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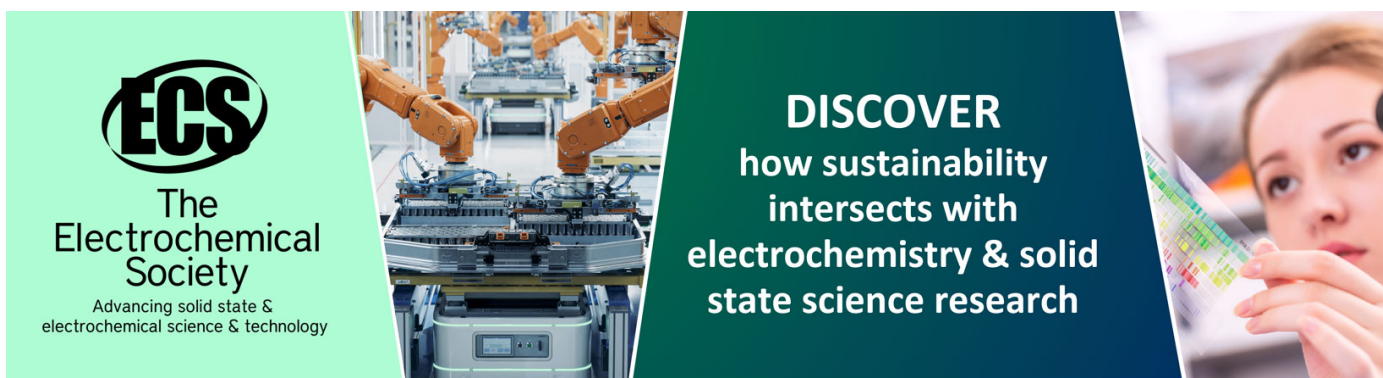
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Traveling wave solutions for the spatial diffusion of bird flu model

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Abstract. We describe mathematical model to study bird flu transmission in bird system and human system. The behaviour of this model was analyzed through stability of constant solutions. Our result shows that these stabilities depend on values of some parameters. Furthermore, the model of bird system is reformulated by adding diffusive term. Traveling wave solutions of the diffusive model were investigated. The positive solutions are numerically illustrated with homogeneous Neumann boundary conditions. The result shows that transmission progress can be expressed in form of a traveling wave solutions.

1. Introduction

Bird flu, caused by H5N1 virus, is carried by wild birds. The disease is very contagious and deadly disease among domesticated birds. Infection of those birds with H5N1 virus is classified into two types, highly pathogenic form and low pathogenic form. The one causes serious disease in domestic birds with a highly mortality rate, but the other one is not a serious threat to domestic birds. Bird flu not only infects through direct contact between infected birds and susceptible birds, but it also infects through indirect contact with equipment, material, and space that have been contaminated with the virus.

Bird flu viruses usually attack poultry. It happened worldwide in 2003, and poultry farmers have a risk from loss of domestic birds due to the disease ever since. Bird flu not only infects poultry, but it also infects human. Cases of transmission of bird flu from bird to human have been occurred since 1997. Although there has not been bird flu infection case from human to human, global vigilance is urgently needed in view of increasing number of infected domestic birds.

Study of bird flu infection in form mathematical model has been discussed in many literatures. Mathematical models to study the bird flu transmission from the bird world to human world were analyzed [1,5,6,7]. They also analyzed stability of equilibrium point. The transmission dynamics and spatial transmission of bird flu was studied [2]. A disease transmission model with diffusive terms to investigate spatial transmission of bird flu among poultry and human was studied [3,4]. Traveling wave solutions associated with bird flu transmission in a poultry farm were investigated [8,9,10].

In this study, susceptible infected model that interprets of bird flu transmission in bird and human system are described and numerical are presented. The model in bird system was developed by adding diffusion terms. The diffusive model is a system of partial differential equations that expresses solution in form of a traveling wave solutions.



2. Mathematical Model

Mathematical model interpreting bird flu epidemic model were proposed [1,2]. In this study, the model was reformulated based on assumption:

- All infected birds remain infected, can never recover, or dead.
- Infected humans cannot transmit the bird flu virus to susceptible humans.
- There is no mutation of the bird flu virus and no recover in human system.

The model describing interaction among birds lead to the following system equations

$$\frac{dX}{dt} = c - bX - \omega XY, \quad (1)$$

$$\frac{dY}{dt} = \omega XY - (b + m)Y. \quad (2)$$

The bird population is grouped into two groups, susceptible birds (X) are healthy birds that can be infected, and infected birds (Y) can transmit the bird flu virus. c is the rate at which new birds are born, b is the rate of natural death for both susceptible birds and infected birds, m is the rate of birds death due to bird flu infection, and ω is the contact rate between susceptible birds and infected birds.

The model describing interaction between birds and humans is given as the following system equations

$$\frac{dS}{dt} = \varepsilon - \mu S - \beta_1 SY, \quad (3)$$

$$\frac{dB}{dt} = \beta_1 SY - (\mu + d)B. \quad (4)$$

The human population is grouped into two groups, susceptible humans (S) and infected humans (B). ε is the rate of human growth are born, μ is the rate of natural death for both susceptible and infected humans, d is the rate of human death due to bird flu infection, and β_1 is the rate of interaction between susceptible humans and infected birds.

Furthermore, the transmission of bird flu model in equations (1), (2), which is the bird system, is modeled by spatial diffusion. Let, a diffusive term is added to the right side of the system equations, the system of equations (1), (2) become

$$\frac{dX}{dt} = c - bX - \omega XY + d_1 \frac{\partial^2 X}{\partial x^2}, \quad (5)$$

$$\frac{dY}{dt} = \omega XY - (b + m)Y + d_1 \frac{\partial^2 Y}{\partial x^2}. \quad (6)$$

where d_1 is positive constant. $X = X(x, t)$ and $Y = Y(x, t)$ denote the population of susceptible birds and infected birds at position x and time t , respectively, where x is the one dimensional coordinate variable.

Solutions $X = X(x, t)$ and $Y = Y(x, t)$ are defined in the plane $\{(x, t) | 0 \leq x \leq l, t > 0\}$. For $t > 0$ and $0 \leq x \leq l$, the system of equations (5), (6) is bounded with homogeneous Neumann boundary conditions

$$\frac{\partial X}{\partial x} = \frac{\partial Y}{\partial x} = 0 \quad (7)$$

and initial conditions

$$X(x, 0) = X_0(x), \quad Y(x, 0) = Y_0(x). \quad (8)$$

3. Analytical Solution

3.1. Constant Solutions

There are two constant solutions of system of equations (1), (2) which is bird system. One solution is the disease free equilibrium $e_0 = \left(\frac{c}{b}, 0\right)$. It means that there are no infected birds in population. The

other solution is endemic equilibrium $e_+ = \left(\frac{b+m}{\omega}, \frac{c}{b+m} - \frac{b}{\omega}\right)$. It means that there are part of the infected bird in population. The stability of those constant solutions depend on the value of the parameter

$$r_0 = \frac{\omega c}{b(b+m)}. \quad (9)$$

The disease free equilibrium e_0 is asymptotically stable if $r_0 < 1$, and it is unstable if $r_0 > 1$, whereas the endemic equilibrium e_+ is asymptotically stable if $r_0 > 1$, and it is unstable if $r_0 < 1$. The endemic equilibrium e_+ is significant if the contact rate between susceptible birds and infected birds ω is big.

For the human system of equations (3), (4), there are two constant solutions. One solution is the disease free equilibrium $E_0 = \left(\frac{c}{b}, 0, \frac{\varepsilon}{\mu}, 0\right)$. It represents that there are no infected birds and infected

humans in population. The other solution is endemic equilibrium $E_+ = (X^*, Y^*, S^*, B^*)$, where

$$X^* = \frac{b+m}{\omega}, Y^* = \frac{c}{b+m} - \frac{b}{\omega}, S^* = \frac{\lambda}{\mu + \beta_1 Y^*}, B^* = \frac{\beta_1 S^* Y^*}{\mu + d}.$$

It represents that part of the bird and the human population is infected in population. The stability of those constant solutions depend on the the parameter

$$R_0 = \frac{\beta_1 \omega c}{b(b+m)}. \quad (10)$$

The disease free equilibrium E_0 is asymptotically stable if $R_0 < 1$, and it is unstable if $R_0 > 1$, whereas the endemic equilibrium E_+ is asymptotically stable if $R_0 > 1$, and it is unstable if $R_0 < 1$. The endemic equilibrium E_+ is significant if the rate of interaction between susceptible humans and infected birds β_1 is big.

3.2. Traveling Wave Solutions

We analyze traveling wave solution of system of equations (5), (6), that represent the propagation of the bird flu virus in the bird system. In particular, traveling wave solutions of these system are solutions that are written in the following form

$$X(x, t) = u(z) = u(x - kt), \quad (11)$$

$$Y(x, t) = w(z) = w(x - kt). \quad (12)$$

where k is the positive constant wave speed. Let $s = x - kt$ (s is called the wave variable), then we get the system in term of just s . By substituting $X(x, t)$ and $Y(x, t)$ into the bird system leads to

$$-ku' = c - bu - \omega uw + d_1 u'', \quad (13)$$

$$-kw' = \omega uw - (b + m)w + d_1 w'', \quad (14)$$

where $' = \frac{d}{ds}$. Let $u' = v$ and $w' = z$, then now we get a first order system of equation

$$\begin{aligned} u' &= v, \\ v' &= -\frac{k}{d_1}v - \frac{c}{d_1} + \frac{b}{d_1}u + \frac{\omega}{d_1}uw, \\ w' &= z, \\ z' &= -\frac{k}{d_1}z - \frac{\omega}{d_1}uw + \frac{b+m}{d_1}w. \end{aligned} \quad (15)$$

There are two critical points of the system (15), one is $(u, v, w, z) = \left(\frac{c}{b}, 0, 0, 0\right)$ and the other one is $(u, v, w, z) = \left(\frac{b+m}{\omega}, 0, \frac{\omega c - b(b+m)}{\omega(b+m)}, 0\right)$. The Jacobian matrix of the system (15) is given as the following matrix

$$J = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{b+\omega w}{d_1} & -\frac{k}{d_1} & \frac{\omega u}{d_1} & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{\omega w}{d_1} & 0 & \frac{-\omega u + b + m}{d_1} & -\frac{k}{d_1} \end{bmatrix}. \quad (16)$$

Eigenvalues of the system (15) at $\left(\frac{c}{b}, 0, 0, 0\right)$ are

$$\begin{aligned} \lambda_1 &= \frac{-k + \sqrt{k^2 + 4bd_1}}{2d_1}, \\ \lambda_2 &= \frac{-k - \sqrt{k^2 + 4bd_1}}{2d_1}, \\ \lambda_3 &= \frac{-k + \sqrt{k^2 - \frac{4d_1}{b}(\omega c - b(b+m))}}{2d_1}, \\ \lambda_4 &= \frac{-k - \sqrt{k^2 - \frac{4d_1}{b}(\omega c - b(b+m))}}{2d_1}. \end{aligned}$$

The eigenvalues at $\left(\frac{c}{b}, 0, 0, 0\right)$ are less than zero except for λ_1 and λ_3 for $\omega c < b(b+m)$, therefore there is a two dimensional stable manifold at $\left(\frac{c}{b}, 0, 0, 0\right)$. All eigenvalues are real for

$k > k^* = \sqrt{\frac{4d_1}{b}(\omega c - b(b+m))}$. If $k < k^*$, then there are two complex conjugate eigenvalues that would cause a spiral around $\left(\frac{c}{b}, 0, 0, 0\right)$.

Jacobi matrix at $\left(\frac{b+m}{\omega}, 0, \frac{\omega c - b(b+m)}{\omega(b+m)}, 0\right)$ leads to the polynomial

$$p(\lambda) = \frac{\lambda^2}{d_1^2}(d_1\lambda + k)^2 + \frac{\lambda}{d_1^2}(d_1\lambda + k)\left(\frac{-\omega c}{b+m}\right) + \frac{b(b+m) - \omega c}{d_1^2}. \quad (17)$$

Let,

$$p(\lambda, d_1^2) = d_1^2 p(\lambda) = \lambda^2(d_1\lambda + k)^2 + \lambda(d_1\lambda + k)\left(\frac{-\omega c}{b+m}\right) + b(b+m) - \omega c, \quad (18)$$

then

$$p(\lambda, 1) = p(\lambda) = \lambda^2(\lambda + k)^2 + \lambda(\lambda + k)\left(\frac{-\omega c}{b+m}\right) + b(b+m) - \omega c. \quad (19)$$

The characteristic equation (19) will be a local minimum when $\lambda_0 = \frac{-k \pm \sqrt{k^2 + \frac{2\omega c}{b+m}}}{2}$, where

$p(\lambda_0, 1) = (b(b+m) - \omega c) - 1/4\left(\frac{\omega c}{b+m}\right)^2$. If $b(b+m) < \omega c$, then $p(\lambda, 1)$ will have two negative real roots. By using the Hurwitz criterion on $p(\lambda)$, there are two negative real part eigenvalues and two positive real part eigenvalues. Therefore, there a two dimensional stable manifold and a two dimensional unstable manifold at $\left(\frac{b+m}{\omega}, 0, \frac{\omega c - b(b+m)}{\omega(b+m)}, 0\right)$.

4. Numerical Simulations

The transmission of bird flu is investigated by some numerical simulations for fix parameters at $c=1, b=1, m=2, \varepsilon=0.01, \mu=0.015, d=1, d_1=2$. Numerical simulations of infected birds (Y) and infected human (B) are presented by choosing different constants ω and β_1 . Let

$$\Omega = \{(X, Y, S, B) | X > 0, Y > 0, S > 0, B = 0\} \quad (20)$$

is region of initial values that infected birds and infected humans do not exist. In this paper, initial values are fixed at $X(0) = 0.9, Y(0) = 0.1, S(0) = 1, B(0) = 0$.

Example 1.

Set $\omega = 2$ and $\beta_1 = 0.2$ then $r_0 = 0.67$ and $R_0 = 0.00067$. It means that the bird system and the human system admit to the disease free equilibrium e_0 and E_0 , respectively. Figure 1.a shows that the solution of system of equations (1), (2) converges to the disease free equilibrium $e_0 = (1, 0)$ and figure 1.b shows that the solution of system of equations (3), (4) converges to the disease free equilibrium $E_0 = (1, 0, 0.67, 0)$.

Example 2.

Set $\omega = 5$ and $\beta_1 = 0.5$ then $r_0 = 1.67$ and $R_0 = 1.167$. It means that the bird system and the human system admit to the endemic equilibrium e_+ and E_+ , respectively. Figure 2.a shows that the solution of system of equations (1), (2) converges to the endemic equilibrium e_+ and figure 2.b shows that the solution of system of equations (3), (4) converges to the endemic equilibrium E_+ . Figure 3 and figure 4 show the positive solutions of system of equations (5), (6) and profile of transmission bird flu of infected birds in spatial coordinate, respectively.

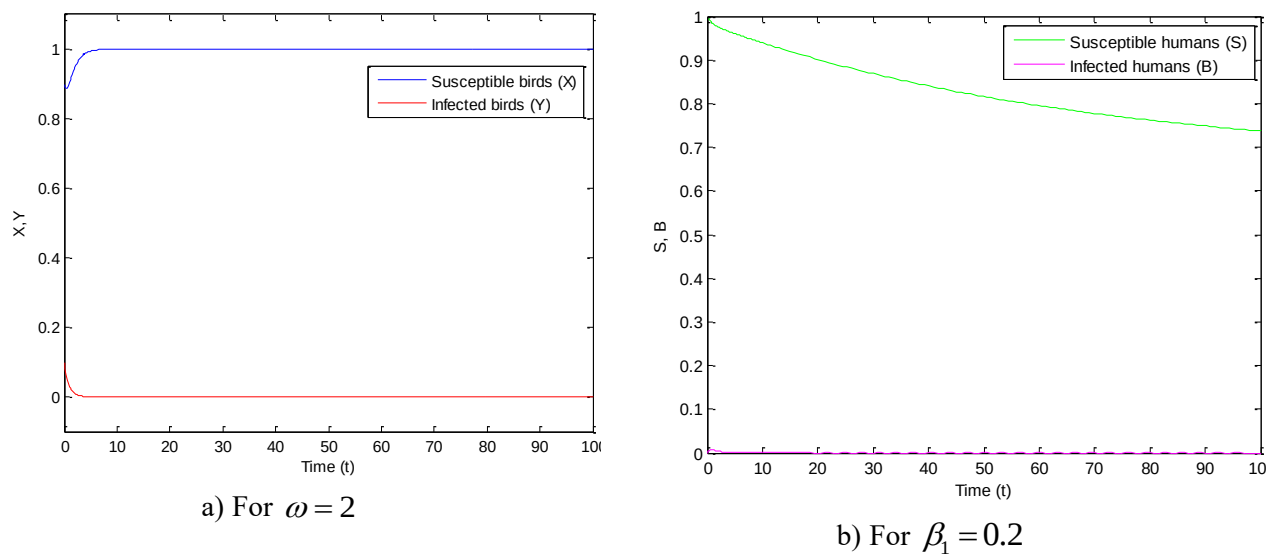


Figure 1. Costant solutions of the bird and human system admit to the disease free equilibrium.

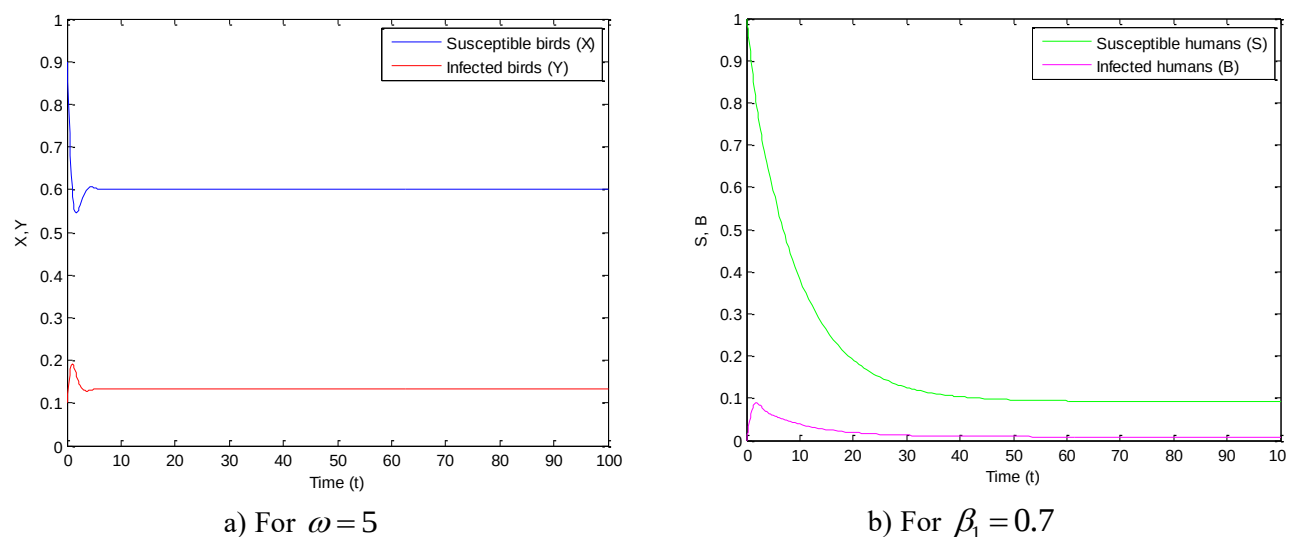


Figure 2. Costant solutions of the bird and human system admit to the endemic equilibrium.

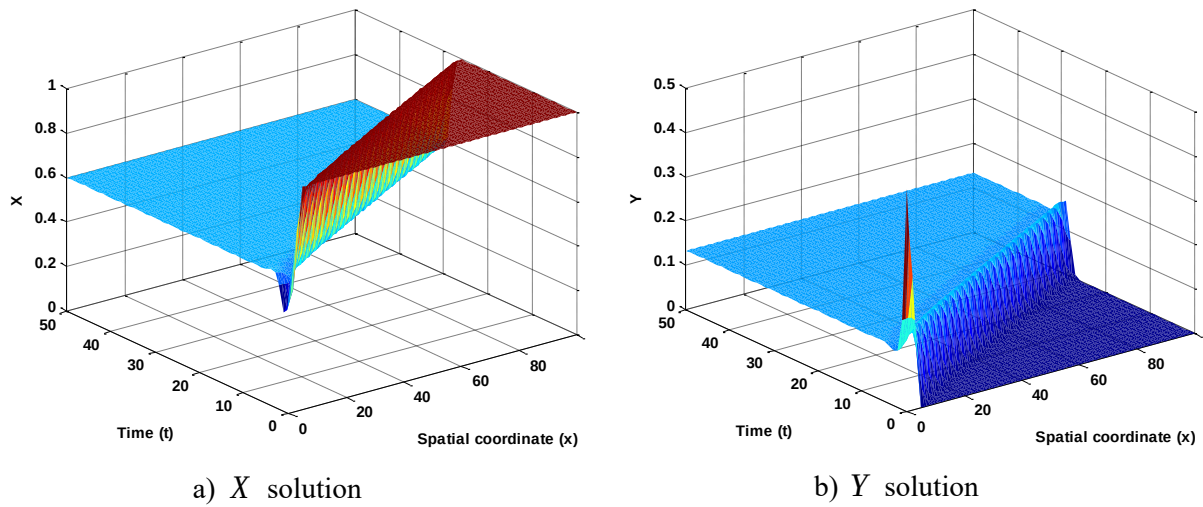


Figure 3. Positive solutions of the system of equations (5), (6).

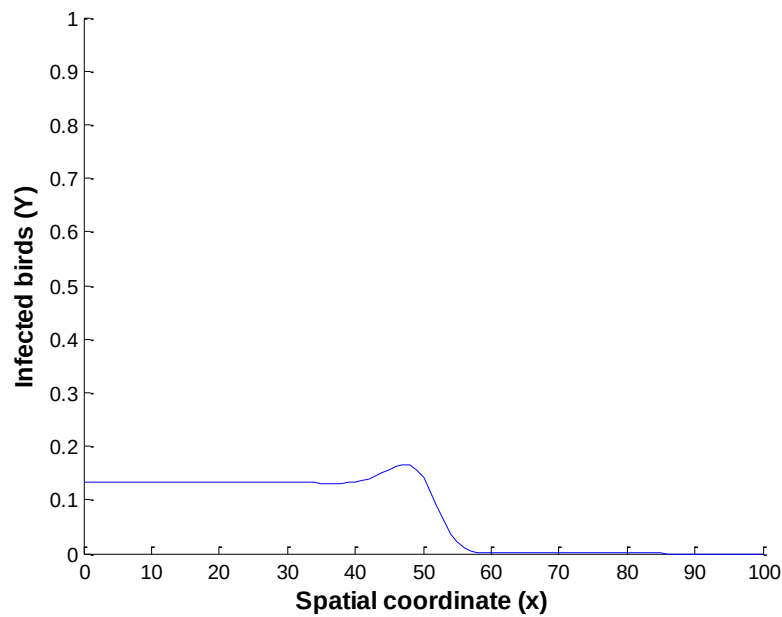


Figure 4. Profile of transmission bird flu.

5. Conclusions

A mathematical model of bird flu transmission in bird system and human system was investigated. Our results have shown that the disease free equilibrium e_0 and E_0 are asymptotically stable if $r_0 < 1$ and $R_0 < 1$. On the other hand, the endemic equilibrium e_+ and E_+ are asymptotically stable if $r_0 > 1$ and $R_0 > 1$. Our result also shows that transmission progress can be expressed in terms of a traveling wave

solutions for condition $\omega c > b(b+m)$ and $k > k^* = \sqrt{\frac{4d_1}{b}(\omega c - b(b+m))}$.

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