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# Modify Variational Iteration Method for Solving Heat Equations

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**Abstract.** The Variational Iteration method (VIM) is one of the semi approximation procedure to solve numerous differential problems. Many literatures shows that this method need to be rectify to get preferable results. This research would like to modify VIM on the replacement the initial value in each steps. Although this method enact the algorithm more complex, but modify VIM gives good outcomes peculiarly in long time intervals. We compare the solutions with the analytical solution using separation variable. The results denotes that both of the VIM and modify VIM have small error in solving heat equation in Dirichlet boundary. It could be seen that the modify VIM give better solutions instead of VIM.

## 1. Introduction

The Heat equations is one of the attractive equation because of the utility in many substantial areas. The heat transfer in liquid, gas and dense media are the example. The research in this topic perennially extend presently. Plentiful works of research extend a new approach for solving heat equations in amount fields. Finite volume was used to solve heat equations, [1] used it for solving in homogenous metal rod problem while [2] in thermal radiation in participating media. Another method is Variational Iteration method (VIM). VIM is one of the semi approximation procedure to solve many differential problems. There is numerous researches related VIM. VIM for solving a system of ordinary differential equations and partial differential equations was done by [3]. It is also powerful for solving the fuzzy differential equations [4]. Research related VIM for solving ode was conducted by [5]. While VIM for solving the model of physical problem in partial differential equations was done by [6,7, 8,9]. VIM for special transform was used by [10]. The stability of VIM was analysed by [ 11]. [ 12, 13] employ VIM for accomplishing fractional equations. Many literatures shows that this method need to be rectify to get preferable results.

This paper aims to provide calculations in new ways to solving heat equations using the VIM modification. The heat equation with Dirichlet boundary is a form of heat propagation with a fire source in the centre of the mass with heat at the first and the end of the boundary maintained zero. The heat equation with the Dirichlet boundary according to the equation below.

$$\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}, 0 < x < l, \quad t > 0 \quad (1)$$

With the initial value



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$$u(x, 0) = x(l - x), 0 < x < l \quad (2)$$

The boundary

$$u(0, t) = u(l, t) = 0, t > 0 \quad (3)$$

## 2. Variational Iteration Method (VIM)

Given the general nonlinear problem as follow

$$L[u(t)] + N[u(t)] = g(t) \quad (4)$$

$L$  is a linear operator,  $N$  is a non linear operator, and  $g(t)$  is a continuous function. The iterative function with the variational iteration method (VIM) is built on the equation below

$$u_{n+1}(x, t) = u_n(x, t) + \int_0^t \lambda_i(s) \{Lu_n(s) + N\tilde{u}_n(s) - g(s)\} ds, n \geq 0 \quad (5)$$

Here  $\lambda$  is a Lagrange multiplier. Contrivance the solutions of the heat equations in Dirichlet boundary using VIM consists of the following steps.

Step 1: Determine how many iterations

Step 2: Create correction with Lagrange multiplier

Step 3: Set initial value condition and verify the boundary value

Step 4: Transform the ordinary equation system with the variational iteration

Step 5: Solve the integral form

Step 6: Check the solutions

Step 7: Repeat from step 3 to step 6 in each iteration

Step 8: Continue until the desired iteration

We would like to discuss the application of the both VIM and modify VIM for solving heat equation. In this paper, based on [3]. We use Lagrange multiplier  $\lambda = -1$ . The following is explained by the process for finding lagrange multiplier.

$$u_{n+1}(x, t) = u_n(x, t) + \int_0^t \lambda(\xi) \left( \frac{\partial u_n}{\partial \xi} - \alpha^2 \frac{\partial^2 u_n}{\partial x^2} \right) d\xi \quad (6)$$

Suppose that  $\delta$  is limited variational.

$$\delta u_{n+1}(x, t) = \delta u_n(x, t) + \delta \int_0^t \lambda(\xi) u'_n(x, \xi) d\xi \quad (7)$$

$$\delta u_{n+1}(x, t) = (1 + \lambda(t)) \delta u_n(x, t) - \int_0^t \lambda'(\xi) \delta u_n(x, \xi) d\xi \quad (8)$$

The equation 8 will be stationary if  $\lambda = -1$ .

The application of VIM to obtain a heat equation solution is given by iterative function

$$u_{n+1}(x, t) = u_n(x, t) - \int_0^t \left( \frac{\partial u_n}{\partial \xi} - \alpha^2 \frac{\partial^2 u_n}{\partial x^2} \right) d\xi \quad (9)$$

With the initial value  $u_0(x, t) = \ell x - x^2$

$$\frac{\partial u_0}{\partial x} = \ell - 2x \quad (10)$$

$$\frac{\partial^2 u_0}{\partial x^2} = -2 \quad (11)$$

$$u_0(\xi, t) = \ell\xi - \xi^2 \quad (12)$$

$$\frac{\partial u_0}{\partial \xi} = \ell - 2\xi \quad (13)$$

For  $n = 0$  we obtain

$$u_1(x, t) = u_0(x, t) - \int_0^t \left( \frac{\partial u_0}{\partial \xi} - \alpha^2 \frac{\partial^2 u_0}{\partial x^2} \right) d\xi \quad (14)$$

$$u_1(x, t) = (\ell x - x^2) - \int_0^t ((\ell - 2\xi) - \alpha^2(-2)) d\xi \quad (15)$$

$$u_1(x, t) = \ell x - x^2 - \ell t + t^2 - 2\alpha^2 t \quad (16)$$

$$\frac{\partial u_1}{\partial x} = \ell - 2x \quad (17)$$

$$\frac{\partial^2 u_1}{\partial x^2} = -2 \quad (18)$$

$$u_1(\xi, t) = \ell\xi - \xi^2 - \ell t + t^2 - 2\alpha^2 t \quad (19)$$

$$\frac{\partial u_1}{\partial \xi} = \ell - 2\xi \quad (20)$$

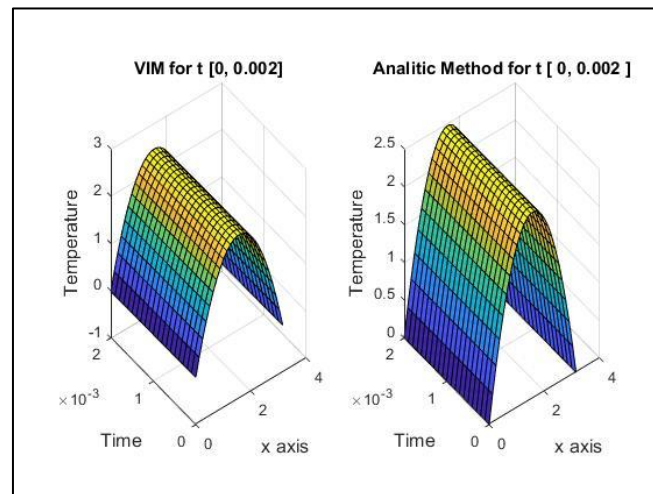
For  $n = 1$  we get

$$u_2(x, t) = (\ell x - x^2 - \ell t + t^2 - 2\alpha^2 t) - \int_0^t (\ell - 2\xi + 2\alpha^2) d\xi \quad (21)$$

$$u_2(x, t) = \ell x - x^2 - 2\ell t + 2t^2 - 4\alpha^2 t \quad (22)$$

In this paper, we discuss the VIM solution until 2 iterations in a closed interval  $[0, 0.002]$ . The simulation of the solution using VIM with 2 iterations is in figure 1. The analytical solutions using separation variable is

$$u(x, t) = \frac{8l^2}{\pi^3} \sum_{m=1}^{\infty} \left( \frac{1}{(2m-1)^3} \right) \sin\left(\frac{(2m-1)\pi x}{l}\right) e^{-\frac{(2m-1)^2 \pi^2 \alpha^2}{l^2} t} \quad (23)$$



**Figure 1.** VIM solutions in interval [0,0.002]

Figure 1 describes that the solutions of Vim method which is compared with the analytical method look similar. The peak using VIM is under 3, it is around 2.5 same as with the analytical solutions. The heat transform spread with the maximum heat is in the center of the rod bars. It in accordance with the source. At short intervals, VIM can describe the solution behavior well. To apply VIM, it is best to use a lot of iterations to get a decent result.

### 3. The Modify Variational Iteration Method

There are some differences between the vim and modify VIM. In the modify vim the interval is not too long. After we get the solutions in the first interval and then update the initial value and govern new formula. Finding the solutions of the heat equations in Dirichlet boundary using modify VIM consists of the following steps.

Step 1: Specify the interval to be resolved

Step 2: Divide intervals into small intervals

Step 3: Do the following steps at each interval, do the following steps

Step 4: Make correction with Lagrange multiplier

Step 5: Set initial value condition and verify the boundary value

Step 6: Determine how many iterations

Step 7: Transform the ordinary equation system with the variational iteration

Step 8: Solve the integral form

Step 9: Check the solutions

Step 10: Repeat from step 4 to step 6 in each iteration

Step 11: Continue until the desired iteration

Step 12: Check the solution in each interval then update the initial value with the last solution and then continue to step 5.

In this paper, modify VIM solution in the first interval same with the VIM. The first interval is about [0, 0.001]. To solve the next interval we update the value in the 0.0001 with the solutions of VIM. We define the initial value in [0.001,0.002] with the

$$u_0(x, t) = \ell x - x^2 - 2\ell t + 2t^2 - 4\alpha^2 t \quad (24)$$

$$u_1(x, t) = u_0(x, t) - \int_0^t \left( \frac{\partial u_1}{\partial \xi} - \alpha^2 \frac{\partial^2 u_1}{\partial x^2} \right) d\xi \quad (25)$$

$$u_1(x, t) = (\ell x - x^2 - 2\ell t + 2t^2 - 4\alpha^2 t) - \int_0^t (\ell - 2\xi + 2\alpha^2) d\xi \quad (26)$$

$$u_1(x, t) = \ell x - x^2 - 3\ell t + 3t^2 - 6\alpha^2 t \quad (27)$$

The modify VIM solution during the interval  $[0, 0.002]$  would be simulated in figure 2.

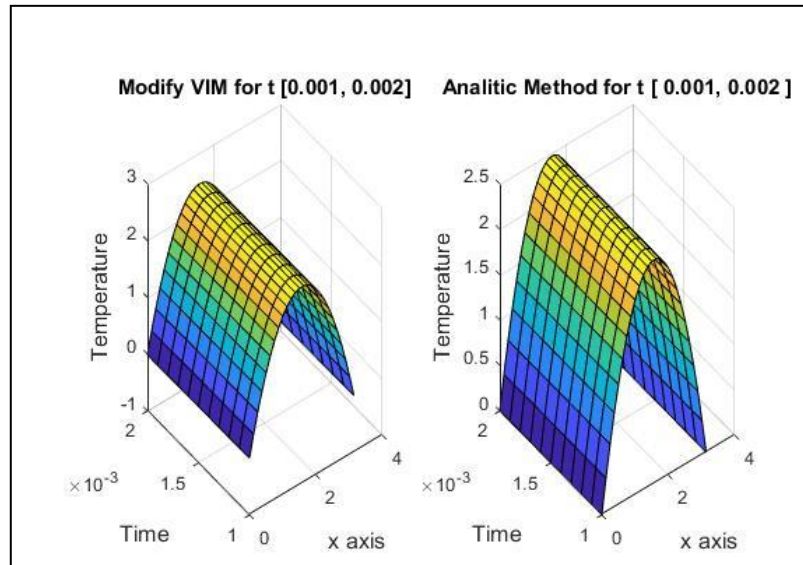


Figure 2. Modify VIM Solution

Figure 2 describes that the solutions of modify VIM method which is compared with the analytical method look similar. The peak using modify VIM is under 3, it is around 2.5 same as with the analytical solutions. The heat transform spread with the maximum temperature is in the center of the rod bars. In this paper we would like to give emphasis that in implementing VIM we need to update the initial value at each interval. In this way the completion of using VIM can be convergent for a long period of time. The example in this paper illustrates that at long intervals using two iterations gives worse results compared to small intervals by using one iteration. In the second interval, the VIM solution and the modify VIM are different. Although in the VIM solutions were build using 2 iteration, while the modify VIM solutions using 1 iteration, the modify VIM solutions give preferable accomplishment than VIM. Modify VIM would rectify the solution pretty well than VIM. VIM modification makes the algorithm complex but gives better results. The difficulty that can arise from VIM modification is that it is not always easy to complete the integral form from the results of updating the initial value, especially when using high iterations. Compared with the VIM, the modify vim give better results. Error analysis is done by calculating the average difference between analytic completion and both methods namely VIM and Modify VIM. The average error of modify VIM is 0.004804 while the VIM is 0.008435.

#### 4. Conclusion

We applied Modify VIM for solving heat equation in Dirichlet Boundary by using another way. Results show that both of VIM and modify VIM presents a powerful and simple calculations. Results clearly show that unlike analytical methods using separation variables, our presented method does not require too long operations. It is obviously indicated that heat equations with Dirichlet boundary can be computed by modify VIM well.

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