S_3(x), [S_3(x'), \rho_{\nu}]],$$
(25)

$$K_{\parallel x} = \frac{1}{2} \int d^3x \int d^3x' \int dx'_0 (\partial_2^2 + \partial_3^2) D_1(x - x') [S_1(x), [S_1(x'), \rho_\nu]],$$
(26)

$$K_{\parallel y} = \frac{1}{2} \int d^3x \int d^3x' \int dx'_0 (\partial_1^2 + \partial_3^2) D_1(x - x') [S_2(x), [S_2(x'), \rho_\nu]],$$
(27)

$$K_{mixed}\rho_{\nu} = \sum_{i \neq j} \frac{1}{2} \int d^3x \int d^3x' \int dx'_0 \partial_i \partial_j D_1(x - x') [S^i(x), [S^j(x'), \rho_{\nu}]].$$
(28)

In this notations superoperators  $K_{\perp}$  and  $K_{\parallel x} + K_{\parallel y}$  can be interpreted as an influence of longitudinal and transversal fields on neutrino propagation respectively.  $K_{mixed}$  has all components of a magnetic field.

In order to study the influence of a longitudinal field we need to set  $S_1$  and  $S_2$  equal to null. In this case it is more interesting to consider neutrino as a superposition of two states:  $|\psi_1 \rangle$ and  $|\psi_2 \rangle$  that describe spiral states of neutrino. Then density matrix  $\rho(x)$  is written in the form  $\rho(t) = \rho_{11}(t) + \rho_{22}(t) + \rho_{12}(t) + \rho_{21}(t)$ , where  $\rho_{ij} = |\psi_i \rangle \langle \psi_j|$ . Since  $|\psi_i \rangle$  is an eigenstate of  $S_3$  it can be shown that  $\rho_{11}$  and  $\rho_{22}$  are not effected by the superoperator  $K_{\parallel}$ . It means that a transversal magnetic field has an influence only on  $\rho_{12}$  and  $\rho_{12}$  which are responsible for coherency between two spiral neutrino states. This influence is representable with using a decoherence functional  $\Gamma$ 

$$\rho(t_f) = \rho_{11}(t_i) + \rho_{22}(t_i) + \exp(\Gamma)\rho_{12}(t_i) + \exp(\Gamma)\rho_{21}(t_i),$$
(29)

where indexes "i" and "f" are responsible for initial state and final states respectively. The decoherence functional  $\Gamma$  is

$$\Gamma = \left(\frac{\mu m}{E}\right)^2 \int_{t_i}^{t_f} d^4x \int_{t_i}^{t_f} d^4x' (\partial_1^2 + \partial_2^2) D_1(x - x'), \tag{30}$$

where m and E are neutrino mass and energy. It is evident that the affect of a longitudinal field is suppressed by a factor  $(\frac{m}{E})^2$ .

# 4. Conclusions

The derived equation (20) describes the effect of neutrino quantum decoherence due to entanglement with the magnetic field fluctuations. In contrast to the approach based on Lindblad's equation, there are no free parameters in equation (20). The developed approach is more appropriate than one based on Lindblad's equation for using in studies of neutrino propagation in astrophysical environments with magnetic fields (for example, in supernovae) because the obtained up to now experimental data on supernovae neutrinos is quite poor in respect, for instance, with the present data on solar neutrinos.

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