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Constructive heuristic for the mixed capacitated arc routing problem with multi capacity

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Abstract. This paper addresses a variant of the capacitated arc routing problems known as the mixed capacitated arc routing problem (MCARP) with multi capacity constraints. The MCARP represents closely real life situations since it takes into account directed and undirected links to serve customers in the associated network. A constructive heuristic based on insertion techniques is proposed to tackle the problem. Computational experiments were conducted using some randomly generated instances and the experiments show competitive results. Some avenues for future research are also highlighted.

1. Introduction

Capacitated arc routing problems (CARP), firstly introduced by [1], are known to be np-hard combinatorial optimization problem where there is a set of customers and their corresponding demand links have to be served by a fleet of vehicles with limited capacity. The objective is to minimize the sum of the travel cost (or distance). In practice, the applications of CARP emerge in a variety of public services such as waste collection, winter gritting and street swiping. For details on these applications, the reader can refer to [2, 3, 4, 5, 6, 7, 8 and 9].

Waste collection can be considered as a logistical activity where municipalities must collect wastes at residential houses in the most cost-effective way. This is usually done by dividing the municipality's service area into collection quarters and then assigning a vehicle to each quarter. However, the astonishing rate at which the countries' urban areas are expanding and the increasing fuel price may lead to solutions involving service cost and capacity constraints. Therefore, the management for waste collection should find the way to improve the waste collection system. However, the absence of adequate planning has led to a serious wastage of expenditure and effort in this problem.

Our research has demonstrated an increasing awareness over the poor quality of service provided in most urban areas, in terms of the quantity of solid wastes collected and the environmental protection provided, making it difficult to justify even by the present levels of expenditure to address these problems. The objective of an efficient service should, therefore, be the minimization of waste collection costs, together with the provision of an adequate and regular service to all of the target areas. Providing an efficient collection service to a city often requires a combination of techniques and equipment, to accommodate the different challenges of the various neighbourhoods within the city. Thus, we design a model of collection systems based on the real life situations and establish rational planning and assessment at this stage. These are crucial for the overall system's efficiency and good performance. It aims to create operational features which are essential to ensure efficiency in waste collection systems.

One of the collection systems that we focus on is based on the Capacitated Arc Routing Problems, in which the pickup or delivery activities occur at links between nodes of a network graph. The aim of CARP is to produce an optimum-routing cost for a fleet of vehicles without violating all constraints. The existing mathematical techniques of CARP are rudimentary and messy that could not support most asymmetric case. Heuristic approaches (i.e.: Construct-Strike algorithm, the Path-Scanning method, and the Augment-Merge algorithm) are the popular methods to solve CARP to find near optimal solutions within reasonable times. The applications of the classical CARP are limited to either undirected or directed graph. In the case of undirected graph, customers can be served in any directions whereas in the case of directed graph, customers have to be served according to the defined one way direction. However, many real life conditions need two-way roads where both sides have to be served simultaneously and in any direction. These variations of the classical CARP have established a new idea of CARP with mixed network known as the mixed capacitated arc routing problems (MCARP). Hence, in this paper, we propose a constructive heuristic to tackle MCARP by taking into account multi capacity constraints.

The remainder of the paper is organized as follows. The next section presents a review of the relevant literatures. The section thereafter introduces the definition and formulation of MCARP. This is then followed by a section on solution methods and computational results. The last section concludes some remarks and highlights some future research avenues.

2. Literature Review

During the last two decades, capacitated arc routing problems have flourished as an active research area in logistic problems. CARP was officially formulated by [1] and ever since numerous optimization and heuristics-type algorithms have been put forward to tackle CARP. The classical arc routing problems are generalizations from routing problems, i.e., the Chinese Postman Problem (CPP) and the Rural Postman Problem (RPP) (see [4, 5]). The CPP commonly relates to mail delivery in urban settings that seek the least cost traversal of all the streets. While, the RPP corresponds to rural settings and can be defined as: The set of streets in some villages/towns which has to be serviced by a postman, and a set of links between the villages/towns that do not have to be served, but may be used for travelling purposes between villages/towns. A study by [5] highlighted that it seeks the least cost traversal of the subset of the required streets. Hence, the RPP, in general, can model real life arc routing applications that can be accurately implemented to the MCARP.

The MCARP also consists of various streets that may not have to be served as they have the task and will just be used to travel to streets that require services. The MCARP also involves additional characteristics such as capacity constraints and multiple vehicles, therefore for this reasons it cannot be modeled as a pure RPP but rather be categorized as an arc routing problem.

This section does not provide a review at the CARP in particular, but the reader may refer to [10] and [11] for details on CARP and this section also include the description about RPP and MCARP. To the best of our knowledge, it was [12] as the pioneer to formulate MCARP. The authors proposed three constructive heuristics based on path–scanning, augment-merge, and Ulusoy’s heuristic [13] and a memetic algorithm was also developed to tackle the problem. Lower and upper bounds were also obtained using two sets of randomly generated instances. A study by [14] put forward heuristic algorithm to solve the MCARP, inspired by the household waste collection problem in Lisbon, Portugal. Research by [9] also studied MCARP based on urban waste collection problem in the municipality of Barcelona where their research resembled the real life application problem involving traffic regulations.

The constructive heuristics and ant colony heuristic algorithm were designed to deal with the problem and their experiments showed better results compared to the previous results found by the municipality. Two compact flow based models for the MCARP was developed by [15]. The idea of compact models were to guarantee the connectivity of the solution and the matching between trips and vehicles. The aggregation version of the original MCARP were also constructed to simplify the problem.

3. Optimization Model for the Mixed Graph CARP

The MCARP is on a mixed graph $\Gamma = (N, E \cup A)$ with a set *N* of *n* nodes. Edges, in *E*, characterize the narrow two way streets that may be served by only one traversal (i.e., zigzag services). Arcs, in *A*, represent either one way or large two way streets that must be served in both directions. A homogenous vehicle fleet is based at a *depot* node, $0 \in N$. Here, two types of links in $E \cup A$ are distinguished: *demand links* or *tasks* and *deadheading links*. Then, it is assumed that any vehicle will use exactly one arc leaving or one arc entering. In our paper, we follow [15] in formulating the MCARP. The following notations are used throughout the formulations.

$\Gamma = (N, E \cup A)$ is the mixed graph, with $A_R \subseteq A$ and $E_R \subseteq E$ the set of required arcs and edges, respectively; and *N* the set of nodes, representing street crossings, or the depot.

$0 \in N$ is the depot node where every vehicle trip must start and end.

$R \subseteq E \cup A$ is the set of required edges or arcs in Γ.

K is the maximum number of trips with, $k \subseteq K$ is the set of required number of trips allowed.

W is the capacity of each vehicle.

c_{ij} is the service cost and deadheading cost of the edge or arc $(i, j) \in R$.

d_{ij} is the distance and deadheading distance of the edge or arc $(i, j) \in R$.

q_{ij} is the demand of edge or arc $(i, j) \in R$.

The objective, basically, is to find a set of vehicle trips to serve each demand link without violating the vehicles' capacity which every trip starts and ends at the depot. Initially, the problem is characterized by finding the feasible region for mixed graph of capacitated arc routing problems with a compact model. Then, the flow variables are used to fit with undirected, directed and mixed graph problem cases. If a feasible solution satisfies the capacity constraint, it means that it serves all the tasks without the dump cost (paid every time a vehicle is emptied at the depot). As in [15], we defined the following variables:

$$x_{ij}^k = \begin{cases} 1 & \text{if } (i,j) \in R \text{ is served by trip } p \\ 0 & \text{otherwise} \end{cases} \qquad \forall (i,j) \in R;$$

y_{ij}^k is the number of deadheaded.

f_{ij}^k is the flow traversing edge or arc $(i, j) \in E \cup A$, it is related to the remaining demand in the tour, or in a sub-tour of it.

This section presents a mathematical model of the problem which is proposed by [15]:

$$\text{Minimize} \quad \sum_{k=1}^{K} \left[\sum_{(i,j)\in R} c_{ij} x_{ij}^k + \sum_{(i,j)\in E\cup A} d_{ij} y_{ij}^k \right] \tag{1}$$

Subject to:

$$\sum_{(i,j)\in E\cup A} y_{ij}^k + \sum_{(i,j)\in R} x_{ij}^k = \sum_{(j,i)\in E\cup A} y_{ji}^k + \sum_{(j,i)\in R} x_{ji}^k \quad ;\forall (i,j) \in R;\ k = 1,\dots,K \tag{2}$$

$$\sum_{k=1}^{K} x_{ij}^k = 1; \quad \forall (i,j) \in A_R \tag{3}$$

$$\sum_{k=1}^{K} \left(x_{ij}^k + x_{ji}^k \right) = 1; \ \forall (i,j) \in E_R \tag{4}$$

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$$\sum_{(j,i)\in E\cup A} f_{ij}^{k} - \sum_{(i,j)\in E\cup A} f_{ij}^{k} = \sum_{(j,i)\in R} q_{ij} x_{ij}^{k} \quad ; \; i=1,\dots,n; \; k=1,\dots,K \tag{5}$$

$$\sum_{(0,j)\in E\cup A} f_{0j}^{k} = \sum_{(i,j)\in R} q_{ij} x_{ij}^{k} \quad ; \; k=1,\dots,K \tag{6}$$

$$\sum_{(i,0)\in E\cup A} f_{i0}^{k} = \sum_{(i,0)\in R} q_{i0} x_{i0}^{k} \quad ; \; k=1,\dots,K \tag{7}$$

$$x_{ij}^{k} \in \{0,1\} \quad \forall (i,j)\in R; \; k=1,\dots,K \tag{8}$$

$$f_{ij}^{k} \geq 0 \quad \forall (i,j)\in E\cup A; \; k=1,\dots,K \tag{9}$$

$$y_{ij}^{k} \geq 0 \text{ integer} \; \forall (i,j)\in E\cup A; \; k=1,\dots,K \tag{10}$$

$$\sum_{(i,j)\in R}^{k} \left(q_{ij} + q_{i+1,j+1}\right) \leq W; \forall (i,j)\in E\cup A; \; k=1,\dots,K \tag{11}$$

The objective function (1) is to minimize the sum of the service costs and the deadheading costs. Constraints (2) impose the continuity of the trips at each node; the service of each required arc and edge is guaranteed by Constraints (3) and (4), respectively. Constraints (5) to (7) are the flow conservation constraints that force the connectivity of the trips. Constraints (8) to (10) are integrality conditions to complete the model. Constraints (11) are imposing upper bounds on the flow variables needed to guarantee the capacity constraints for each vehicle. They also imply that a flow variable can be positive only if the corresponding arc is traversed by the vehicle trip.

4. Similarity and Variation

In this section, the similarity and variation of our model and previous model will be discussed to foresee the modelling feasibility. The concept of flows to guarantee the connectivity of the solution becomes a basic idea to design the mathematical formulation. The similarity of our model compared to [15] is the flow variables are done separately and only work for tasks that have been set for it. The uniqueness of the model are:

- It formulates the mixed case.
- The flow variables have different interpretations.
- Additional constraints are included to ensure that trips start at the depot.
- Extra valid inequalities are considered to strengthen the linear programming relaxation.

The difference between our mathematical models compared to [15] are:

- The model formulation is suitable for undirected, directed and mixed cases.
- Dump cost is not taken into consideration. There is no concern on the dump cost activity because it is already included in the total cost for the whole operation.
- For the model in the directed graph, every edge is replaced by one or two arcs.
- The capacity of vehicles is considered for every trip process.

5. Constructive Heuristic Algorithm

Constructive Heuristics (CH) is classified as a modified path-scanning heuristics by [16]. Other heuristics that are related in the modification of path-scanning include the randomized highest demand or cost (RHDCC), highest demand/cost ratio (HDCR), switching rule (SR), route compactness rule (RCR) and shortest route rule (SRR). These heuristics, though simple, are powerful in solving the routing problem and it requires less parameters involved.

Global design of the proposed heuristics algorithm that control the searching process is given in **Figure 1**. In general, the proposed algorithm builds a feasible route by inserting at every iteration an

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unrouted customer into a previous connected serviced routes. This process is performed one route a time and is gradually improves in each iteration.

Step 1: Compute the mixed network graph $G = (V, E \cup A)$ with a set of initial value for all variables such as deadheading cost, y_{init}, trip for vehicles, k_{init}, capacity/demand,q_{init} and cost service, c_{init} equal to zero. Determine the value for demand, q_{ji} and cost, c_{ji} according to the pre-specified distribution.

Step 2: The first node, $V_n = \{V_i\}$ *will be chosen with edge,* $E_{(i,j)}$ or arc, $A_{(i,j)}$ along the link between nodes. Using the RHDCC to determine the good and best $E_{(i,j)}$ *or* $A_{(i,j)} \in V_n$ and choose variables at link whether $q_{ij} \in E_{(i,j)}$ *or* $q_{ij} \in A_{(i,j)}$ and $c_{ij} \in E_{(i,j)}$ *or* $c_{ij} \in A_{(i,j)}$. For the next cycle, find the new value for capacity, $q_{new} = q_{init} + q_{ij}$ and $q_{balnew} = W - q_{new}$ and new value for cost $c_{new} = c_{init} + c_{ij}$. Mark the covered $E'_{(i,j)} = E'_{(j,i)} = 1$ *or* $A'_{(i,j)} \neq A'_{(j,i)} = 1$.

Step 3: **Repeated** choosen next edge or arc, $E_{(i+1,j+1)}$ *or* $A_{(i+1,j+1)}$ using RHDCC **until** $q_{new} \geq W$ or $q_{balnew} \leq 0$. **If** $q_{balnew} \leq 0$, then assigned $q_{new} := \sum_{(i,j)\in R'} q_{ijk}$, $q_{balnew} := W - \sum_{(i,j)\in R'} q_{ijk}$ and $c_{new} := \sum_{(i,j)\in R'} c_{ijk}$ to repeat another cycle **or** $q_{new} \geq W$ **then** the operation will stop and for all $E'_{(i,j)}$ *or* $A'_{(i,j)} \in R'$ will be check. **If** ($E_{(i,j)} \in R = E'_{(i,j)} \in R'$) and **or** ($E_{(i,j)} \in R = E'_{(j,i)} \in R'$) or ($A_{(i,j)} \in R$)– ($A'_{(i,j)} \in R'$) $= 0$ **then** stops Step 3 and continue with Step 4. **If** ($E_{(i,j)} \in R = E'_{(i,j)} \in R'$) > 0 **or** ($E_{(i,j)} \in R = E'_{(j,i)} \in R'$) > 0 or ($A_{(i,j)} \in R - A'_{(i,j)} \in R'$) > 0 **then** assigned $c_{gencost}$ as a new cost, c_{new}. Label for $E'_{(i,j)} \in R'$ *or* $A'_{(i,j)} \in R'$ as covered and then terminate $\forall q_{ij}^k \in E'_{(i,j)}$ *or* $\forall q_{ij}^k \in A'_{(i,j)}$ and go back to Step 2.

Step 4: All the operations will be terminated and the total cost at edges, E or arcs, A and value for variables will be counted.

Figure 1. The step by step of the proposed Constructive Heuristic.

Step 1 is the first process to start the cycle. An initial value for certain variables such as deadheading cost, y_{init}, trip for vehicles, k_{init}, capacity/demand, q_{init} and cost service, c_{init} is set zero value. In addition, pre-specified distribution is determined for demand, q and cost, c. Step 2 is the process designated to identify good types of link (whether edge, $E_{(i,j)}$ or arc, $A_{(i,j)}$) in the network graph using the RHDC method. The basic concept for the RHDC is nearly similar to that for the nearest neighbour (NN) path-scanning method. The set of solutions is currently generated using a specific neighbourhood operator which is called the neighbourhood.

After the evaluation of the neighbourhood, one solution that emerges in the neighbourhood is NN. Similar to the NN, the RHDC is also in one of the family scope neighbourhood methods. In general, the RHDC builds up a path by choosing or adding a new feasible edge or arc in each cycle. For step 3, the ongoing process to search for the edge or arc using the RHDC will be continued until the cycle is completed. This ongoing process will take into consideration the limitation of the capacity of vehicles. At this stage, the process of works will be related with another step (Step 1, Step 2 and Step 4) for the simulation to back their functions. This is to ensure that all links between nodes in the network graph are served. Finally, Step 4 is the final process, where at this stage all the processes for developing cycles and variables are stopped.

6. Computational Results

In this section, we present the results from computational experiments that performed on a Intel(R) Pentium(R) Dual CPU T2370 @1.73GHz, 1.99 GB of RAM. The program developed in C# language by Microsoft Visual Studio environment. The computational experiments are to tests the performance of the proposed heuristic based on different capacity constraints. For this purpose, capacity testing is to measure and compare the solution of every problem. In this solution, the vehicle capacity is assumed as intermediate (W = 5000 kg and 10000 kg) with a random value range. For the initial process, intermediate instances with maximum 40 edges or arcs are tested. **Table 1** simplify the outcomes of the capacity test.

Table 1. Computational result for different types of link and capacity.

Capacity	Nodes	Links	Edge	Arc	Total cost	CPU time (ms)
5000	10	Undirected	13	0	158	00:00.01
5000	10	Directed	0	13	292	00:00.01
5000	10	Mixed	9	4	168	00:00.02
5000	25	Undirected	40	0	764	00:00.08
5000	25	Directed	0	40	620	00:00.09
5000	25	Mixed	20	20	695	00:00.12
5000	50	Undirected	85	0	1310	00:05.26
5000	50	Directed	0	85	1312	00:31.38
5000	50	Mixed	43	42	1358	00:02.97
10000	10	Undirected	13	0	223	00:00.01
10000	10	Directed	0	13	265	00:00.01
10000	10	Mixed	9	4	231	00:00.02
10000	25	Undirected	40	0	612	00:00.19
10000	25	Directed	0	40	648	00:00.16
10000	25	Mixed	20	20	633	00:00.17
10000	50	Undirected	85	0	1347	00:00.83
10000	50	Directed	0	85	1256	00:00.16
10000	50	Mixed	43	42	1276	00:00.62

Table 1 represents the computational result including the CPU time in second and maximum capacity in kilogram (i.e. 5000 kg and 10000 kg). The number of nodes is set for 10, 25 and 50 nodes which used for different link type. The analysis of Table 1 is described in Section 7.

7. Analysis and Discussion

The result of **Table 1** is described as follows. The analysis is prepared to test our new heuristics in order to determine initial results with different capacities of vehicles and types of links. From the capacity initial test, generally the result of different capacity problems yielded reasonable and feasible cost within very fast computation time (less than 1 second). Therefore, the initial solution for our new heuristics is capable to produce near-optimal result, but might not mean it is also the best one.

As evidenced from our computational result, the algorithm found a lot of good solutions especially in terms of the computational time and speed result (less than 1 minute) for small and intermediate networks especially in the mixed graph problem. For example, in the case of small networks (10 and 25 nodes), the heuristics produced very fast but vary results. In the case of undirected 13 edges with 10 nodes, algorithms comfortably looping with relax 5000 capacity. When tested with mixed graph, it produced a midst cost compared to undirected and directed links, but still in very fast time. However, the loops is tighter when tested with directed edge. This is understandable as we restricted the reversal and the algorithm can only moves to the next dedicated node. The same pattern happened for 50 nodes when the directed edge produced longer computation time (31 seconds) but considerably acceptable with very reasonable cost.

Our heuristic works in more stable and too relax within these same networks when tested with 10000 capacity. This condition is due to larger spaces to assign in the demand and cost into q_{new} and c_{new} parameter respectively. Additionally, q_{balnew} also works simultaneously to ensure the capacity constraint is not violated. In other words, our new heuristic algorithm is convincing to be comprehended in the undirected, directed and mixed graph problem for various capacity constraints.

8. Conclusion

In this paper, the problem of MCARP is described by undirected, directed and mixed where some of its edges and arcs have associated demand and cost. This type of problems aims to draw the attention to obtain a feasible optimum solution containing all the required edges and arcs in various capacity constraints. The biggest challenge in this research is the mathematical model used in this work was built based on some unique network characteristics. Through the study of this model, we are able to obtain a better understanding and sensitivity about the restrictions associated with this kind of routing problem.

The methodology used develops a new constructive heuristics from the existing heuristics. The new heuristics method was constructed where later on it is capable of providing a feasible solution in an acceptable capacity constraint. The development of the new method was applied to different sets of links. Previously, the existing heuristics method was always used to solve undirected problems but this research tried the new constructive method is built to offer solution for directed and mixed graph problems. The computational results show that the established mathematical modelling is comfortable with mixed graph problems with various capacity constraints. It shows a successful development of the new heuristics method, thus enable to get a good and reasonable solution. For future research, these can be extended using the metaheuristics method to get more accurate and good feasible solutions such as Tabu Search (TS), Genetic Algorithm (GA) or others. As represented in many articles, metaheuristics method can obtain the solution for larger instances, but with huge computation time.

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References

[1] Golden B L and Wong, R T (1981). Capacitated Arc Routing Problems. Networks, 11, 305-315.

[2] Assad A A and Golden B L (1995). Arc routing methods and applications. In: Ball M O, Magnanti T L, Monma C L, Nemhauser G L, Editors. *Handbooks in Operations Research and Management Science,* vol. 8, Network Routing. Amsterdam: Elsevier, 375-483.

[3] Dror M E (2000). *Arc Routing: Theory, Solutions and Applications.* Boston: Kluwer Academic Publishers.

[4] Eiselt H A, Gendreau M and Laporte G (1995a). Arc Routing Problems, Part I: the Chinese Postman Problem. Operations Research, 43, 231–242.

[5] Eiselt H A, Gendreau M and Laporte G (1995b). Arc Routing Problems, Part II: the Rural Postman Problem, Operations Research, 43, 339-414.

[6] Irnich S (2008). Solution of real-world postman problems. European Journal of Operational Research. 190, 52–67.

[7] Amposah S K and Salhi S (2004). The Investigation of a Class of Capacitated Arc Routing Problems: the Collection of Garbage in Developing Countries, Waste Management, 35, 711-721.

[8] Amaya A, Langevin A and Trépanier M (2007). The Capacitated Arc Routing Problem with Refill Points. Operations Research Letters, 35, 45-53.

[9] Bautista J, Fernández E and Pereira J (2008). Solving an urban waste collection problem using ants heuristics. Computers & Operations Research, 35, 3020–3033.

[10] Wøhlk S (2008). A Decade of Capacitated Arc Routing. In: Golden B, Raghavan S and Wasil E, Editors. *The Vehicle Routing Problem: Latest Advances and New Challenges pp 29-48*. Berlin. Springer.

[11] Liu M, Singh H K and Ray T (2014). Application Specific Instance Generator and a Memetic Algorithm for Capacitated Arc Routing Problems. Transportation Research Part C 43, 249–266.
[12] Belenguer J, Benavent E, Lacomme P and Prins C (2006). Lower and Upper Bounds for the Mixed Capacitated Arc Routing Problem. Computers & Operations Research, 33(12), 3363-3383.
[13] Ulusoy G (1985). The Fleet Size and Mixed prolem for Capacitated Arc Routing. European Journal of Operational Research, 22, 329-337.
[14] Mourão M C and Amado L (2005). Heuristic Method for a Mixed Capacitated Arc Routing Problem: A Refuse Collection Application. European Journal of Operational Research, 160, 139-153.
[15] Gouveia L, Mourão M C and Pinto L S (2010). Lower Bounds for the Mixed Capacitated Arc Routing Problem. Computers & Operations Research, 37, 692-699.
[16] Fadzli M R (2012). *Constructive Heuristics for Weather-Type Models of Capacitated Arc Routing Problems in Waste Collection*. PhD Thesis, Universiti Teknologi Malaysia, Skudai Johor.