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Chaos prediction and control based on time series analysis

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Abstract. The nonlinear time-series analysis method is used to study the nonlinear large-scale measurement system and the discrete nonlinear system. For the instability of the linear steady-scale large-scale pulse system, a class of discrete Lipschitz nonlinear system dimensionality observer is used. The suboptimal control method for chaotic discrete systems with time-delay is obtained. The mathematical model of the large-scale pulsed system and the nonlinear discrete system is established. The instability of the large-scale steady-turbulence-type impulsive large-scale system is discussed by using the concept of the metric impulse system. The chaos of the nonlinear large-scale pulsating large-scale system with perturbation is proved by using the Lyapunov V function and the comparison principle. Bifurcation, using differential dynamic programming method to transform the suboptimal control problem of nonlinear discrete systems with time delay into the optimal control problem of nonlinear discrete systems without time delay, and realize the control of chaos.

1. Introduction

A system that is non-linear if its output is not proportional to its input. For a single pendulum motion, its behavior is linear only when its angular displacement is small. In fact, almost all known systems in the natural or social sciences are non-linear when the input is large enough. Therefore, nonlinear systems are much more numerous than linear systems. The objective world is inherently nonlinear, and linearity is only a very special case.

Linear science with linear systems as research objects is far from being able to adapt to the needs of modern science and technology development. With the deepening of human research on nature, more problems we face are nonlinear. With powerful analytical methods and tools, including linear algebra, linear differential equations, Fourier analysis, linear operator theory, and linear theory of stochastic processes, we have achieved very good results in the study of current problems.

In fact, linear systems are idealized representations of natural phenomena, and the natural phenomena, normal states, and essential features of nature are nonlinear.

It is well known that the Hamiltonian system has two important conservation properties, namely symplectic structure and energy conservation. A large number of experimental results show that the conservation of discrete symplectic structures or energy can be maintained in terms of numerical stability and accuracy of long-term calculations. The non-conservative format is excellent. However, in the sense of the B-order method, the numerical method of simultaneously maintaining the symplectic structure and energy of any Hamiltonian system does not exist. Therefore, the numerical schemes for solving the Henon-Heiles system and a class of Boussinesq systems are designed from the perspective



of energy conservation [1-7]. The classical Henon-Heiles system can be written as a finite-dimensional Hamiltonian system, and its chaotic phenomenon is closely related to the system energy. So is the numerical algorithm satisfying the conservation of energy is especially important to investigate the trajectory of the system? We can use the mean vector field method to construct the energy conservation format of the Henon-Heiles system. By studying the Poincaré section, we can investigate the chaotic and ordered phenomena of the system. The results show that chaos is not only related to energy, but also related to initial value state and disturbance. Both energy conservation algorithm and symplectic algorithm simulate chaos and order well, but the energy conservation algorithm better preserves the Henon-Heiles system. Energy, and allows for a larger time step.

The general Boussinesq system consists of four parameters. This Boussinesq system can be written in the form of an infinite dimensional Hamiltonian system, so it has energy conservation. Similarly, whether the numerical format satisfies the energy conservation has a great influence on the waveform propagation of the numerical solution. We can The Fourier pseudospectral method is used to construct the energy conservation algorithm of the Boussinesq system using the mean vector field method. We also use the midpoint method to construct the symplectic format, and compare the numerical results with experimental results. It shows that the energy conservation algorithm and the symplectic algorithm both simulate the chaotic condition of the nonlinear sequence. In addition, the energy conservation algorithm also maintains the energy of such Boussinesq system well.

The nonlinear scientific research with chaos, fractal, soliton and complexity as the main body unifies simplicity and complexity, order and disorder, certainty and randomness, inevitability and contingency in a new and colorful in the natural picture. Since the complexity of the real world mostly originates from nonlinearity, it is necessary to study nonlinear problems in exploring complexity, and nonlinear science is a new science established in the process of studying nonlinear problems.

Nonlinear science currently has six main research areas, namely chaos, fractals, pattern formation, solitons, cellular automata, and complex systems. The birth of nonlinear science has made people realize that Newtonian mechanics is not only applicable to the study of celestial bodies and microscopic particles, but also to the study of complex systems composed of multiple elements. Like quantum mechanics and relativity, nonlinear science represents the future of scientific development. It covers systems of various scales and involves objects moving at arbitrary rates. The research methods include: (1) Analytical methods: such as normal perturbation, Linz Ted-Poincaré method, multi-scale method, average method, KBM method and so on. There are not many nonlinear problems that can be solved analytically at present. Therefore, the analytical method has limited use; (2) Phase plane method: By qualitative analysis of the phase curve of the integral curve in the phase space, the motion law of the system is judged. It is the most intuitive qualitative analysis method, which can reflect the change of the topological structure of the system solution when the parameters or initial values change. The limitation is that the quantitative law cannot be obtained. It can not only obtain intuitive qualitative results, but also other research. The method provides theoretical basis; (3) Numerical method: Through the numerical solution, the motion law of the nonlinear system under certain parameter conditions and initial conditions is obtained, which can calculate the time history of various motions of the system (balance, periodic motion and Aperiodic motion, etc., can also determine the impact of parameters on the system, and determine the impact of initial conditions on system motion by calculating the attractors and their boundaries.

Chaos theory has been regarded as the best language and tool for studying nonlinear complex problems, and has become a hot topic in the world. For example, problems related to turbulence, air flowing over the wing, weather, and blood flowing through the heart. They are described by nonlinear equations, which are usually impossible to solve, and chaos is the description of such problems.

2. Prerequisite Knowledge

The discovery of chaos is considered to be one of the three major achievements of physics in the 20th century. It can be said that "relativity eliminates the illusion of absolute space and time. Quantum mechanics eliminates the Newtonian dream of controllable measurement processes. Chaos eliminates

Laplace's illusion about the predictability of determinism." There are only three things that science in the 20th century will remember forever, that is, relativity, quantum mechanics, and chaos. Its role in the whole science is equivalent to the influence of calculus on the mathematical science in the 18th century [8-11]. The establishment of chaos theory bridges the two scientific systems of determinism and probability theory. It opens a new chapter in the development of physics, mathematics and even modern science.

In recent years, chaotic science and other sciences have penetrated each other, whether in biology, physiology, psychology, mathematics, physics, electronics, information science, or astronomy, meteorology, economics, and even music, art, etc. Chaos has been widely used.

2.1. Li-Yorke Definition [3]

The continuous self-mapping $f(x)$ on the interval I can determine that it has chaos if the following conditions are met:

- (1) The period of the periodic point of f has no upper bound;
- (2) There is an uncountable subset S on the closed interval I , which satisfies:
 - (i) $\limsup_{n \rightarrow \infty} |f^n(x) - f^n(y)| > 0$, for $x, y \in S, x \neq y$.
 - (ii) $\liminf_{n \rightarrow \infty} |f^n(x) - f^n(y)| = 0$, for $x, y \in S$.
 - (iii) $\limsup_{n \rightarrow \infty} |f^n(x) - f^n(y)| > 0$, for $x \in S$ and y (Any cycle point of f).

This definition accurately depicts several important features of chaotic motion:

- (1) There are countable infinitely stable periodic orbits
- (2) There are uncountable infinitely stable aperiodic orbits
- (3) There is at least one unstable aperiodic orbit.

2.2. Devaney Definition [4]

Let V be a metric space, mapping $f: V \rightarrow V$. If the following three conditions are met, then f is said to be chaotic on V .

- (1) Initial value sensitive dependence. There is $\delta > 0$, for any $\varepsilon > 0$ and any $x \in V$, there is a y and a natural number n in the neighborhood of x , so that $d(f^n(x) - f^n(y)) > \delta$.
- (2) Topological transitivity. For any open set X, Y on V , there is $K > 0$, $f^K(X) \cap Y \neq \emptyset$.
- (3) The periodic point set of f is dense in V .

2.3. Definition of physics: So-called Chaos Refers to A Class of Phenomena with the Following Characteristics

- (1) Produced by certainty.
- (2) It is bounded.
- (3) Has a non-periodic.
- (4) Extreme sensitivity to initial conditions.

In the study of chaos, people are concerned about the following basic issues:

- (1) Can you assert that a given system will exhibit deterministic chaotic motion?
- (2) Can you explain the chaotic motion in mathematical language and make some quantitative descriptions of it?
- (3) The existence of chaotic motion indicates that it is impossible to make long-term predictions for some nonlinear systems, so whether some useful information can be obtained from chaotic signals.

2.4. Qualitative and Quantitative Methods of Chaotic Motion

In the study of chaos, chaos can be characterized by qualitative and quantitative angles, mainly to analyze the qualitative and quantitative methods of chaotic motion of the system.

- (1) Direct observation method
- (2) Frequency division sampling method
- (3) Poincaré section method
- (4) Phase space reconstruction method
- (5) Lyapunov exponential analysis
- (6) Self-power spectral density analysis paper.

According to the Fourier analysis theory, any periodic signal $x(t)$ with a period T can be expanded into a Fourier series. The physical meaning is that any periodic motion can be regarded as a superposition of the fundamental frequency $\omega_0 = 2\pi/T$ and a series of pan-resonance $n\omega_0$.

$$x(t) = \sum_{n=-\infty}^{\infty} a_n e^{in\omega_0 t} \quad (1)$$

$$a_n = \frac{1}{T} \int_{-T/2}^{T/2} x(t) e^{-in\omega_0 t} dt \quad (2)$$

$$x_{n+1} = f(x_n) \quad \int_{-\infty}^{\infty} |x(t)| < \infty \quad (3)$$

$$\delta x_n = |f^{(n)}(x_0 + \delta x_0) - f^{(n)}(x_0)| = \frac{df^{(n)}(x_0)}{dx} \delta x_0 \quad (4)$$

The Burg algorithm in the maximum spectrum analysis is used to estimate the A R model parameter. The estimation criterion of the algorithm is that the sum of the variances of the forward prediction error and the backward prediction error is the smallest, which is consistent with the target of the prediction problem, and is a fast recursive algorithm.

$$x(t) = (1/2\pi) \int_{-\infty}^{\infty} X(\omega) e^{i\omega t} d\omega \quad (5)$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-i\omega t} dt \quad (6)$$

$$S_{XX}(f) = \int_{-\infty}^{\infty} R_{xx}(\lambda) e^{-i\omega\pi\lambda} d\lambda \quad (7)$$

$$R_{XX}(\lambda) = \int_{-\infty}^{\infty} S_{xx} e^{-i\pi\lambda\sigma} d\sigma \quad (8)$$

In the process of continuous iteration, after two iterations of the original two points, the distance between the two points is:

$$ne^{n\sigma(x_0)} = |g^{(n)}(x_0 + \varepsilon) - f^{(n)}(x_0)| \quad (9)$$

$$\sigma = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \ln \left| \frac{df^{(k)}(x)}{dx} \right|_{x=x_k} \quad \varepsilon \rightarrow 0, k \rightarrow \infty \quad (10)$$

$$\delta(x_0) = \lim_{n \rightarrow \infty} \lim_{\sigma \rightarrow 0} \frac{1}{n} \ln \left| \frac{f^{(n)}(x_0 + \sigma) - f^{(n)}(x_0)}{\tau} \right| = \lim_{\sigma \rightarrow 0} \frac{1}{n} \ln \left| \frac{df^{(n)}(x)}{dx} \right| \quad (11)$$

Through the study of chaos control and its application, we gradually grasp the instability, uncontrollability and unreliability of chaotic motion, and have a further understanding of the topological structure of chaotic strange attractors.

3. Organization of the Text

We consider the following control model for time series analysis

$$\ddot{y}_i(n+i|n) = \ddot{y}_0(n+i|n) + \alpha_i \Delta u(n), \quad i = 1, 2, \dots, N \quad (12)$$

$$\ddot{y}_N(M+i|M) = \ddot{y}_0(M+i|M) + \sum_{k=1}^M \rho_{i+j-1} \Delta u(i+j-1) \quad (13)$$

$$\max P(i) = \sum_{j=1}^K p_i [\omega(j+1) - \bar{x}(k+i)]^\lambda + \sum_{\lambda=1}^P q_i \Delta u(p+q-1) \quad (14)$$

$$\tilde{x}_t = \alpha_t - \tau_1 \alpha_{t-1} - \tau_2 \alpha_{t-2} - \dots - \tau_p \alpha_{t-p} \quad (15)$$

$$\sigma(B) = 1 - \delta_1 B - \delta_2 B^2 - \dots - \delta_p B^p \quad (16)$$

Combining the above (12)-(16), using the moving average model method, introducing the sliding average operator

$$\ddot{x}_{MP}(N) = \ddot{x}_{M0}(N) - \tau \Delta u_P(k) \quad (17)$$

$$\tilde{x}_t = \Theta(\Omega) \alpha_t \quad (18)$$

Using the sliding autoregressive mixing method to fit the time series with the autoregressive model, you can get

$$b_m(n) = 2 \sum_{k=1}^n \alpha_{k-1}(m) f_{k-1}(m-i) + \varepsilon \sum_{k=1}^n \alpha_{k-1}^2(m) f_{k-1}^2(m-i)^2 \quad (19)$$

$$\sigma_k^2(m) = (1 - \delta_k^2(m)) \sigma_k^2(m-1) - \alpha_k(m) f_{k-1}(n+1) \quad (20)$$

In which, $\delta_i(k)$, $\alpha_i(k)$, $\sigma_i(k)$, $i = 1, 2, \dots, K$ is the Parameter valuation of A R (k), The residual sequence $\{e(n)\}$ of the model is used as the prediction error to replace the unpredictable white noise component $x_{kp}(n)$. The $p+q+2$ parameters are used, and the generalized features of these sequences are similar. Under the premise of allowing horizontal error, the level of fluctuation has a certain degree of stability.

$$\tilde{x}_{TM}(m) = \begin{bmatrix} \tilde{x}_M(p+1)k \\ \vdots \\ \tilde{x}_T(m+k)p \end{bmatrix}, \quad \tilde{x}_{TP}(m) = \begin{bmatrix} \tilde{x}_0(k+1)m \\ \vdots \\ \tilde{x}_T(n+k)p \end{bmatrix} \quad P = \begin{bmatrix} \lambda_1 & \dots & \lambda_M \\ \dots & \dots & \dots \\ 0 & \dots & \lambda_k \\ \dots & \dots & \dots \\ \lambda_p & 0 & 0 \end{bmatrix} \quad (21)$$

Here A is the dynamic matrix formed by the step response coefficients. The objective function can be described as:

$$\max \rho(\Omega) x_t = \omega(\Omega) \Phi^\alpha X_t \quad (22)$$

Where: $x(i), i = 1, 2, \dots, k$ is the parameter estimate of the AR, $\sigma(k)$ is the variance estimate of the residual sequence, k is the model order, and $x_k(n)$ $b_x(n)$ are the forward prediction error and the backward prediction error, respectively. The statistical test of the model occupies an important position when fitting the actual data sequence. For a random sequence $\{x(n)\}$, it includes both predictable and unpredictable parts, $x(n) - x_p(n) = x_{kp}(n)$. For the prediction problem, the predictive component is extracted as much as possible. $X_p(n)$. The unmeasurable component x_u is theoretically a white noise signal and can only be used as an inherent prediction error. The statistical relationship between the sequence data is quantitatively expressed by the model, so that the control and prediction of chaos can be realized by using the past sequence values.

$$V(k) = \frac{1}{\sqrt{N-1}} \frac{\left(\sum_{i=1}^{N-1} (Q_k(f_i) - \sigma_i^2(k)) \right)^{P/N}}{\sum_{i=1}^{N-1} \sigma_i^2(k)} \quad (23)$$

$$\sigma_{i(k)} = (\psi(k+1) + \dots \psi(K+P))^T \quad (24)$$

$$V = \begin{bmatrix} p_1 & 0 & \dots & 0 \\ 0 & p_2 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & p_r \end{bmatrix}, \quad P = \begin{bmatrix} r_1 & 0 & \dots & 0 \\ 0 & r_2 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & r_N \end{bmatrix} \quad (25)$$

4. Chaotic Parameter Estimation and Feedback Correction

Substituting (21)-(25) into (26) according to the above-mentioned error weight matrix and control smoothing weight matrix, at time K , the state space model of the stochastic process is based on the so-called Markov property, that is, the future state of the process only Depends on the current state, and

has nothing to do with the past state. The state space model consists of a state equation and a measurement equation. In many cases, the coefficient matrix of the state space model can be obtained according to the actual background of the problem, or can be obtained by deriving other models. For example, the smooth model and the ARMA model can be written in the form of a state space model (3)-(24). But it makes more sense to study how to directly identify the coefficient matrix based on the observed data. This can usually be done using Akiake's canonical correlation analysis and Aoki's singular value decomposition method. In order to minimize the identification parameters, the coefficient matrix is often taken as a §canonical structure. The canonical structure representation is the least number of parameters, but is sensitive to data errors.

The Bayesian prediction method is mathematically complex, especially the transition probability is difficult to calculate, but this method can reflect human subjective information into the model, and can predict by means of the model with few data, and the model can handle structural changes. The intervention analysis model was mentioned earlier, but the intervention model requires certain observation data before and after the “intervention”, otherwise it cannot be modeled, and the Bayesian prediction method does not have such a restriction.

For $\sigma_k(p)$, $\tilde{\alpha}_{p0}(k)$ are all known, with the necessary condition of extremum value, we can get $\Delta u_k(m)$ from $P(k)$.

$$\Delta u_k(m) = \frac{A^T Q((\omega_p(k) - y_{po}(k)))}{C^T (\Delta u_M(K))} \quad (26)$$

Apply rolling optimization strategy, and directly calculate the control rate, one can get

$$\Delta u_k(m) = C(\Delta u_M(K))d((\omega_p(k) - y_{po}(k))) \quad (27)$$

We can use the following steps to calculate its predicted value:

$$P_m(1) = R_p(k) \sum_{k=1}^M \alpha_i x(n+1-k) \quad (28)$$

$$P_m(k) = R_p(k) \sum_{k=1}^{M+k-1} \alpha_i x(n+1-k) + \lambda \sum_{k=1}^M \alpha_i^{\eta-1} P(n-k) \quad (29)$$

From this, the prediction result of actual chaos can be obtained

$$Y_t = \lambda_0 X_t + \lambda_1 X_{t-1} + \lambda_2 X_{t-2} + \dots = \sigma^{-1} v(B) X_t + \lambda(B) \tau(B) a_t \quad (30)$$

It can be proved that the optimal prediction of chaos can be achieved by using the results calculated by the above formula.

5. Conclusion

Chaos control and synchronization have attracted widespread attention. Some countries have invested huge amounts of research funding and a lot of manpower and resources to intensify the exploration of various experimental research and key technologies in chaos control and synchronization and its applications in many sectors, such as military, energy, industry, medicine, electronic engineering, etc. Research in this area has been increasingly integrated with the development of high-tech. The discovery

of chaos is the third physics revolution after the discovery of quantum mechanics and relativity in the century. The first two scientific revolutions have brought unparalleled material and spiritual civilizations to human beings, such as nuclear energy, aerospace, computers, lasers and Superconductivity and so on are shocking and endless achievements. We believe that the third scientific revolution, which is being carried out with the application of chaos control and synchronization, will increasingly show great potential for application and further contribute to the high civilization of mankind.

Compared with the actual results, the prediction results in this paper have maintained good consistency in terms of trend, sequence structure and numerical values. At the same time, this time series analysis method is more effective for short-term prediction, and the effect is also ideal for medium and long-term prediction.

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