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Adaptive iterative learning control for a class of fractional-order nonlinear systems

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Abstract. Based on the discussions on the properties of fractional integral and Caputo fractional derivative, an adaptive iterative learning control approach is proposed for a class of fractional-order nonlinear system (FONS) with unknown time-varying parameter. With the design of learning controller and adaptive learning law for unknown parameter, the tracking error sequence converges to zero in the iteration domain while all the closed-loop signals remain bounded. Finally, a numerical example is given to verify the validity of the designed method.

1. Introduction

Fractional calculus is a generalization of the integer-order integration and differentiation to non-integer order operator. The fractional-order system (FOS) is a system which is described by fractional differential equations. Some scholars believed that the employment of fractional-order derivative operators into the system model is an improvement in accuracy. Since the 1960s, the fractional calculus operator theory [1] has been widely applied to design the control schemes for the fractional-order systems [2-4]. The fractional-order model reference adaptive control was mentioned in [2]. A feedback controller was designed for the stabilization of fractional-order chaotic system in [3]. In [4], the authors proposed an internal model control scheme for fractional-order systems.

Iterative learning control (ILC) is an approach for improving the transient performance of systems which operate repetitively over a fixed time interval. The ILC system improves its control performance by updating the control input iteratively with the information of errors and inputs in the preceding trials. There are two main branches of ILC schemes: one is the proportion integral derivative (PID) control, the other is adaptive iterative learning control (AILC). As we know, the adaptive scheme is effective for the uncertainties which are inevitable in all real systems. Hence, the AILC schemes have been proposed for nonlinear systems with some uncertainties [5-8].

Just as mentioned above, ILC can improve the transient performance of systems, the application of ILC into the fractional-order systems is useful. Up to now, the ILC of FOS has received more and more attentions [9-11]. However, almost all of the ILC schemes in the current literatures belong to $PI^{\lambda}D^{\mu}$-control, and the convergence of tracking error can be obtained by using the well-established Mittag-Leffler functions. We find that the adaptive scheme is rarely adopted in the ILC of the FOS. In view of the advantage of adaptive control, it is essential to take it into account in the future work.

Motivated by the above discussions, an adaptive iterative learning control approach is presented for fractional-order nonlinear systems with unknown time-varying parameter in this work. The rest of this paper is organized as follows. In Section 2, some basic definitions and properties of fractional calculus are introduced. Section 3 designs an AILC for a class of FONS of fractional order $\alpha(\alpha \geq 1)$. In Section

4, an illustrative example shows the effectiveness of designed method. Finally, conclusions are given in Section 5.

2. Preliminaries

In this section, some basic definitions and properties are introduced, which will be used in the following sections.

Definition 1. [1] The Riemann-Liouville integral is defined as:

$$ {}_{t_0}D_t^{-\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_{t_0}^{t} \frac{f(\tau)}{(t-\tau)^{1-\alpha}} d\tau \, , \alpha > 0 \, , \tag{1} $$

where $f(\cdot)$ is a continuous function, $\Gamma(\cdot)$ is the gamma function satisfying the following functional equation:

$$ \Gamma(z+1) = z\Gamma(z) \, . $$

There are some different definitions for fractional derivatives, including Grunwald-Letnikov (GL), Riemann-Liouville (RL) and Caputo definitions. The initial conditions for fractional differential equations with Caputo derivatives take on the same form as for integer-order differential equations [1], which have real physical interpretations. The Caputo approach is therefore adopted in this paper which is written in the following.

Definition 2. [1] The Caputo fractional derivative is expressed as:

$$ {}_{t_0}D_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \int_{t_0}^{t} \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau \, , n-1 < \alpha < n \, . \tag{2} $$

where $n \in Z^+$, Z^+ denotes the set of non-negative integers, $f^{(n)}(\cdot)$ stands for the *n*th-order derivative of function $f(\cdot)$. This symbol is also used for the description of derivatives later.

In what follows $t_0 = 0$, the Caputo fractional derivative is expressed as ${}_0D_t^{\alpha} f(t)$. The following properties can be found in Chapter 2 of [1].

Property 1. The Caputo fractional differentiation is a linear operation:

$$ {}_0D_t^{\alpha}[\lambda f(t) + \mu g(t)] = \lambda \, {}_0D_t^{\alpha} f(t) + \mu \, {}_0D_t^{\alpha} g(t) \, , $$

where λ and μ are two real constants.

Property 2. If $\alpha > 0$ and $\beta > 0$, then ${}_0D_t^{-\alpha} \, {}_0D_t^{-\beta} f(t) = {}_0D_t^{-\alpha-\beta} f(t)$.

To prove the main results in the next section, the following two lemmas are needed.

Lemma 1. α is a real constant which is not less than one. Without loss of generality, we assume that $n-1 \le \alpha < n$ with $n \in Z^+$ and $n \ge 2$. If the function $f(t)$ is *n*-times continuously differentiable and satisfies $f^{(n-1)}(0) = 0$, let $s(t) = {}_0D_t^{\alpha-1} f(t)$, then $s'(t) = s^{(1)}(t) = {}_0D_t^{\alpha} f(t)$.

Proof. Considering the condition $f^{(n-1)}(0) = 0$ and integrating by parts [12], we have

$$ s(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t \frac{f^{(n-1)}(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau = \frac{f^{(n-1)}(0)}{(n-\alpha)\Gamma(n-\alpha)} t^{n-\alpha} + \frac{1}{(n-\alpha)\Gamma(n-\alpha)} \int_0^t (t-\tau)^{n-\alpha} f^{(n)}(\tau) d\tau $$

$$ = \frac{1}{\Gamma(n+1-\alpha)} \int_0^t (t-\tau)^{n-\alpha} f^{(n)}(\tau) d\tau \, . $$

Calculating the derivative of *s*(*t*), we obtain

$$ s'(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau = {}_0D_t^{\alpha} f(t) \, . $$

Hence, the conclusion of the lemma is proved.

Lemma 2. For $n-1 < \alpha < n (n \in Z^+)$, if the function $f(t)$ has *n*-times derivatives and satisfies $f^{(m)}(0) = 0 \; (m = 0,1,2,\cdots,n)$, then ${}_0D_t^{-\alpha} \, {}_0D_t^{\alpha}[f(t)] = f(t)$.

Proof. ${}_0D_t^{-\alpha} \, {}_0D_t^{\alpha}[f(t)] = {}_0D_t^{-\alpha} \, {}_0D_t^{\alpha-n}[f^{(n)}(t)] = {}_0D_t^{-\alpha+\alpha-n}[f^{(n)}(t)] = {}_0D_t^{-n}[f^{(n)}(t)] = f(t)$.

3. AILC for a class of FONS

3.1. System description and control objective

Consider a fractional-order system as follows

$$ {}_0D_t^{\alpha} x(t) = \theta(t)\xi(x(t)) + u(t)\,, \tag{3}$$

where $x(t) \in R$ is the state and the output， $\xi(x(t))$ is a known smooth nonlinear function, $\theta(t)$ stands for an unknown continuous time-varying parameter, $\alpha \geq 1$ denotes the fractional order of the system, $u(t)$ is the control input, respectively.

Let the system (3) run repetitively over the fixed time interval $[0,T]$, the equation of the system can be expressed as:

$$ {}_0D_t^{\alpha} x_k(t) = \theta(t)\xi(x_k(t)) + u_k(t)\,, \tag{4}$$

where *k* denotes the iteration index.

The desired trajectory $r(t) \in R$ is the output of the fractional-order system ${}_0D_t^{\alpha} r(t) = h(t)$, where *h*(*t*) is a known smooth function of variable *t* ($t \in [0,T]$). Then, the tracking error at the *k*-th iteration is defined as $e_k(t) = x_k(t) - r(t)$.

The control objective is to find a sequence of control input $u_k(t)$ on the time interval $[0,T]$ and the updating law for unknown time-varying parameter, such that the tracking error $e_k(t)$ converges to zero while *k* tends to infinity, that is,

$$ \lim_{k\to\infty} e_k(t) = \lim_{k\to\infty}[x_k(t) - r(t)] = 0\,. \tag{5}$$

In order to achieve the above objective, the following assumption is needed.

Assumption 1. The initial conditions of system (4) satisfy $x_k^{(m)}(0) = r^{(m)}(0)$ $(m = 0,1,2,\cdots,[\alpha])$, that is, $e_k^{(m)}(0) = 0$ $(m = 0,1,2,\cdots,[\alpha])$, where $[\alpha]$ stands for the integer part of α.

3.2. Design of the control input and adaptive learning law

The learning controller at the *k*-th iteration is designed as:

$$ u_k(t) = -\hat{\theta}_k(t)\xi(x_k(t)) - cs_k(t) + h(t)\,, \tag{6}$$

where $s_k(t) = {}_0D_t^{\alpha-1} e_k(t)$, *c* is a positive constant, $\hat{\theta}_k(t)$ stands for the estimation to $\theta(t)$, respectively.

Let $\tilde{\theta}_k(t) = \theta(t) - \hat{\theta}_k(t)$ denote the parameter estimation error at the *k*-th iteration, the equation of tracking error at the *k*-th iteration can be written as:

$$ {}_0D_t^{\alpha} e_k(t) = \tilde{\theta}_k(t)\xi(x_k(t)) - cs_k(t)\,. \tag{7}$$

The estimated parameter $\hat{\theta}_k(t)$ is updated iteratively with the information in the preceding trials. In this updating process, the estimated parameter may become too large to affect the stability of the system. Therefore, the saturation function $sat(\cdot)$ is used to design the adaptive learning law.

For a scalar *a*, the saturation function is defined as:

$$ sat(a) = \begin{cases} a_1, a < a_1 \\ a, a_1 \leq a \leq a_2 \\ a_2, a > a_2 \end{cases}, \tag{8}$$

where $\{a_1, a_2\}$ are the lower and upper bounds of *a* and satisfy $a_1 < a_2$.

By using the saturation function, we can make the estimated parameter $\hat{\theta}_k(t)$ always lie within the interval $[a_1, a_2]$.

The proposed adaptive learning law for $\hat{\theta}_k(t)$ is designed as:

$$\begin{cases} \hat{\theta}_k(t) = sat(\hat{\theta}_k^*(t)), \\ \hat{\theta}_k^*(t) = sat(\hat{\theta}_{k-1}^*(t)) + \gamma\xi(x_k(t))s_k(t), \\ \hat{\theta}_{-1}^*(t) = 0, \end{cases} \tag{9}$$

where γ is a positive design parameter, $\hat{\theta}_k^*(t)$ is the estimation without saturation, $\hat{\theta}_k(t)$ is the estimation with saturation, respectively.

Without loss of generality, we assume that $\theta(t)$ lies within the saturation bounds. In order to analysize the relationship between the real parameter and estimated parameter, the following lemma is needed which can be found in the Appendix of [6].

Lemma 3. [6] Given scalars a and b, suppose that $a_1 < b < a_2$, where $\{a_1, a_2\}$ are the lower and upper bounds of a, then $[b - sat(a)][a - sat(a)] \le 0$.

Using Lemma 3, we can obtain the following inequality

$$[\theta(t) - sat(\hat{\theta}_k^*(t))][\hat{\theta}_k^*(t) - sat(\hat{\theta}_k^*(t))] \le 0 . \tag{10}$$

In order to obtain the convergence of the closed-loop system, the following lemma is needed.

Lemma 4. [6] If the sequence of derivatives $s_k'(t)$ is uniformly bounded on the time interval $[0,T]$ and $\lim_{k\to\infty}\int_0^T s_k^2(\tau)d\tau = 0$, then $\lim_{k\to\infty} s_k(t) = 0$ uniformly on $[0,T]$.

3.3. Convergence analysis

The convergence property of the proposed control scheme is summarized in the following theorem.

Theorem 1. Based on Assumption 1, all signals in the closed-loop system, consisting of system (4), the control law (6) and adaptive learning law (9), remain bounded on $[0,T]$, and the tracking error $e_k(t)$ converges to zero while the iteration index k tends to infinity, that is, $\lim_{k\to\infty} e_k(t) = 0$.

Proof. We design a Lyapunov-Krasovskii functional at the k-th iteration as follows:

$$W_k(t) = V_k(t) + \frac{1}{2\gamma}\int_0^t \tilde{\theta}_k^2(\tau)d\tau \,,\; V_k(t) = \frac{1}{2}s_k^2(t)\,, \tag{11}$$

where $s_k(t) = {}_0D_t^{\alpha-1}e_k(t)$. Then, applying Lemma 1, we have $s_k'(t) = {}_0D_t^{\alpha}e_k(t)$.

The derivative of $V_k(t)$ is expressed as:

$$V_k'(t) = s_k(t)s_k'(t) = s_k(t)[\tilde{\theta}_k(t)\xi(x_k(t)) - cs_k(t)] = s_k(t)\tilde{\theta}_k(t)\xi(x_k(t)) - cs_k^2(t) + s_k(t)e_k(t). \tag{12}$$

The difference of $W_k(t)$ at the k-th iteration can be derived as:

$$\begin{aligned} \Delta W_k(t) = W_k(t) - W_{k-1}(t) &= V_k(t) + \frac{1}{2\gamma}\int_0^t \tilde{\theta}_k^2(\tau)d\tau - V_{k-1}(t) - \frac{1}{2\gamma}\int_0^t \tilde{\theta}_{k-1}^2(\tau)d\tau \\ &= V_k(t) + \frac{1}{2\gamma}\int_0^t [\tilde{\theta}_k^2(\tau) - \tilde{\theta}_{k-1}^2(\tau)]d\tau - V_{k-1}(t)\,, \end{aligned} \tag{13}$$

where

$$\begin{aligned} V_k(t) = \int_0^t V_k'(\tau)d\tau + V_k(0) &= \int_0^t [s_k(\tau)\tilde{\theta}_k(\tau)\xi(x_k(\tau)) - cs_k^2(\tau)]d\tau \\ &= \int_0^t [s_k(\tau)\tilde{\theta}_k(\tau)\xi(x_k(\tau))]d\tau - c\int_0^t s_k^2(\tau)d\tau \,. \end{aligned} \tag{14}$$

Applying (9) yields

$$\begin{aligned} \frac{1}{2\gamma}\int_0^t [\tilde{\theta}_k^2(\tau) - \tilde{\theta}_{k-1}^2(\tau)]d\tau &= \frac{1}{2\gamma}\int_0^t \{[\theta(\tau) - \hat{\theta}_k(\tau)]^2 - [\theta(\tau) - \hat{\theta}_{k-1}(\tau)]^2\}d\tau \\ &= \frac{1}{2\gamma}\int_0^t \{-2\tilde{\theta}_k(\tau)[\hat{\theta}_k(\tau) - \hat{\theta}_{k-1}(\tau)] - [\hat{\theta}_k(\tau) - \hat{\theta}_{k-1}(\tau)]^2\}d\tau \end{aligned}$$

$$=\frac{1}{\gamma}\int_0^t[\theta(\tau)-\hat{\theta}_k(\tau)][\hat{\theta}_{k-1}(\tau)-\hat{\theta}_k(\tau)]d\tau-\frac{1}{2\gamma}\int_0^t[\hat{\theta}_k(\tau)-\hat{\theta}_{k-1}(\tau)]^2d\tau$$

$$=\frac{1}{\gamma}\int_0^t[\theta(\tau)-sat(\hat{\theta}_k^*(\tau))][sat(\hat{\theta}_{k-1}^*(\tau))-sat(\hat{\theta}_k^*(\tau))]d\tau-\frac{1}{2\gamma}\int_0^t[\hat{\theta}_k(\tau)-\hat{\theta}_{k-1}(\tau)]^2d\tau$$

$$=\frac{1}{\gamma}\int_0^t[\theta(\tau)-sat(\hat{\theta}_k^*(\tau))][\hat{\theta}_k^*(\tau)-sat(\hat{\theta}_k^*(\tau))]d\tau$$

$$-\int_0^t\tilde{\theta}_k(\tau)\xi(x_k(\tau))s_k(\tau)d\tau-\frac{1}{2\gamma}\int_0^t(\hat{\theta}_k(\tau)-\hat{\theta}_{k-1}(\tau))^2d\tau\,. \tag{15}$$

Using the inequality (10), we obtain

$$\frac{1}{2\gamma}\int_0^t[\tilde{\theta}_k^2(\tau)-\tilde{\theta}_{k-1}^2(\tau)]d\tau\le-\int_0^t\tilde{\theta}_k(\tau)\xi(x_k(\tau))s_k(\tau)d\tau\,. \tag{16}$$

Substituting (16) and (14) into (13) yields

$$\Delta W_k(t)\le-c\int_0^t s_k^2(\tau)d\tau\le 0\,. \tag{17}$$

According to the definition of $W_k(t)$, we have

$$W_k(t)\le-c\int_0^t s_k^2(\tau)d\tau+\frac{1}{2\gamma}\int_0^t\tilde{\theta}_{k-1}^2(\tau)d\tau\le\frac{1}{2}s_{k-1}^2(T)+\frac{1}{2\gamma}\int_0^T\tilde{\theta}_{k-1}^2(\tau)d\tau\,. \tag{18}$$

Therefore

$$W_k(t)\le W_{k-1}(T), t\in[0,T], \tag{19}$$

it follows from (17) that

$$W_k(T)-W_{k-1}(T)\le-c\int_0^T s_k^2(\tau)d\tau\le 0\,, \tag{20}$$

which shows that

$$W_k(T)\le W_{k-1}(T), k=1,2,3,\cdots, \tag{21}$$

$W_k(T)$ is a non-increasing sequence.

In order to show the boundedness of $W_k(T)$, we calculate the derivate of $W_0(t)$ as follows:

$$W_0'(t)=V_0'(t)+\frac{1}{2\gamma}\tilde{\theta}_0^2(t)=s_0(t)\tilde{\theta}_0(t)\xi(x_0(t))-cs_0^2(t)+\frac{1}{2\gamma}\tilde{\theta}_0^2(t)\,. \tag{22}$$

Applying (9), we obtain

$$\frac{1}{2\gamma}\tilde{\theta}_0^2(t)=\frac{1}{2\gamma}[\tilde{\theta}_0^2(t)-\tilde{\theta}_{-1}^2(t)]+\frac{1}{2\gamma}\tilde{\theta}_{-1}^2(t)\le-s_0(t)\tilde{\theta}_0(t)\xi(x_0(t))+\frac{1}{2\gamma}\theta^2(t)\,. \tag{23}$$

Substituting (23) into (22) yields

$$W_0'(t)\le-cs_0^2(t)+\frac{1}{2\gamma}\theta^2(t)\le\frac{1}{2\gamma}\theta^2(t)\,. \tag{24}$$

Based on the properties of continuous function on a closed interval, $\theta(t)$ is continuous on $[0,T]$, then bounded on $[0,T]$. There is a positive constant M which satisfies $W_0'(t)\le M=\max_{t\in[0,T]}\frac{1}{2\gamma}\theta^2(t)<\infty$, so that $W_0(t)-W_0(0)\le Mt\le MT$. Then we obtain that $W_0(t)$ is bounded on $[0,T]$. It follows from (20) that $W_k(T)$ is bounded, and from (19) $W_k(t)$ is uniformly bounded on $[0,T]$. According to the definition of $W_k(t)$, $s_k(t)$ is uniformly bounded on $[0,T]$, that is, there is a positive constant M_1, such that $|s_k(t)|\le M_1$. Then, applying Assumption 1 and Lemma 1, for $t\in[0,T]$ and $\alpha\ge 1$, we have

$$|e_k(t)|=\left|{}_0D_t^{1-\alpha}s_k(t)\right|=\left|{}_0D_t^{-(\alpha-1)}s_k(t)\right|=\left|\frac{1}{\Gamma(\alpha-1)}\int_0^t\frac{s_k(\tau)}{(t-\tau)^{2-\alpha}}d\tau\right|\le\frac{1}{\Gamma(\alpha-1)}\int_0^t\frac{|s_k(\tau)|}{(t-\tau)^{2-\alpha}}d\tau$$

$$\le \frac{1}{\Gamma(\alpha-1)}\int_0^t \frac{M_1}{(t-\tau)^{2-\alpha}}d\tau \le \frac{M_1}{(\alpha-1)\Gamma(\alpha-1)}t^{\alpha-1} \le \frac{M_1}{\Gamma(\alpha)}T^{\alpha-1}. \quad (25)$$

Let $M_2 = \frac{M_1}{\Gamma(\alpha)}T^{\alpha-1}$, we obtain $|e_k(t)| \le M_2$, that is, $e_k(t)$ is uniformly bounded on $[0,T]$. Then, the uniform boundedness of $x_k(t)$, $\xi(x_k(t))$ on [0, T] is obviously shown. In accordance with (6), (7), (9) and (11), $\hat{\theta}_k(t)$, $u_k(t)$ and $s_k'(t)$ are all uniformly bounded on $[0,T]$. Hence, all signals in the closed-loop system remain bounded.

It follows from (20) that $\int_0^T s_k^2(\tau)d\tau \le \frac{1}{c}[W_{k-1}(T) - W_k(T)]$. Since $W_k(T)$ is monotone decreasing but lower bounded, $W_k(T)$ converges. Therefore, $\lim_{k\to\infty}\int_0^T s_k^2(\tau)d\tau = 0$. By using Lemma 4, $\lim_{k\to\infty} s_k(t) = 0$ uniformly on $[0,T]$. Based on the definition of limit of sequence, $\forall\varepsilon > 0$, $\exists K(K \in Z^+)$ such that $|s_k(t)| < \varepsilon$ for all $k > K$, then $|e_k(t)| = |{}_0D_t^{1-\alpha}s_k(t)| \le \frac{1}{\Gamma(\alpha-1)}\int_0^t \frac{|s_k(\tau)|}{(t-\tau)^{2-\alpha}}d\tau < \frac{t^{\alpha-1}}{\Gamma(\alpha)}\varepsilon$ for all $k > K$. Hence, $\forall\varepsilon_1 = \frac{t^{\alpha-1}}{\Gamma(\alpha)}\varepsilon > 0$, $\exists K(K \in Z^+)$ such that $|e_k(t)| < \varepsilon_1$ for all $k > K$.

From the above calculations we obtain $\lim_{k\to\infty} e_k(t) = 0$ on $[0,T]$.

4. Simulation example

In this section, a numerical example is given to show the effectiveness of the designed method.

Example 1. Consider a fractional-order nonlinear system with repetitive operation over a fixed time interval [0, 7] as follows

$${}_0D_t^{1.9}x_k(t) = \theta(t)\xi(x_k(t)) + u_k(t). \quad (26)$$

The parameters in the system (26) are $\theta(t) = \cos(2t)$, $\xi(x_k(t)) = x_k^2(t) + 2x_k(t)$.

The desired trajectory $r(t)$ is the output of the system as follows

$${}_0D_t^{1.9}r(t) = \cos t + \sin t. \quad (27)$$

According to the designed method in this paper, the control law (6) can be expressed as:

$$u_k(t) = -\hat{\theta}_k(t)\xi(x_k(t)) - cs_k(t) + \cos t + \sin t, \quad (28)$$

where $s_k(t) = {}_0D_t^{\alpha-1}e_k(t) = {}_0D_t^{\alpha-1}[x_k(t) - r(t)]$, $\hat{\theta}_k(t)$ is designed in the formula (9), respectively.

As mentioned in section 3, the design parameters of control law (28) and adaptive learning law (9) are positive constant, which are selected as $c = 1$ and $\gamma = 0.5$. Based on Assumption 1, the initial conditions of system (26) and (27) can be chosen as $x_k(0) = r(0) = 0$ and $x_k'(0) = r'(0) = 0$.

The adaptive iterative learning control method is simulated by MATLAB numerical value method and SIMULINK toolbox. In the simulation, fractional-order systems are approximated by integer-order systems. In order to ensure that the parameter $\theta(t)$ lies within the saturation bounds, we select $\{-2, 2\}$ as the lower and upper bounds of the saturation function, which is required in Theorem 1.

The numerical simulation results show that a perfect tracking performance is achieved while the iteration number increases. Figure 1 shows that the maximum absolute value of tracking error between 0 and 7 seconds decreases with the iteration k, and it is almost equal to zero as iteration index $k = 20$. Figure 2 gives the output of the system (26) and the desired trajectory (27) as iteration index $k = 100$. We can find that the close-loop error system converges to zero while the iteration index k tends to infinity, and the state of the system (26) asymptotically tracks the desired trajectory. Figure 3 and Figure 4 give the evaluation of estimated time-varying parameter and the curve of control input as iteration index $k = 100$, in which the signals in the close-loop systems remain bounded.

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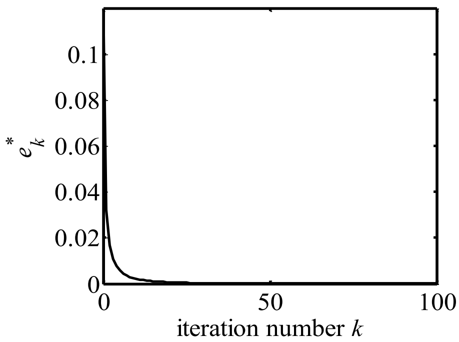

Figure 1. The change of $e_k^* = \max_{t\in[0,7]} |e_k(t)|$ with iteration index k.

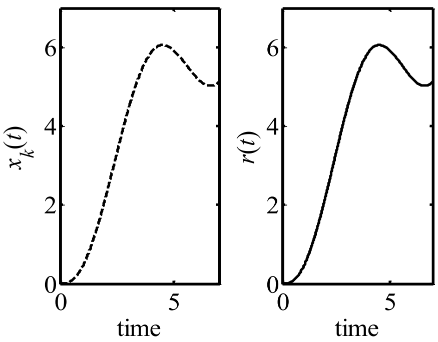

Figure 2. The output of the system (26) and the desired trajectory (27) as iteration index $k = 100$.

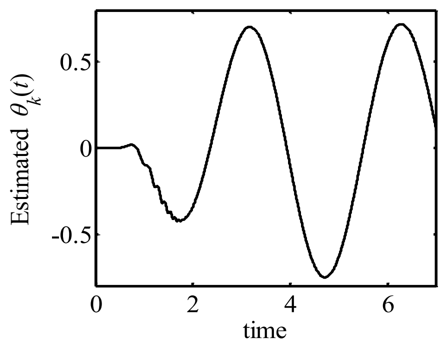

Figure 3. The evaluation of estimated parameter $\theta_k(t)$ as iteration index $k = 100$.

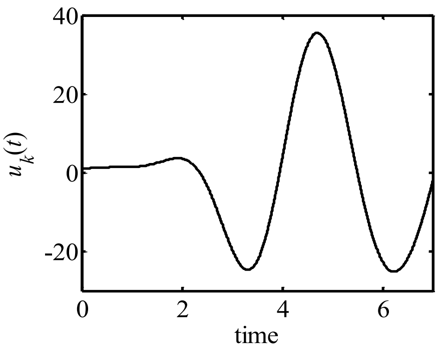

Figure 4. The curve of control input $u_k(t)$ as iteration index $k = 100$.

5. Conclusions

In this paper, we present an adaptive iterative learning control scheme for a class of fractional-order nonlinear systems of fractional order $\alpha(\alpha \geq 1)$. When the iteration index tends to infinity, the output of the system can track the desired trajectory, and the signals in the closed-loop system remain bounded. A simulation example verifies our theoretical results.

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