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Neutrino spin precession and oscillations in transversal matter currents

A. Popov[1], **P. Pustoshny**[1] **and A. Studenikin**[1,2]

[1]Department of Theoretical Physics, Faculty of Physics, Lomonosov Moscow State University, Moscow, 119991, Russia

[2] Joint Institute for Nuclear Research, Dubna, Moscow Region, 141980, Russia

E-mail: `ar.popov@physics.msu.ru`, `kfrepp@gmail.com`, `studenik@srd.sinp.msu.ru`

Abstract. In this paper we considered semiclassical neutrino spin precession in a transversal matter current and quantum case of the neutrino spin oscillations in the mass and flavor bases induced by a moving media. In the first part we have verified, that even without presence of an electromagnetic field, in presence of a matter, when the transverse matter term is not zero, the neutrino spin oscillations can be induced. In the second part there were some calculations, that in the end led us to the evolution equations which include an effective Hamiltonian of the weak interactions. From these equations one can see the influence of the transverse component of a matter velocity on a spin oscillations and of the longitudinal component of a matter velocity on the neutrino energy spectrum.

1. Introduction

It is well known that massive neutrinos have nontrivial electromagnetic properties [1] and at least magnetic moments of neutrinos should be nonzero . Even before convincing theoretical arguments in favour of nonzero neutrino magnetic moments were obtained [2] the effect of mixing between two neutrino helicity states in the transversal magnetic field and the corresponding neutrino spin oscillations due to the magnetic field interaction with the neutrino magnetic moment were proposed in [3]. After a detailed investigations [4, 5] of this phenomenon in vacuum and in matter of constant density, the resonance amplification on neutrino spin (spin-flavour) oscillations in matter, the effect similar to the MSW effect in neutrino flavour oscillations, was studies in [6, 7].

For many years, until 2004, it was believed that neutrino helicity precession and the corresponding spin oscillations can be induced by the neutrino magnetic interactions with the transversal magnetic field. A new and very interesting possibility for neutrino spin (and spin-flavour) oscillations engendered by the neutrino interaction with matter background was proposed and investigated in [8]. It was shown that neutrino spin oscillations can be induced not only by the neutrino interaction with a magnetic field but also by neutrino interactions with matter in the case when there is the transversal matter current (or the transversal matter matter polarization). There is no need for neutrino magnetic moment interaction in this case. The origin of the oscillations $\nu_L \Leftrightarrow \nu_R$ in the transversal matter currents $j_\perp$ is the neutrino weak interactions with moving matter and the corresponding mixing between neutrino states ν_L and ν_R is determined by $G_F j_\perp$. This new effect has been explicitly highlighted in [8, 9],

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recently the existence of this effect was confirmed in [10]. For historical notes reviewing studies and derivation of the discussed effect see [11, 12] and [13].

2. Neutrino spin oscillations in transversal matter current: semiclassical treatment

Consider, as an example, an electron neutrino spin precession in the case when neutrinos with the Standard Model interaction are propagating through moving and polarized matter composed of electrons (electron gas) in the presence of an electromagnetic field given by the electromagnetic field tensor $F_{\mu\nu} = (\boldsymbol{E}, \boldsymbol{B})$. Following the discussion in [8, 9] to derive the neutrino spin oscillation probability in the transversal matter current we use the generalized Bargmann-Michel-Telegdi equation that describes the evolution of the three-dimensional neutrino spin vector $\boldsymbol{S}$,

$$\frac{d\boldsymbol{S}}{dt} = \frac{2}{\gamma}\left[\boldsymbol{S} \times (\boldsymbol{B_0} + \boldsymbol{M_0})\right], \tag{1}$$

where the magnetic field $\boldsymbol{B_0}$ in the neutrino rest frame is determined by the transversal and longitudinal (with respect to the neutrino motion) magnetic and electric field components in the laboratory frame,

$$\boldsymbol{B_0} = \gamma\left(\boldsymbol{B}_\perp + \frac{1}{\gamma}\boldsymbol{B}_\parallel + \sqrt{1-\gamma^{-2}}\left[\boldsymbol{E}_\perp \times \frac{\boldsymbol{\beta}}{\beta}\right]\right) \tag{2}$$

$\gamma = (1-\beta^2)^{-\frac{1}{2}}$, $\boldsymbol{\beta}$ is the neutrino velocity. The matter term $\boldsymbol{M_0}$ in (2) is also composed of the transversal $\boldsymbol{M_{0\perp}}$ and longitudinal $\boldsymbol{M_{0\parallel}}$ parts,

$$\boldsymbol{M_0} = \boldsymbol{M_{0\perp}} + \boldsymbol{M_{0\parallel}}, \tag{3}$$

$$M_{0\parallel} = \gamma\boldsymbol{\beta}\frac{n_0}{\sqrt{1-v_e^2}}\left\{\rho_e^{(1)}\left(1 - \frac{\boldsymbol{v_e}\boldsymbol{\beta}}{1-\gamma^{-2}}\right)\right\} - \frac{\rho_e^{(2)}}{1-\gamma^{-2}}\left\{\boldsymbol{\zeta_e}\boldsymbol{\beta}\sqrt{1-v_e^2} + \left(\boldsymbol{\zeta_e}\boldsymbol{v_e}\frac{(\boldsymbol{\beta}\boldsymbol{v_e})}{1+\sqrt{1-v_e^2}}\right)\right\}, \tag{4}$$

$$M_{0\perp} = -\frac{n_0}{\sqrt{1-v_e^2}}\left\{\boldsymbol{v_{e_\perp}}\left(\rho_e^{(1)} + \rho_e^{(2)}\frac{(\boldsymbol{\zeta_e}\boldsymbol{v_e})}{1+\sqrt{1-v_e^2}}\right) + \boldsymbol{\zeta_{e_\perp}}\rho_e^{(2)}\sqrt{1-v_e^2}\right\}. \tag{5}$$

Here $n_0 = n_e\sqrt{1-v_e^2}$ is the invariant number density of matter given in the reference frame for which the total speed of matter is zero. The vectors $\boldsymbol{v_e}$ and $\boldsymbol{\zeta_e}$ $(0 \le |\boldsymbol{\zeta_e}^2| \le 1)$ denote, respectively, the speed of the reference frame in which the mean momentum of matter (electrons) is zero, and the mean value of the polarization vector of the background electrons in the above mentioned reference frame. The coefficients $\rho_e^{(1,2)}$ calculated within the extended Standard Model supplied with $SU(2)$-singlet right-handed neutrino R are respectively, $\rho_e^{(1)} = \frac{\tilde{G}_F}{2\sqrt{2}\mu}$, $\rho_e^{(2)} = -\frac{G_F}{2\sqrt{2}\mu}$, where $\tilde{G}_F = (1 + 4\sin^2\theta_W)$.

For neutrino evolution between two neutrino states $\nu_e^L \leftrightarrow \nu_\mu^R$ in presence of the magnetic field and moving matter we get [8] the following equation

$$i\frac{d}{dt}\begin{pmatrix}\nu_e^L \\ \nu_e^R\end{pmatrix} = \mu\begin{pmatrix}\frac{1}{\gamma}|\boldsymbol{M_{0\parallel}} + \boldsymbol{B_{0\parallel}}| & |\boldsymbol{B}_\perp + \frac{1}{\gamma}\boldsymbol{M_{0\perp}}| \\ |\boldsymbol{B}_\perp + \frac{1}{\gamma}\boldsymbol{M_{0\perp}}| & -\frac{1}{\gamma}|\boldsymbol{M_{0\parallel}} + \boldsymbol{B_{0\parallel}}|\end{pmatrix}\begin{pmatrix}\nu_e^L \\ \nu_e^R\end{pmatrix} \tag{6}$$

Thus, the probability of the neutrino oscillations in the adiabatic approximation is given by

$$P_{\nu_e^L \to \nu_e^R}(x) = \sin^2 2\theta_{eff}\sin^2\frac{\pi x}{L_{eff}}, \sin^2 2\theta_{eff} = \frac{E_{eff}^2}{E_{eff}^2 + \Delta_{eff}^2}, L_{eff} = \frac{2\pi}{E_{eff}^2 + \Delta_{eff}^2}, \tag{7}$$

$$E_{eff} = \mu|\boldsymbol{B}_\perp + \frac{1}{\gamma}\boldsymbol{M_{0\perp}}|, \Delta_{eff} = \frac{\mu}{\gamma}|\boldsymbol{M_{0\parallel}} + \boldsymbol{B_{0\parallel}}|. \tag{8}$$

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Thus, it follows that even without presence of an electromagnetic field, $\boldsymbol{B}_\perp = \boldsymbol{B}_{\mathbf{0}\|} = 0$, neutrino spin oscillations $\nu_e^L \leftrightarrow \nu_\mu^R$ can be induced in the presence of matter when the transverse matter term $\boldsymbol{M}_{\mathbf{0}\perp}$ is not zero. If we neglect possible effects of matter polarization then the neutrino evolution equation (6) simplifies to

$$i\frac{d}{dt}\begin{pmatrix}\nu_e^L\\ \nu_e^R\end{pmatrix} = \frac{\mu}{\gamma}\begin{pmatrix}M_{0\|} & M_{0\perp}\\ M_{0\perp} & -M_{0\|}\end{pmatrix}\begin{pmatrix}\nu_e^L\\ \nu_e^R\end{pmatrix} \tag{9}$$

$$\boldsymbol{M}_{\mathbf{0}\|} = \gamma\boldsymbol{\beta}\rho_e^{(1)}\left(1 - \frac{\boldsymbol{v_e}\boldsymbol{\beta}}{1-\gamma^{-2}}\right)\frac{n_0}{\sqrt{1-v_e^2}}, \boldsymbol{M}_{\mathbf{0}\perp} = -\rho_e^{(1)}\boldsymbol{v}_{\boldsymbol{e}\perp}\frac{n_0}{\sqrt{1-v_e^2}}. \tag{10}$$

The effective mixing angle and oscillation length in the neutrino spin oscillation probability (7) now are given by

$$\sin^2 2\theta_{eff} = \frac{M_{0\perp}^2}{M_{0\|}^2 + M_{0\perp}^2}, L_{eff} = \frac{2\pi}{\mu M_0}\gamma. \tag{11}$$

The above considerations can be applied to other types of neutrinos and various matter compositions. It is also obvious that for neutrinos with nonzero transition magnetic moments a similar effect for spin-flavour oscillations exists under the same background conditions.

3. Neutrino spin oscillations in mass and flavor bases

Here below we further develop [14] the description of neutrino spin oscillations in the transversal matter current that implies corresponding evaluations in both the mass and flavour neutrino basis. In a sense the approach we develop here and in [14] is analogous to one [15] we applied in calculations the neutrino spin oscillation probability in the presence of the magnetic field.

We consider two flavour neutrinos with two possible helicities $\nu_f = (\nu_e^+, \nu_e^-, \nu_\mu^+, \nu_\mu^-)^\tau$ in moving media composed of neutrons. The neutrino interaction Hamiltonian reads

$$H_{int} = f^\mu \sum_l \left(\bar{\nu}\gamma_\mu \frac{1+\gamma^5}{2}\nu\right), \tag{12}$$

where $l = e,\ \mu$, $f^\mu = G_F j^\mu/\sqrt{2}$, $j^\mu = n_n(1, \boldsymbol{v})/\sqrt{1-v^2}$ - medium current, G_F - Fermi constant of weak interaction. Each of flavour neutrinos is a superposition of the neutrino mass states,

$$\nu_e^\pm = \nu_1^\pm\cos\theta + \nu_2^\pm\sin\theta, \nu_\mu^\pm = -\nu_1^\pm\sin\theta + \nu_2^\pm\cos\theta, \tag{13}$$

The neutrino evolution equation in the flavour basis is

$$i\frac{d}{dt}\nu_f = H^{eff}\nu_f, \tag{14}$$

where the effective Hamiltonian consists of the vacuum and interaction parts:

$$H^{eff} = H_0^{eff} + \Delta H^{eff}. \tag{15}$$

ΔH^{eff} can be expressed as

$$\Delta H^{eff} = \begin{pmatrix}\Delta_{ee}^{++} & \Delta_{ee}^{+-} & \Delta_{e\mu}^{++} & \Delta_{e\mu}^{+-}\\ \Delta_{ee}^{-+} & \Delta_{ee}^{--} & \Delta_{e\mu}^{-+} & \Delta_{e\mu}^{--}\\ \Delta_{\mu e}^{++} & \Delta_{\mu e}^{+-} & \Delta_{\mu\mu}^{++} & \Delta_{\mu\mu}^{+-}\\ \Delta_{\mu e}^{-+} & \Delta_{\mu e}^{--} & \Delta_{\mu\mu}^{-+} & \Delta_{\mu\mu}^{--}\end{pmatrix}, \tag{16}$$

where

$$\Delta_{kl}^{ss'} = \langle \nu_k^s | H_{int} | \nu_l^{s'} \rangle, \tag{17}$$

$k, l = e, \mu,\ s, s' = \pm$. In evaluation of $\Delta_{kl}^{ss'}$ we have first introduced the neutrino flavour states ν_k^s and $\nu_l^{s'}$ as superpositions of the mass states $\nu_{1,2}^{\pm}$. Then, using the exact free neutrino mass states spinors,

$$\nu_\alpha^s = C_\alpha \begin{pmatrix} u_\alpha^s \\ \frac{\boldsymbol{\sigma p_\alpha}}{E_\alpha + m_\alpha} u_\alpha^s \end{pmatrix} \sqrt{\frac{(E_\alpha + m_\alpha)}{2E_\alpha}} \exp(i\boldsymbol{p_\alpha x}), \tag{18}$$

where the two-component spinors define neutrino helicity states, and are given by

$$u_\alpha^{s=+1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, u_\alpha^{s=-1} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \tag{19}$$

define neutrino helicity states, we have performed calculations that are analogous to those, performed in [15]. The difference in calculations is that here we consider not electromagnetic neutrino interaction with a magnetic field but the neutrino weak interaction with moving media given by (12). For typical term $\Delta_{\alpha\alpha'}^{ss'} = \langle \nu_\alpha^s | H_{int} | \nu_{\alpha'}^{s'} \rangle$, that by fixing proper values of α, s, α' and s' can reproduce all of the elements of the neutrino evolution Hamiltonian ΔH^{eff} that accounts for the effect of matter motion, we obtain,

$$\Delta_{\alpha,\alpha'}^{s,s'} = n\tilde{G} u_\alpha^{sT} \{(1 - v_\parallel) \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix} + v_\perp \begin{pmatrix} 0 & \gamma_{\alpha\alpha}^{-1} \\ \gamma_{\alpha'\alpha'}^{-1} & 0 \end{pmatrix} \} u_{\alpha'}^{s'}, \tag{20}$$

where $\frac{n_n G_F/\sqrt{2}}{2\sqrt{1-v^2}} \equiv n\tilde{G}$, $v_\parallel$ and $v_\perp$ are the longitudinal and transversal velocities of the matter current and

$$\gamma_{\alpha\alpha'}^{-1} = \frac{1}{2}(\gamma_\alpha^{-1} + \gamma_{\alpha'}^{-1}), \tilde{\gamma}_{\alpha\alpha'}^{-1} = \frac{1}{2}(\gamma_\alpha^{-1} - \gamma_{\alpha'}^{-1}), \gamma_\alpha^{-1} = \frac{m_\alpha}{E_\alpha}. \tag{21}$$

One can put spinors in this formula and easily find for example

$$\begin{aligned} \Delta_{\alpha\alpha'}^{+-} &= n\tilde{G} v_\perp \gamma_{\alpha\alpha}^{-1}, \\ \Delta_{\alpha\alpha'}^{--} &= 2n\tilde{G}(1 - v_\parallel). \end{aligned} \tag{22}$$

so that the effective interaction Hamiltonian in the flavour basis has the following structure,

$$\Delta H^{eff} = n\tilde{G} \begin{pmatrix} 0 & v_\perp(\frac{\eta}{\gamma})_{ee} & 0 & 0 \\ v_\perp(\frac{\eta}{\gamma})_{ee} & 2(1 - v_\parallel)\eta_{ee} & 0 & 0 \\ 0 & 0 & 0 & v_\perp(\frac{\eta}{\gamma})_{\mu\mu} \\ 0 & 0 & v_\perp(\frac{\eta}{\gamma})_{\mu\mu} & 2(1 - v_\parallel)\eta_{\mu\mu} \end{pmatrix}. \tag{23}$$

Here we introduce the following formal notations:

$$\begin{aligned} (\frac{\eta}{\gamma})_{ee} &= \frac{\cos^2\theta}{\gamma_{11}} + \frac{\sin^2\theta}{\gamma_{22}} + \frac{\sin 2\theta}{\gamma_{12}} \\ (\frac{\eta}{\gamma})_{\mu\mu} &= \frac{\sin^2\theta}{\gamma_{11}} + \frac{\cos^2\theta}{\gamma_{22}} - \frac{\sin 2\theta}{\gamma_{12}} \\ \eta_{ee} &= 1 + \sin 2\theta \\ \eta_{\mu\mu} &= 1 - \sin 2\theta \end{aligned} \tag{24}$$

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Let us consider a particular case. For example, one would have two states (ν_e^+, ν_e^-) mixed in accordance with the equation

$$i\frac{d}{dt}\begin{pmatrix}\nu_e^+\\ \nu_e^-\end{pmatrix} = \begin{pmatrix}-\frac{\Delta m^2}{4E}\cos 2\theta & n\tilde{G}v_\perp(\frac{\eta}{\gamma})_{ee}\\ n\tilde{G}v_\perp(\frac{\eta}{\gamma})_{ee} & -\frac{\Delta m^2}{4E}\cos 2\theta + 2n\tilde{G}(1-v_\parallel)\eta_{ee}\end{pmatrix}\begin{pmatrix}\nu_e^+\\ \nu_e^-\end{pmatrix}, \tag{25}$$

where $\Delta m^2 = 0$ in the considered case. From this it follows that the neutrino interacts with moving media and can generate the neutrino spin mixing (an additional mixing to the usual effect due to neutrino mixing angle θ) with changing neutrino chirality. For the spin neutrino oscillation probability in the adiabatic case we get

$$P_{\nu_e^+\to\nu_e^-} = \frac{\left(2n\tilde{G}v_\perp(\frac{\eta}{\gamma})_{ee}\right)^2}{\left(2n\tilde{G}v_\perp(\frac{\eta}{\gamma})_{ee}\right)^2 + \left(2n\tilde{G}(1-v_\parallel)\eta_{ee}\right)^2}\sin^2\left(\frac{1}{2}\sqrt{D}x\right), \tag{26}$$

where $D = \left(2n\tilde{G}v_\perp(\frac{\eta}{\gamma})_{ee}\right)^2 + \left(2n\tilde{G}(1-v_\parallel)\eta_{ee}\right)^2$. It follows that $\boldsymbol{v}$ components not only generates spin neutrino mixing but also can produce the resonance amplification of the corresponding oscillations.

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