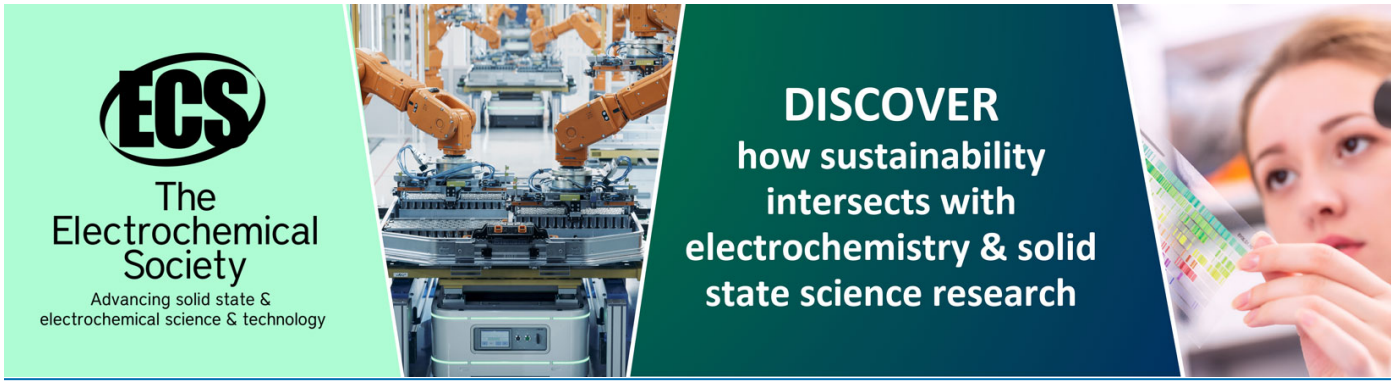
Analysis of torch deployment models



Analysis of torch deployment models

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Abstract. Toxoplasmosis, Other Agents, Rubella, Cytomegalovirus and Herpes Simplex stands for TORCH infection. It is highly contagious and attacks humans of all ages and genders. Transmission of this infection in 2 ways: actively (acquired) and passively (congenital). Active transmission through sexual intercourse, eating oosista-containing foods, blood transfusions, breast milk, organ transplants, saliva, and sweat. Passive transmission occurs from an infected mother to her baby through the placenta. The fatal danger caused by this infection is miscarriage or babies born handicapped and infertile for men and women. This research discusses the formation of mathematical models of TORCH infections transmission and then analyzes the model. The method used is the study of literature. The results of the research obtained are mathematical models of torch transmission in the form of differential equation systems and the results of the model analysis are obtained 2 equilibrium points with their stability properties.

1. Introduction

TORCH is an infection caused by parasites and viruses. TORCH stands for Toxoplasmosis, Other Agents, Rubella, Cytomegalovirus, and Herpes Simplex. Toxoplasmosis is an infectious disease caused by the Toxoplasma gondii parasite while Other Agents, Rubella, Cytomegalovirus, and Herpes Simplex are an infectious disease caused by virus [7].

In general, transmission of TORCH in 2 ways: actively (acquired) and passively (congenital). Active transmission can be caused by several things including sexual intercourse, eat raw or undercooked foods that contain oocysts, breast milk, saliva, blood transfusions, organ transplants, and sweat. Passive transmission occurs from an infected mother to her baby through the placenta [5].

TORCH infections are very contagious, attacking various ages and genders. Therefore, if there is a family infected with a TORCH, the others will be infected. The impact of infections: cause fertility problems in both women and men, which make pregnancy difficult. In pregnant women can result in miscarriage or birth defects. Some fetal defects caused by this infection: brain damage, inflammation, seizures, too much fluid in the brain (hydrocephalus).

The government has made an effort to overcome the spread of this infection. The effort made is to provide vaccinations and medicine for sufferers. The type of medicine that is believed to be medically capable of curing the TORCH isisoprinocin, repomicine, valtrex, spiromicine, spiradan, and aciclovir. However, the drug has not been able to eliminate the virus to a negative level. Not only that, treatment takes a long time and is carried out continuously.

Based on the problem, a mathematical model of TORCH transmission is formed by analyzing theories that are relevant to the problem based on literature review [1]-[10]. Furthermore, the model is analyzed to see the dynamics of TORCH infection transmission.

2. Discussion

2.1. Formulating Epidemiology Model

The population is divided into 3 subpopulations. They are subpopulations vulnerable to TORCH infection (S), subpopulations infected with TORCH (I), and subpopulations recovering from a TORCH infection (R). Model assumption: every individual born from the S and R subpopulation is susceptible to contracting a TORCH infection, every individual born from subpopulation I was immediately infected with a TORCH, TORCH is transmitted through: direct contact between S and I subpopulations, and from pregnant women to children in their womb, and individuals in the R subpopulation can be re-infected with TORCH if their immune system is weakened. Parameter of the model: level of transmission (β), level of birth (α), level of death (μ), level of recovery (γ_1), and the level of displacement from the R to I subpopulation (γ_2).

Based on the problem, we can make a flow diagram of model:

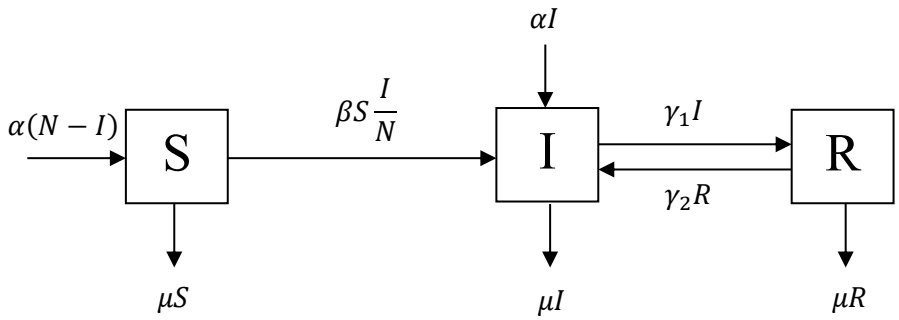

Figure 1. Flow diagram of model

and the formula is system (1)

$$\frac{dS}{dt} = \alpha(N-I) - S\left(\beta\frac{I}{N} + \mu\right),$$
$$\frac{dI}{dt} = \beta S\frac{I}{N} + (\alpha - \mu - \gamma_1)I + \gamma_2 R, \quad (1)$$
$$\frac{dR}{dt} = \gamma_1 I - (\mu + \gamma_2)R,$$

Where $N = S + I + R$ and $\frac{dN}{dt} = (\alpha - \mu)N$. N constant if and only if $\alpha = \mu$

2.1.1. Fixed Point of Model. Based on definition of fixed point, system (1) to be

$$f := \alpha(N-I) - S\left(\beta\frac{I}{N} + \mu\right) = 0, \quad (2)$$

$$g := \beta S\frac{I}{N} + (\alpha - \mu - \gamma_1)I + \gamma_2 R = 0, \quad (3)$$

$$h := \gamma_1 I - (\mu + \gamma_2)R = 0. \quad (4)$$

From equation (2)-(4), we find out 2 kinds fixed points: none-endemic and endemic fixed points. None-endemic fixed point is

$$E_0 = (S, I, R) = \left(\frac{\alpha N}{\mu}, 0, 0\right)$$

and endemic fixed point is

$$E_1 = (S, I, R) = \left(\frac{\alpha N(N-I)}{\beta I + \mu N}, I, \frac{\gamma_1}{\mu + \gamma_2}I\right)$$

where

$$I = \frac{N\left[(\mu+\gamma_2)\left(\alpha(\beta+\mu)-\mu(\mu+\gamma_1)\right)+\gamma_1\gamma_2\mu\right]}{\beta\mu(\mu+\gamma_1+\gamma_2)}$$

2.1.2. Stability Fixed Point of Model. Fixed point stability is used to study system dynamics. Furthermore System (1) is converted into a linear differential equations system using the Jacobi matrix and obtained:

$$J = \begin{bmatrix} \frac{\partial f}{\partial S} & \frac{\partial f}{\partial I} & \frac{\partial f}{\partial R} \\ \frac{\partial g}{\partial S} & \frac{\partial g}{\partial I} & \frac{\partial g}{\partial R} \\ \frac{\partial h}{\partial S} & \frac{\partial h}{\partial I} & \frac{\partial h}{\partial R} \end{bmatrix} = \begin{bmatrix} -\left(\beta\frac{I}{N}+\mu\right) & -\left(\alpha+\beta\frac{S}{N}\right) & 0 \\ \beta\frac{I}{N} & \beta\frac{S}{N}+(\alpha-\mu-\gamma_1) & \gamma_2 \\ 0 & \gamma_1 & -(\gamma_2+\mu) \end{bmatrix}$$

Fixed point stability of the model is as follows:

2.2. Stability of none-endemic fixed point

The stability of none-endemic fixed point can be seen from the Eigen value of Jacobi matrix around the E_0 point, as follows:

$$\left|\lambda I - J\left(f(E_0)\right)\right| = 0$$

$$\begin{vmatrix} \lambda+\mu & \alpha+\frac{\alpha\beta}{\mu} & 0 \\ 0 & \lambda-\left[\frac{\alpha\beta}{\mu}+(\alpha-\mu-\gamma_1)\right] & -\gamma_2 \\ 0 & -\gamma_1 & \lambda+(\gamma_2+\mu) \end{vmatrix} = 0$$

Let $a = \gamma_2+\mu$, $b = \frac{\alpha\beta}{\mu}+(\alpha-\mu-\gamma_1)$, and we get

$$(\lambda+\mu)\left(\lambda^2+(a-b)\lambda+(-ab-\gamma_1\gamma_2)\right) = 0$$

$$\lambda_1 = -\mu \text{ or } \left(\lambda^2+(a-b)\lambda+(-ab-\gamma_1\gamma_2)\right) = 0$$

Consider $\lambda^2+(a-b)\lambda+(-ab-\gamma_1\gamma_2) = 0$

Let $a_1 = a-b$ and $a_2 = -ab-\gamma_1\gamma_2$, obtained

$$\lambda^2+a_1\lambda+a_2 = 0 \quad (5)$$

According Routh-Hurwitz criteria, Eigen value of equation (5) is negative if and only if

$$a_1 > 0 \text{ and } a_1a_2 > 0$$

$$a_1 > 0 \Leftrightarrow (\gamma_2+\mu)-\left[\frac{\alpha\beta}{\mu}+(\alpha-\mu-\gamma_1)\right] > 0$$

$$\Leftrightarrow \frac{\mu(2\mu+\gamma_1+\gamma_2)}{\alpha(\beta+\mu)} > 1$$

Because $a_1 > 0$ then $a_1a_2 > 0$will be satisfied if $a_2 > 0$.

$$a_2 > 0 \Leftrightarrow -(\gamma_2 + \mu)\left[\frac{\alpha\beta}{\mu} + (\alpha - \mu - \gamma_1)\right] - \gamma_1\gamma_2 > 0$$

$$\Leftrightarrow \frac{\mu^2(\gamma_1 + \gamma_2 + \mu)}{\alpha(\gamma_2 + \mu)(\beta + \mu)} > 1$$

In other words, the stable point of disease free is stable with all the eigenvalues negative. So, for a long time, the disease does not develop in the population.

2.3. Stability of endemic fixed point

The stability of endemic fixed point also can be seen from the Eigen value of Jacobi matrix around the E_1 point, as follows:

$$\left|\lambda I - J\left(f(E_1)\right)\right| = 0$$

$$\Leftrightarrow \begin{vmatrix} \lambda + \left(\beta\frac{I}{N} + \mu\right) & \left(\alpha + \frac{\alpha\beta(N-I)}{\beta I + \mu N}\right) & 0 \\ -\beta\frac{I}{N} & \lambda - \left(\frac{\alpha\beta(N-I)}{\beta I + \mu N} + (\alpha - \mu - \gamma_1)\right) & -\gamma_2 \\ 0 & -\gamma_1 & \lambda + (\gamma_2 + \mu) \end{vmatrix} = 0$$

Let $a = \gamma_2 + \mu$, $c = \beta\frac{I}{N}$, and $d = \frac{\alpha\beta(N-I)}{\beta I + \mu N}$

$$\lambda^3 + b_1\lambda^2 + b_2\lambda + b_3 = 0$$

where

$b_1 = a + 2\mu + \gamma_1 + c - d - \alpha,$

$b_2 = -ad - a(\alpha - \mu - \gamma_1) - \gamma_1\gamma_2 + (c + \mu)(a + \mu + \gamma_1 - d - \alpha) + c(\alpha + d),$

$b_3 = (c + \mu)(-ad - a(\alpha - \mu - \gamma_1) - \gamma_1\gamma_2) + ac(\alpha + d).$

According Routh-Hurwitz criteria, Eigen value of equation is negative if and only if $b_1 > 0$ and $b_1 b_2 - b_0 b_3 > 0$

Consider,

$b_1 = a + 2\mu + \gamma_1 + c - d - \alpha,$

$$b_1 = \frac{(\beta I + \mu N)[(3\mu + \gamma_1 + \gamma_2)N + \beta I] - \alpha N^2(\mu + \beta)}{N(\beta I + \mu N)}. \quad (6)$$

$b_2 = -a(d + \alpha - \mu - \gamma_1) - \gamma_1\gamma_2 + (c + \mu)(a + \mu + \gamma_1 - d - \alpha) + c(\alpha + d),$

$$b_2 = \{(\beta I + \mu N)[(\gamma_2 + 2\mu)(\mu N + \gamma_1 N + \beta I) + \mu N(\gamma_2 + \mu) + \gamma_1\beta I] - (\gamma_2 + 2\mu)\alpha N^2(\mu + \beta) - \gamma_1\gamma_2(\beta I + \mu N)\}/N(\beta I + \mu N). \quad (7)$$

$b_3 = (c + \mu)(-ad - a(\alpha - \mu - \gamma_1) - \gamma_1\gamma_2) + ac(\alpha + d),$

$$b_3 = \frac{\{(\gamma_1 + \mu)(\gamma_2 + \mu)(\beta I + \mu N)^2 - [\gamma_1\gamma_2(\beta I + \mu N)^2 + \mu(\gamma_2 + \mu)\alpha N^2(\mu + \beta)]\}}{N(\beta I + \mu N)} \quad (8)$$

After that, from equation (6), (7), and (8) obtained

$$b_1 > 0 \text{ if and only if } \frac{(\beta I + \mu N)[(3\mu + \gamma_1 + \gamma_2)N + \beta I] - \alpha N^2(\mu + \beta)}{N(\beta I + \mu N)} > 0$$

$$\frac{[(3\mu + \gamma_1 + \gamma_2)N + \beta I](\beta I + \mu N)}{N\alpha N(\mu + \beta)} > 1$$

And $b_1 b_2 - b_3 > 0$ if and only if $b_1 b_2 > b_3$ or $\frac{b_1 b_2}{b_3} > 1$. This means that, in this condition for a long time there will always be infected individuals.

3. Conclusion

The research obtained a mathematical model formula for TORCH transmission. The model is a nonlinear differential equations system. The results of the model analysis obtained two kinds fixed points. It is none-endemic fixed point and endemic fixed point. Point of E_0 stable if $a_1 > 0$ and $a_2 > 0$. Furthermore, E_1 point stable if $b_1 > 0$ and $b_1 b_2 > b_3$

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