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Mathematical model of red sludge sedimentation in single-level circular thickener

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Abstract. As a controllable facility, a thickener features multiple interrelated and correlated input and output parameters. Thickening is a two-phase process comprising sedimentation and consolidation. During sedimentation, particles settle in a liquid with the pressure, friction, gravity-forcing collision. At a certain concentration, particles begin to collide and transform the suspension into a sediment of particulate matter. Now the inter-particle forces are transmitted directly particle-to-particle. If the settling particles to reach the tank bottom and lay upon each other are incompressible, such as glass balls, the process is over; however, in case of compressible particles such as red-sludge floccules, the weight of the sediment compresses the flakes under it and forces water out of the pores. Such retrieval of water by compression is referred to as consolidation. This paper describes the basic equations of red-sludge consolidation and sedimentation in a circular thickener.

1. Introduction

Most aluminum factories produce waste comprising mostly the oxides of iron, aluminum, titanium, and other metals suitable for further use. Such waste is referred to as red sludge (RS); it is fine-grained and contains a lot of valuable components, retrieving which might be profitable. RS is generated when cleaning bauxite (the basic raw material for aluminum production) while making alumina by the Bayer process, which produces pure aluminum oxide. Deprived of aluminum oxide, RS is alkali-contaminated, making it hazardous for the environment and for human beings, as it can damage the skin. Sludge wastes are red as they contain iron oxide (III), titanium dioxide, and silicon dioxide, all of which are toxic. For each ton of aluminum oxide, 360 to 800 kg of sludge is produced.

Many specialists believe such sludge is not actually waste, as it is suitable for recycling—in theory. However, this is not cost-efficient, which is why sludge is often stored in isolated areas, the so-called sludge depots. The today's world is facing the challenge of developing various ways too improve the RS recycling and disposal technologies; such technologies are being proposed by experts from all over the world [1-6].

Paper [2-4] presents a multidimensional statistical estimation of how much valuable RS components is lost when using the Bayer method. These data are a booster to further research in the improvement of bauxite recycling, etc.

This research is to address a relevant problem of creating and improving the RS thickening models and control systems to maximize pre-extraction of alkali from the RS for further use in the basic production, which will prevent such alkali from seeping into ground waters. Costs can be cut by reducing the loss of alkali when using the Bayer method to process bauxite, i.e. ensuring full reuse of this reagent



and minimizing the loss of valuable components.

2. Solid-Liquid Interaction

Thickening is a complex process comprising multiple parameters. This paper focuses on such thickening properties as the suspension permeability, suspension compressibility, and free settling velocity, or Stokes' velocity.

3. RS Sediment Permeability in Thickener bronze

Gravity-induced settling of particles is inhibited by hydrodynamic resistance, with the settling velocity being drastically reduced as the fraction of particulate matter increases. This effect can be expressed by various equations that describe hindered settling velocity, free settling velocity (Stokes' velocity), hindered settling factor (the Richardson-Zaki index), and permeability as a function of the concentration of solids.

Table 1. Symbols and units used herein

U	Hindered settling velocity (m s ⁻¹)
ψ	Solid-phase flow settling velocity (kg m ⁻² s ⁻¹)
R(φ)	Hindered settling factor (kg m ⁻³ s ⁻¹)
K(f)	Suspension permeability (m ²)
μ	Suspension viscosity (Pa)
ρ_s	Solid-phase density (kg m ⁻³)
ρ_l	Liquid density (kg m ⁻³)
φ	Volume fraction of particulate matter
g	Acceleration of gravity (9.8 m s ⁻²)
d	Stokes equivalent particle diameter (m)
HSI	Hindered settling index
U _{St}	Stokes free settling velocity (m s ⁻¹)

The hindered settling velocity is based on Stokes' law and written as formulas (1) and (2):

$$U = \frac{d^2 g (\rho_s - \rho_l)}{18\mu} (1 - \varphi)^{HSI} \quad (1)$$

$$U = U_{St} (1 - \varphi)^{HSI} \quad (2)$$

The effective Stokes diameter of particles (d), the hindered settling index (HSI), and the liquid viscosity are empirical parameters. Let us note that the effective particle diameter is a suspension-wide average; equations (1) and (2) need to be modified to take into account the polydispersity, the porosity, and the shape of particles.

The hindered settling index (HSI) is usually assumed to equal ~ 4.65 for non-flocculated particulate matter. Higher values (up to 100) are used for flocculated suspensions; in such cases, the floccule diameter is used to compute Stokes' velocity (U_{St}). Approximate HSI and U_{St} values can be found experimentally in a cylinder cup for suspensions with different solid-phase concentration ranges.

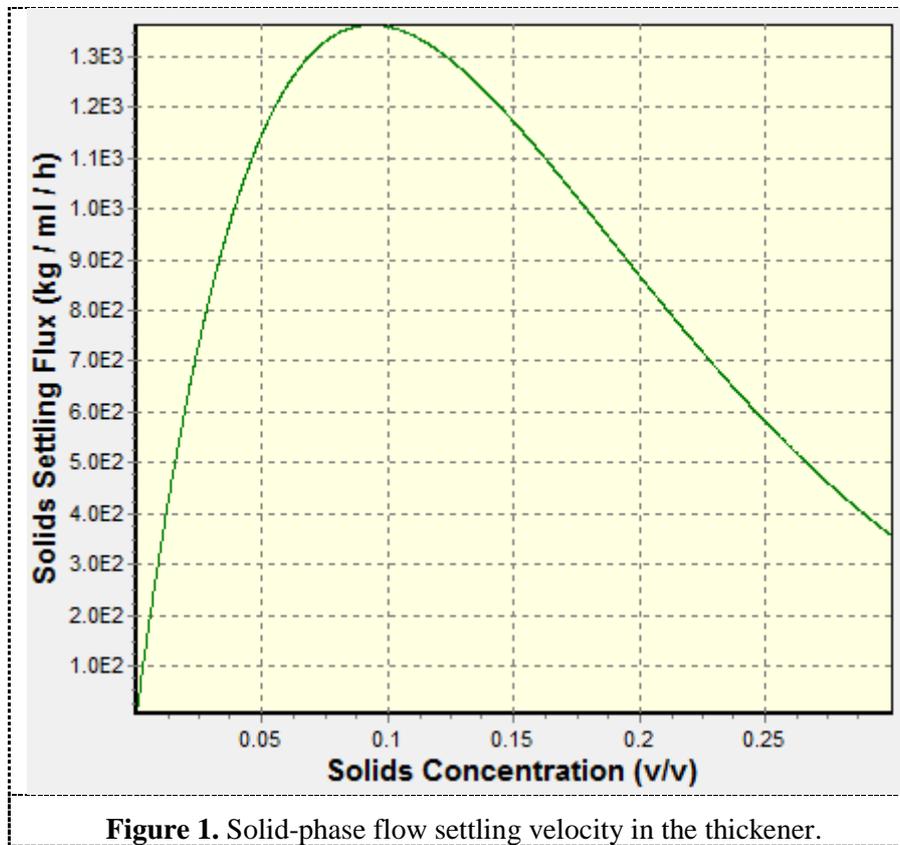
The expression (1) for the settling velocity is a simplistic equation that ignores the settling of real flocculated suspensions as particles move upwards (the rake effect).

Solid-phase flow settling velocity (ψ) is calculated by multiplying the hindered settling velocity by the solid-phase density (ρ_s) and by the volumetric fraction of the solid phase in the pulp (φ) (3):

$$\psi = \rho_s \varphi \frac{d^2 g (\rho_s - \rho_l)}{18\mu} (1 - \varphi)^{HSI}. \quad (3)$$

As the solid phase increases, the mass flow first peaks then drops as the settling velocity goes down

as in Figure 1. Particulate-matter mass flow per sedimentation area unit is often expressed in tons per square meter per hour ($t\ m^{-2}\ h^{-1}$).



The above expressions can be modified to derive the hindered settling factor $R(\varphi)$ (4):

$$R(\varphi) = \frac{18\mu}{d^2} (1 - \varphi)^{-HSI} \quad (4)$$

or the permeability $K(\varphi)$ (5):

$$K(\varphi) = \frac{\varphi d^2}{18} (1 - \varphi)^{HSI-1}. \quad (5)$$

4. RS Sediment Compressibility in Thickener

As the particles reach the bed height, they reach a concentration referred as the gel phase (φ_{gel}), where particles are clustered. The gel point is the solid-phase concentration on the top of the sediment bed in the steady state; it is assumed to equal the lowest solid-phase concentration, with which the yield strength is $P_y(\varphi)$ identifiable.

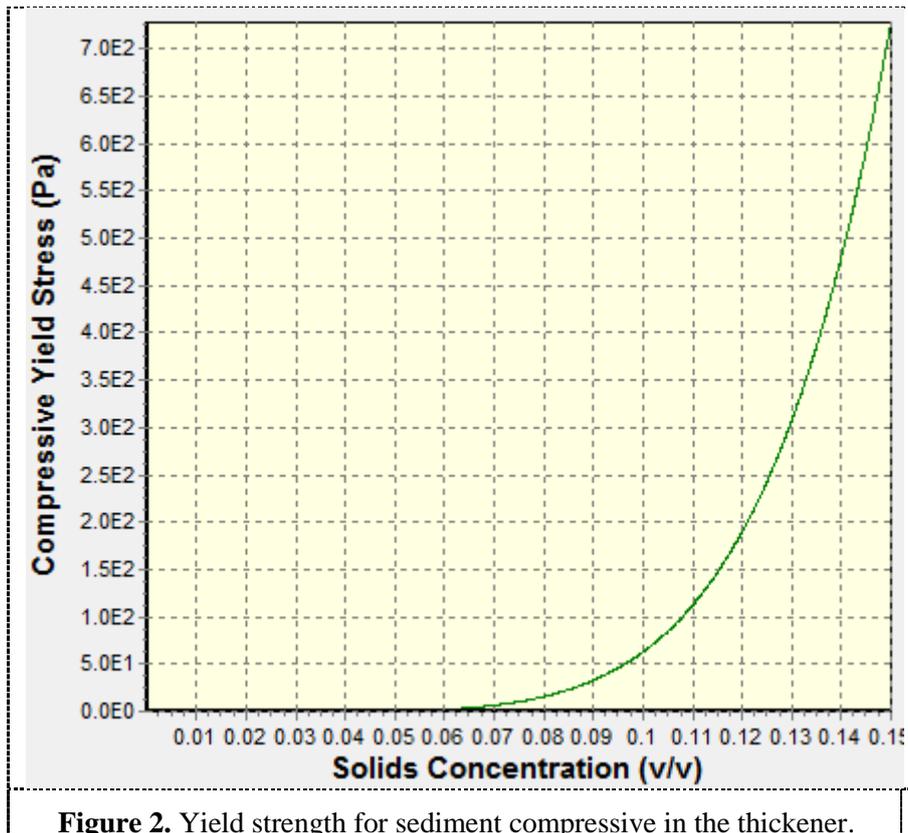
Table 2. Symbols and units used herein

$P_y(\varphi)$	Compressive yield strength ($N\ m^{-2}$)
k	Empirical parameter ($N\ m^{-2}$)
φ	Solid-phase fraction
φ_{gel}	Solid-phase fraction in the gel point
n	Empirical parameter

As the sediment is compressed in the thickener while the concentration of solids increases, the yield strength can be described by the equation (6):

$$P_y(\varphi) = k \left(\left(\frac{\varphi}{\varphi_{gel}} \right)^n - 1 \right) \quad (6)$$

where φ is greater than or equal to φ_{gel} . If φ is less than φ_{gel} , $P_y(\varphi)$ equals zero. Figure 2 shows the yield strength for sediment compression in the thickener.



The RS sediment compression yield strength depends on the flocculant dosage and type, flocculation conditions, pulp rheology that affects the shear history, the particle size/shape/chemical properties, pulp temperature, pH, ionic strength, the rake effect (the rake work in the thickener), etc. As of today, these factors are taken into account by means of two empirical parameters, k and n .

There is currently no experimental method to find k and n . These can be found by experimental data on an individual basis using specialized equipment; however, such methods ignore the rake effect (the work of rakes in the thickener, i.e. the rake height and rotation speed).

Optimum k (e.g. 0 to 10) and n (e.g. 4 to 30) values have been found empirically. To account for raking, let us use the least n value and adjust k to set a significant yield strength. Multiple k and n combinations may result in identical steady-state profiles; the values are greatly dependent on the flocculation conditions (in particular, the dosage and concentration of solids during flocculation) and on the particle distribution by size; this is why such combinations must be analyzed with caution. If unacceptable parameters are selected, the model will not match a real facility under various conditions.

5. Conclusion

This paper describes the basic equations of red-sludge consolidation and sedimentation in a circular thickener. Equations fundamental to the thickening process are: Stokes' equation, which describes the settling of spherical particles in a liquid medium; the Richardson-Zaki equation, which describes the

hindered settling velocity (the hindered settling index value for aggregated particles (or particles that are irregular in shape, have a wider size and density range, and are structurally diverse) will be significantly different from the same value for non-aggregated, or non-cohesive particles).

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