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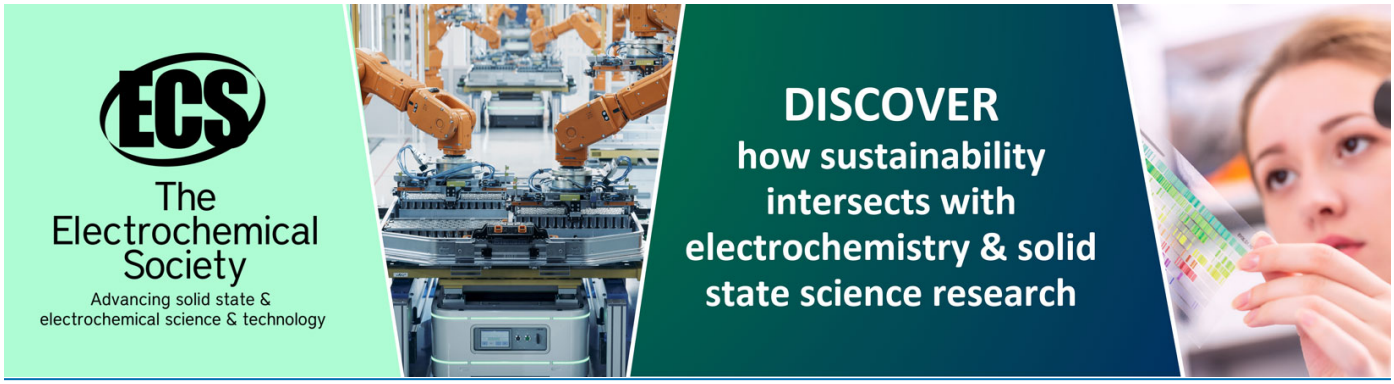

An analysis mathematical problem solving and mathematical critical thinking skills of junior high school students

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Abstract. This article describes mathematical problem solving and critical thinking skills and examines the relationship between junior high school students' problem solving skills and critical thinking skills in learning mathematics. The research samples consisted of thirty-six students at the age of 13 to 14 having mathematical abilities on above average who were selected through purposive random sampling technique from one public junior high school of Yogyakarta. Data were obtained from tests of problem solving skills and critical thinking skills. Both instruments are essay questions. Quantitative analysis is used to determine level of problem solving skills, level of critical thinking skills, and relationship between them. The results revealed that the average of students' problem solving skills and mathematical critical thinking skills was in the medium criteria where critical thinking skills were slightly higher than problem solving skills. Besides, the relationship between them belonged to medium category.

1. Introduction

21st century which is being 'rocked' by a paradigm shift causes development throughout the life. Existing developments not only offer convenience but also create macro opportunities and macro problems. Existing changes will be an opportunity if someone has capital and readiness in mastering main skill of 21st century. Instead, development should be seen as a problem that will continue for those who do not acquire the skills demanded from the recent century [1].

Education is the main role in building knowledge and skills for the 21st century [2]. More importantly, we must note how important the education in that the rapid technology does not make higher education institutions lose their primary goals as important intervention in filling gaps of knowledge [3] but assists education in preparing the nation's generation to able to participate in network society [4]. Thus, it is very important for teachers to create an environment that encourages the development and implementation of desirable thinking skills in the 21st century [2,5].

Learning partnerships of 21st century have identified critical thinking and problem solving, creativity, communication, and collaboration as main learning skills that students must possess in solving problems amid the complexities of life [6,7,8,9]. The skills are also emphasized in learning mathematics [10]. Furthermore, the need for these skills is in accordance with the main objective of learning mathematics namely preparing students to deal with real life situations effectively [11].

The expected achievement in learning mathematics is that students have good problem solving skills. This skill is believed to be the core and foundation that underlies the entire mathematics curriculum since the machine of mathematics knowledge is not an axiom but a problem [12]. A problem is when an individual cannot immediately recognize the required procedure but has to put all his knowledge as an attempt to solve the problems [13]. Moreover, problem solving can be seen as a process that gives students the opportunity to experience the power of mathematics in the world around them [14].

Problem solving is an activity that requires individuals to engage in a variety of cognitive actions, each of which requires some knowledge and skills, and some of which are non-routine [15]. Problem solving has four characteristics, namely organized and flexible knowledge that supports mathematical ideas, a series of problem solving strategies, awareness of choosing and exploiting strategies and resources, and belief that the problems can be solved. These characteristics bring a person to success in solving his problems [16]. The stages of problem solving then have been formulated by Polya, namely understanding the problem; planning to solve problem; carrying out the plan; and interpreting the answers [17]. The success of students in problem solving will lead them to an understanding about structures of basic mathematics [10].

Education gives students a mandate not only on problem solving skills but also on mathematics learning to emphasize the learning of how to think, particularly how to be a critical thinker [18]. Critical thinking is certain and rational thinking [19] and is focused on deciding what to believe or do [20]. This thought refers to students' ability to identify problems and assumptions, recognize relationships, evaluate evidence, and make correct conclusions [21]. Critical thinkers have the tools to deal with new situations. As suggested by Mark Twain, if all you have is a hammer, lots of things will look like nails [22]. Hence, critical thinking allows one to face reality in a reasonable way [18].

Critical thinking enables students to process information logically and assists students to prepare for independent learning. Critical thinking also facilitates students in determining what information is important and is not useful, choosing the right response, and making things more directed [23]. More importantly, there are aspects of critical thinking, namely interpretation which is the ability to understand and express the meaning of various experiences, situations, data, events, rules, procedures, or beliefs; analysis which is the ability to identify the intended and actual inferential relationships; evaluation which is the ability to assess the credibility of a statement or other representation; and inference which is the ability to draw reasonable conclusions [24].

Both problem solving abilities [25] and critical thinking skills [18] are the foundation of success in every profession. The sharper the students' mathematical critical thinking skills, the more they are able to solve problems and formulate arguments by utilizing a broad base of knowledge. While solving problems, students develop mathematical thinking skills related to advanced mathematics, namely critical thinking [23]. Although this skill is a complex form of high-order processing [26], it is very understandable to see those skills as part of 21st century skills that are expected to have schools achieve it [9]. Particularly, achievement motivation becomes a strong predictor of using these skills in learning. Students with above-average grades in math exams more often apply higher-order thinking processes [26]. Therefore, this study wants to analyze problem solving skills and critical thinking skills and to examine the relationship between these skills, particularly of students with above-average mathematical skills. The findings of this study are expected to provide information about the achievement of the two skills, so that the further development of them can be endeavored and can be maximally facilitated in learning mathematics.

2. Research methods

Thirty-six students at the age of 13 to 14 were selected through purposive random sampling technique from one junior high school in Yogyakarta. The school meets the criteria of the research subjects in which the school has above-average mathematical skills. The level of mathematical skills is based on the results of 2017 mathematics national examination categorized to high and 'A' school accreditation.

To measure students' problem solving skills and critical thinking skills, the instruments of this study were tests using essay questions about polyhedron (a solid with flat faces) material that students have learned. Test for problem solving skills was used to indicate problem solving indicators, namely understanding the problems, designing plans, implementing plans, and interpreting the results. Meanwhile, to assess critical thinking skills, a test was used to measure indicators of critical thinking skills, namely interpreting, analyzing, evaluating, and inferring. The instrument of mathematical problem solving skill was adopted from study of Leohani [27**]** and the instrument of mathematical critical thinking skill was adopted from study of Agus [28**]**. These instruments have a good level of validity and reliability and are in the same subject material so that they were valid to be used.

To achieve the objectives of the study, analysis of problem solving skills, critical thinking skills, and the relationship between them were carried out. The steps to analyze the data were (a) checking the test of problem solving skills by analyzing the steps of students' answers; (b) examining test of critical thinking skills by analyzing the steps of students' answers; (c) analyzing level of the relationship between problem solving skills and critical thinking skills by looking at r value, an output of Pearson's product moment; and (d) analyzing descriptively more in depth.

According to Widoyoko, criteria used in categorizing problem solving skills and critical thinking skills are formulated based on categories. The criteria are presented in Table 1.

Table 1. Criteria of mathematical skills

Score interval	Category	Criteria
$x > x_i + 2{,}4sd_i$	$x > 90$	Very high
$x_i + 1{,}5sd_i < x \leq x_i + 2{,}4sd_i$	$75 < x \leq 90$	High
$x_i + 0{,}6sd_i < x \leq x_i + 1{,}5sd_i$	$60 < x \leq 75$	Average
$x_i - 0{,}3sd_i < x \leq x + 0{,}6sd_i$	$45 < x \leq 60$	Low
$x \leq x_i - 0{,}3sd_i$	$x \leq 45$	Very low

3. Results and discussion

3.1. Students' mathematical problem solving

According to Polya, problem solving skills test was analyzed based on the stages. The results obtained from mathematical problem solving skills test are illustrated in Table 2.

Table 2. The results of mathematical problem solving

Aspect	Mean	Criteria
Understanding the problem	84,03	High
Designing and implementing the plan	62,96	Average
Interpreting the result	41,67	Low

Based on the data in Table 2, in the stage of understanding the problem, dominant students succeeded in understanding the problem in general. However, there were still students who found difficulties in understanding which side is meant from the size given. Understanding at this stage becomes the 'capital' for the next stage namely designing and implementing the plan. At this stage, students who fail to understand the problem were not able to automatically design and implement the problem correctly. Besides, some of them who understand the problem were found to have been able to write the draft correctly but were not fully successful in implementing the plan. In the third stage of implementing the plan, most students failed to relate mathematical concepts. Many students were not aware of using Phytagoras theorem in problem solving. The last stage of interpreting the data, the students were expected to be able to examine the given solution. The students who successfully implement the plan dominantly appear to be successful in interpreting the data. However, some students were found to be careless in interpreting, so what they concluded were not in line with the results.

In doing problem solving skills test, the students had different solution's techniques. Interestingly, the problem given about geometry materials was indirectly able to give an illustration of students'

spatial ability although not all students showed their spatial ability. Solving the problem through drawing seemed to help the students.

3.2. Students' mathematical critical thinking

Critical thinking skills test measured four aspects which are interpretation, analysis, evaluation, and inference. Each aspect had different questions. Students' critical thinking skills in general are presented in Table 3.

Table 3. The result of mathematical critical thinking skills

Aspect	Mean	Criteria
Interpretation	86,45	High
Analysis	42,70	Very low
Evaluation	80,20	High
Inference	81,25	High

Analysis of critical thinking skills test revealed that average students had high interpretation, evaluation, and inference skills but had very low analysis. This result is in line with previous research which demonstrated that one of the reasons for students' low mathematical performance is students' inability to analyze mathematical concepts systematically [29].

Aspects of interpretation, evaluation, and inference successively require students to interpret a problem clearly, assess the truth of a statement, and make conclusions by considering any perspective. The tests given a solution to these three aspects involved calculation while in aspect of analysis students were required to analyze relevant or irrelevant statements with logical reasons. In this aspect students were asked to analyze whether the cube belongs to block. This problem is solved by analyzing the elements on the cube and block without any calculations. Mostly, the students only answered the test by looking at different sides of cube and block. The results illustrate that the students are predominantly less skilled at interpreting the meaning of mathematics.

The results become the researchers' reason to recommend a learning process that more emphasize students' in-depth understanding. Assessment is given not only through questions that use calculations but also through critical thinking particularly analyzing a statement. Students need intentional practice in practicing critical thinking skills [30].

3.3. Relationship between students' mathematical problem solving and mathematical critical thinking

From the tests, the average students' critical thinking skills was higher than the average students' problem solving skills. However, the differences between the two are relatively small which is 70.83 of average problem solving skills and 72.65 of average critical thinking skills. Both of skills are in the same category which is average category. Although there were no research subjects belonged to high category, the results obtained were quite in line with their level of mathematical ability. A score of 70 achieved by students in both skills is believed to have quite commensurate with the students' characteristics.

As seen from minimum and maximum achievements there are relatively small differences. No students are in very low category on the two tests. In addition, small number of students succeed to be in very high category of critical thinking but not of problem solving skills. Furthermore, the results of problem solving skills test and critical thinking skills test are presented in Table 4 and the achievements for each category are presented in Table 5.

Table 4. Result's information

	Minimum	Maximum	Mean	Criteria	SD	Correlation	
Mathematical Problem Solving	50	90	70,83	Average	11,30	0,489	Average
Mathematical Critical Thinking	53,13	96,88	72,65	Average	10,78		

Table 5. Percentage of students' achievement based on students' ability categories

	Very low	Low	Average	High	Very high
Mathematical Problem Solving	0%	30,56%	33,33%	36,11%	0%
Mathematical Critical Thinking	0%	15,62%	53,13%	37,50%	6,25%

Based on Table 4, there was a correlation between students' problem solving skills and critical thinking skills at average category. The students' achievement of problem solving skills and critical thinking skills are in medium, high, low, and very high categories. Through further identification, 44.44% of students were in the same ability category in problem solving ability test and critical thinking ability. An analysis was also carried out on two groups of students with high problem solving abilities and with high critical thinking skills. 53.85% of students with high problem solving abilities also have high critical thinking abilities.

The results of the analysis of problem solving abilities and critical thinking abilities provide information that the level of students' ability is classified as good but their abilities still need to be maximized in the process of learning mathematics. Development of both abilities is inseparable from pedagogy [21]. Learning that does not lead students to think, argue and ask is able to inhibit the development of students' abilities [31] but learning is able to lead the students to achieve high scores in examinations [14]. In addition, students' perceptions of mathematics are also believed to affect students' abilities [32] Therefore, the teacher must take a main role in creating meaningful learning so that it influences students' abilities and perceptions of mathematics. The development of high-order mathematical abilities such as problem solving abilities and critical thinking skills can certainly be achieved if students are intentionally accustomed to learning [14]. Furthermore, the researchers believe that the level of students' mathematical abilities has contributed to the development of two main cognitive abilities. Talented students have a greater chance of success to both skills. Thus, the teachers' efforts to maximize them are highly expected, so that students are able to achieve maximum performance.

4. Conclusion

The results of this research were revealed to support existing theories. The research subjects who are above average students with mathematical abilities succeed in showing problem solving skills and critical thinking which are in the medium category. Although the average ability of students is not in the high category but this result can be concluded quite in line with the level of students' mathematical abilities. Besides, the relationship between problem solving ability and critical thinking ability is medium category. 44.44% of students are in the same category of ability between problem solving abilities and critical thinking abilities. 53.85% of students with high mathematical problem solving abilities have high critical thinking skills.

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