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# Students' Thinking in Solving Geometric Problems Based on PISA Levels

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# Students' Thinking in Solving Geometric Problems Based on PISA Levels

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**Abstract.** The latest results of Programme for International Students assessment (PISA) shows that Indonesia is still on the bottom place among all participating countries. Therefore, this study aims at analysing students' thinking in solving geometric problems based on PISA levels. This is a qualitative descriptive study involving 25 8th graders of a private junior high school in Surakarta, Indonesia. The data collection is conducted using test and documentation. The data analysis is conducted in three stages namely data reduction, data display, and conclusion drawing/verification. Based on the analysis, it can be concluded that most students' are categorizing in medium level of thinking (about 72%), less students are categorizing in high level of thinking (20%), and least students are categorizing in high level of thinking (8%). PISA level 1 and 2 are levels which are easily achieved by students. Whereas, PISA level 3-6 are levels which are difficultly achieved by students. This implies the importance of increasing students' levels of thinking in solving mathematics problems, particularly geometric problems.

## 1. Introduction

Mathematical thinking is an important skill which should be mastered by students. This is because the ability to think mathematically and to use mathematical will support science, technology, economic life and any other fields [1]. However, the latest results of Programme for International Students Assessment (PISA) shows that in the field of mathematics, Indonesia is still in the 63rd place among 72 participating countries [2]. The results indicate that Indonesian students still miss one of the important goals of schooling.

PISA is a study aimed to measure 15-year-old students' performance in mathematics, science, and reading. In assessing students' performance in mathematics, the PISA frameworks divided mathematics into four contents namely space and shape, change and relationship, uncertainty and data, and quantity [3]. In Indonesian Curriculum, the content of space and shape is related to geometry. Those are both plane geometry and solid geometry.

Based on the characteristics, learning geometry should be easier than other topics. It is because the ideas of geometry are familiar with students' daily lives. For instance, in a house, tiles can be the representation of squares, a wardrobe can be the representation of a rectangular prism, and gallon can be the representation of a cylinder. However, the study of PISA mentioned above gives information that the 15-year-old students' performance in mathematics in Indonesia is still have to be improved.

There have been several studies aiming at improving Indonesian students' performances in mathematics. To support Indonesian mathematics textbooks, the studies of some researchers [4–6] have been conducted in developing PISA-like problems. That was because of some previous researchers which reported students' errors in solving either mathematics problems adopted from

PISA or PISA-like problems [7–9]. Moreover, it was also because the lack of PISA-like problems in Indonesian mathematics textbooks, particularly mathematics textbook for grade 8 [10].

Based on PISA levels [3], in the topic of geometry, in general, researchers found that students' difficulties are solving the 3rd level up to 6th level. It is because students are mainly solve problems of the 1st and 2nd levels. Therefore, this study aims at analyzing students' thinking in solving geometric problems based on PISA levels.

## 2. Research Method

This is a descriptive qualitative study involving a class of students of grade 8 in a private school in Surakarta, Indonesia. Furthermore, three students were chosen for the deep analysis. Each of them is student with high, moderate, and low level of mathematics thinking. To ensure the validity, the researchers applied data triangulation based on students' written works, interview, and documentation. The data analysis is conducted in three stages namely data reduction, data display, and conclusion drawing/verification. Based on students' written works and interview, the students' thinking are categorized based on PISA levels.

The test is firstly given to 25 students of grade 8 at the school. Afterwards, students' answers of the test given are scored and categorized based on the rule displayed in Table 1. Based on the categories, one student of each level are chosen for further analysis.

**Table 1.** The Categories of Students' Levels of Thinking in Mathematics

| Score | Students' Levels of Thinking in Mathematics |
|---|---|
| 1-30 | Low |
| 31-70 | Moderate |
| 71-100 | High |

The levels of students' thinking are categorized into six levels based on the indicators of PISA levels [3]. The indicators are used to map students' level of thinking based on both their written works and interviews. The test given to students involving all geometric problems learned in grade 8. The geometric contents have been learned by students before the test. Table 2 shows the geometric topics used for the test. The complete problems are showed in the Appendices section.

**Table 2.** The Geometric Problems used for The Test

| Question Indicator | Geometric Topic |
|---|---|
| Finding the surface area of a three dimensional figure | Cube, Rectangular Prism |
| Finding the area of a triangle | Isosceles triangle, Pythagorean theorem |
| Finding the surface area of a pyramid | Pyramid, Pythagorean theorem. |
| Finding the volume of a triangular prism | Triangular Prism |
| Identifying geometric figures | Pyramid |

## 3. Result and Discussion

Based on the results of the test, most students' are categorizing in medium level of thinking (about 72%), less students are categorizing in high level of thinking (20%), and least students are categorizing in high level of thinking (8%). For the deep analysis, three students were chosen based on the score of the test. Subject 1 is a representative of students with low score, Subject 2 is a representative of students with moderate score, and Subject 3 is a representative of students with high score.

Based on the analysis of students' written works and the interview, Student 1 could only solve one problem correctly among the five problems given. He could only reach level 4 of PISA which is constructing and communicating explanations and arguments based on his interpretations, arguments and actions.

Student 2 could solve three problems correctly (finding the surface area of three dimensional figure, finding the surface area of pyramid, finding the volume of triangular prism). This student could reach level 1 up to level 6 with 60% among indicators in the six levels. Whereas, Student 3 could solve four problems correctly (finding the surface area of three dimensional figure, finding the area of triangle, finding the surface area of pyramid, finding the volume of triangular prism). This student could reach level 1 up to level 6 with 90% among indicators in the six levels.

Based on that findings, it can be concluded that the three students experienced difficulties to reach three to five indicators on level four to six of PISA. Those indicators are related to capability of advanced mathematical thinking and reasoning; applying insight and understandings along with a mastery of symbolic and formal mathematical operations and relationships to develop new approaches and strategies for attacking novel situations; formulating and precisely communicating actions and reflections regarding their findings, interpretations, arguments and the appropriateness of these to the original situations [3].

These results are in accordance with the findings of [11] that there are no students at level 4 and 5. In solving mathematical problems, most students are still at level 1 and 2. Musa [12] also states that female subjects with high geometrical ability are located at level 2. The level can be said as pre sequencing stage where students do not understand the relationship of figures in the given problems. Moreover, they are not able to construct definitions that correspond to the given problems. The problem used in the study is similar with problem about identifying geometric figures applied in this study, where all three subjects unable to answer. Furthermore, a study in Turkey found that Turkish students are also stand in the 2$^{nd}$ level of mathematics literacy based on PISA levels [13].

The factors causing students' errors in solving geometric problems are understanding of geometry is still lacking, there is no preparation for the test, misunderstanding what is the problems mean, and forgetting formula. This is in accordance with [14] who stated that the factors that cause students' mistakes in solving geometry problems (surface area and pyramid volume) are (1) less thorough solving problems, (2) less skilled in solving problems, (3) lack of understanding of the concept of surface area and pyramid volume, (4) lack of prerequisite knowledge. The study of [15] emphasized that for the topic of geometry, students experience reading difficulties at about 34.1%, understanding difficulties are around 35.1%, transformation difficulties were 51.5%, process skills difficulties were 70.1%, and difficulties in drawing 70.1%. Whereas, [16] also revealed that the factors that cause student difficulties in general are students' tendency to memorize formulas without understanding concepts, misunderstanding what is the problems mean and lack of practice working on math problems.

Most students experience difficulties in solving geometric problems because they do not know the concept applied in the problems. This was also expressed by [17] which concluded that the causes of errors made by students in solving geometry problems because students made misconceptions, made operating errors, and made errors in analysis and were unable to recall concepts or operations related to geometrical material that has been studied before. Whereas [18] stated that based on the results of research on the factors causing low mastery of material, it was reached 60%. Misconceptions and lack of understanding of the material were also expressed by [19] in the results of their research obtained the percentage of concept errors 67.77%, procedural errors 17.27% and calculation errors of 13.95%. The results showed that students did more misconceptions, the influencing factors were students who did not understand the material contained in the problem.

Many students find it difficult to visualize or to define geometric concepts. Therefore, it causes many conceptual errors due to lack of students' understanding of geometric problems. Including students' geometry skills in solving geometry problems, students level on visual skills, can only determine the type of quadrilateral based on the appearance of its shape [20]. Whereas, according to

[21] the lack of understanding of geometrical concepts in schools was suspected because the approach to geometry learning did not consider the level of student development and geometry learning materials did not match the level of students' thinking or the construction of learning materials was not in accordance with formal geometrical construction. Consequently, geometric learning should consider these following factors [22]: (1) for mathematics learning required the use of geometry props; (2) the use of geometry teaching aids increases teaching-learning interaction; (3) the constraints of the time, material, and students' ability; (4) recommendations: the use of geometry props is expected to be applied in schools for learning mathematics.

## 4. Conclusion and Suggestion

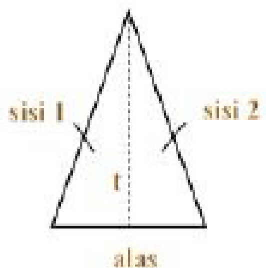
Most students' are categorizing in medium level of thinking (about 72%), less students are categorizing in high level of thinking (20%), and least students are categorizing in high level of thinking (8%). Based on the analysis of students' written works and the interview, Student 1 could only solve one problem correctly among the five problems given. He could only reach level 4 of PISA which is constructing and communicating explanations and arguments based on their interpretations, arguments and actions. Student 2 could solve three problems correctly (finding the surface area of three dimensional figure, finding the surface area of pyramid, finding the volume of triangular prism). This student could reach level 1 up to level 6 with 60% among indicators in the six levels. Whereas, Student 3 could solve four problems correctly (finding the surface area of three dimensional figure, finding the area of triangle, finding the surface area of pyramid, finding the volume of triangular prism). This student could reach level 1 up to level 6 with 90% among indicators in the six levels.

PISA level 1 and 2 are levels which are easily achieved by students. Whereas, PISA level 3-6 are levels which are difficultly achieved by students. However, they could achieve the levels within stimulations given by teachers. Furthermore, the results of this study implies the importance of designing mathematics learning to increase students' levels of thinking in solving mathematics problems, particularly geometric problems.

## 5. Appendix

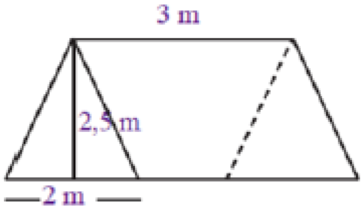
This following are the geometric problems used for this study:

- Problem 1
  Volume sebuah kubus sama dengan volume balok yaitu 1.000 $cm^3$. Diketahui panjang balok dua kali panjang kubus dan tinggi balok setengah kali lebar balok. Tentukan luas seluruh permukaan balok.
  Translation:
  The volume of a cube is 1.000 $cm^3$. The volume is exactly the same as the volume of a rectangular prism. Given the length of the rectangular prism is two times of the length of the cube and the height of the rectangular prism is a half of the width of the rectangular prism. Find the surface area of the rectangular prism.
- Problem 2
  Sebuah segitiga sama kaki mempunyai keliling 98 cm, jika panjang alasnya 24 cm, hitung luas segitiga tersebut.




  Translation:
  Given an isosceles triangle which the perimeter is 98 cm. If the base is 24 cm, evaluate the area of the triangle.

- Problem 3
  Alas sebuah limas segi empat beraturan berbentuk persegi. Jika tinggi segitiga 17 cm dan tinggi limas 15 cm, tentukan luas permukaan limas.
  Translation:
  The base of a 4-gon pyramid is a square. If the height of the triangles is 17 cm and the height of the pyramid is 15 cm, find the surface area of the pyramid.
- Problem 4
  Perhatikan gambar tenda di bawah berikut.
  Sebuah tenda memiliki ukuran seperti pada gambar di atas, tentukan volume tenda tersebut.




  Translation:
  Consider the following figure.
  Given a tent as shown on that figure. Find the volume of the tent.
- Problem 5
  Perhatikan gambar berikut dan selesaikan


  Translation:
  Consider the following figure and answer the questions.
  a. What can you say about the figure?
  b. What kinds of geometric shapes dominant on the figure?
  c. Why?

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