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Experiments and representations in quantum physics: teaching module on the photoelectric effect and the Franck-Hertz experiment

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Abstract. A teaching module, centered on the photoelectric effect and the Franck-Hertz experiment, was designed for upper secondary school students. The activities have been built upon three design principles, brought out from an analysis of the current research literature about laboratories, learning/teaching quantum physics and representations in physics education.

1. Introduction

This paper aims at describing the design of laboratory activities on the *photoelectric effect* and the *Franck-Hertz experiment*, realized with classes of Italian secondary school students. The activities have been built with the overarching goal of creating an interactive space where students develop critical knowledge about the construction/refinement of physical models when they construct their knowledge, and reflect on where, how and why quantum physics challenges classical-like reasoning categories.

To this extent, three main aims have been pointed out to guide the design of the activities:

- to discuss in detail the experiment design so as to make it explicit the *modeling processes* that stay behind the experimental set-up.
- to problematize hyper-simplistic descriptions of quantum phenomena that could enforce resonance with classical-like explanation categories.
- to compare different *representational forms* (pictures, drawings, schemes, applets, verbal descriptions, graphs) that are usually used to present, realize, schematize, analyze and communicate the experiments and their results so as to stress both their pros and their limits.

In the next section, we report a brief overview of the literature and of the evidence that enabled us to turn the previous aims into specific criteria to design the lab activities. In section 3 the overall structure of the activities is outlined and in section 4 the lab on the *photoelectric effect* is described in detail. In section 5 some conclusive remarks are suggested, together with some possible future directions.

2. From a review of the research literature to design principles

Three design criteria have been pointed out both from a critical analysis of the research literature in physics education and from results we obtained in previous teaching experiences concerning students' expectations and stances about different representations of physical phenomena. In the following, we



present an overview of the issues of *didactical laboratories*, *quantum physics teaching*, and *representations in physics education*.

2.1. *The role of laboratories in physics education*

Research in physics education has shown that not all practices used to design lab activities enhance students' conceptual understanding [1]. The two extremes are, on the one hand, the use of laboratory as a simple confirmation of knowledge already acquired and, on the other hand, the so-called 'discovery learning', based on an oversimplified inductive use of experiments [2].

To overcome these extremes, recent papers discuss lab strategies based on *inquiry* [3] and/or on the development of epistemological knowledge about the design of experiments and the interconnections of experimental data and theoretical inferences [3, 4], also through explicit modelling activities [5]. Etkina et al [4] have proposed a classification of the different typologies of experiments that can be used in instruction on the base of their goals: *observational*, *testing* and *classification* experiments, distinguishing between *qualitative* and *quantitative* for each category; the researchers argue that being aware of a taxonomy of experiments can be helpful for teachers, and for students as well. Zwickl et al [5] have instead recently revised the consolidated use of *models* and *modelling* in labs for including the upper-division labs, which commonly "*do not seek to inductively develop new fundamental principles, but more commonly to apply known principles to explain observable phenomena or test predictions*"; for this purpose, they have proposed to explicitly include activities where students are guided to model also the *measurement process*, and they noticed that students ended up spending even more time productively in analyzing and modeling the used measurement tools than in modeling the physical systems. The framework for modeling in the laboratory became an iterative and conscious process of construction and refinement of the models of both the physical system and the measurement tools. On the basis of the previous results, the first design principle is:

DP1: to structure the lab activity in order to provide ways and time to let students to explicitly *model the physical systems and the measurement processes*, so as to enable them to analyze consciously the passage from the theoretical design of the experiment to the reality of instruments.

2.2. *Quantum physics teaching*

Quantum physics laboratories are not so common in Italian high school programs, even though the national guidelines require the Old Quantum Physics to be taught in the fifth year of the scientific *licei*. The previous remarks take a crucial role in the context of quantum physics, whose teaching/learning issues have been deeply debated within the community of physics education. Independently of the conceptual approach chosen, quantum physics raises epistemic and cognitive requirements that often produce deep skepticism, and sometimes even a difficulty to accept the theory [6]; in fact, it has been shown that also at graduate and undergraduate levels, students' epistemic and cognitive needs are often not satisfied by only confidence in the mathematical formalism [7, 8].

Baily and Finkelstein [8] pointed out the relevance of teachers' choices about interpretative issues, founding that an 'agnostic' stance can produce naïve *realist* interpretations in students and that addressing epistemological issues seems to play a crucial role for students' understanding. The *photon* concept, for example, can be seen from a realist perspective (as Einstein's one) as being localized, even if we do not know its position, or can be considered only as "*a click on the detector*" [9], taking an instrumentalist stance. As Klassen suggests, "*the dominant picture of photons as particles of light is misleading, [...] what should be emphasized, rather, is the quantum-mechanical nature of the interaction of light with matter*" [10]. All these remarks led us to the second design principle:

DP2: to focus the attention on the models of *interaction processes* more than on the physical *objects*, so as to avoid inappropriate hyper-simplistic object-based interpretations of quantum phenomena.

2.3. Representations in physics education

The abstract character of quantum models turns their external representations (pictures, drawings, schemes, applets, verbal descriptions, graphs) to be tools of great didactical importance. Their imaginative and analogical role, indeed, is a powerful tool for stimulating and inquiring students' conceptions and ideas about physical phenomena. Podolefski and Finkelstein [11] studied the influence of different representations of oscillating phenomena on students' understanding, showing that when the phenomenon represented has a high degree of *abstraction* – where with *abstract* they intend that students do not have a “*phenomenological grounded knowledge*” about it (as sound propagation, in the article) – they tend to read the representations at hand quite literally. In other words, the more this experiential knowledge grounding is unstable, the more representations tend to cue students to focus their attention on details, or particular aspects of the phenomena represented [11], assuming a bi-univocal relationship between representations characteristics and reality. Elby [12] has described this as *What-You-See-Is-What-You-Get* (WYSIWYG), a *read-out strategy* [13] eventually involved and activated. Learning quantum physics requires a great work of abstraction, and representations take a decisive but delicate role. Every scientific image, in fact, is a cognitive hybrid [14], a mixture of pictorial and verbal elements built upon implicit models of phenomena. In this sense, representations always hide theoretical or personal perspectives on physical reality, and have to be interpreted.

These claims coming from the literature resonate with some results emerging from a *questionnaire* we built to look at students' attitude towards different representations of quantum phenomena. The questionnaire, regarding the photoelectric effect and Bohr's atomic model, was submitted to a class of 25 high school students, who had been briefly exposed to the arguments in the classroom, but only verbally, without any use of representations. It consisted of two ranking exercises, one with 4 different representations of the photoelectric effect, or of some of its features, and the other with 4 regarding the Bohr's atomic model – followed by two open questions: (1) “*Which image do you prefer as a representation of the phenomenon? Please rank the images and justify your answer*”; (2) “*What image wouldn't you choose as a representation of the phenomenon? Please rank the images and justify your answer*”.

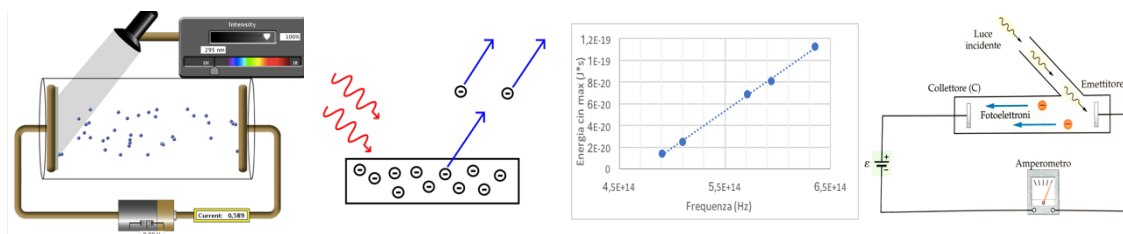


Figure 1. The four representations of the photoelectric effect used in the questionnaire, from left to right: 1.A [15], 1.B [16], 1.C, 1.D [17]

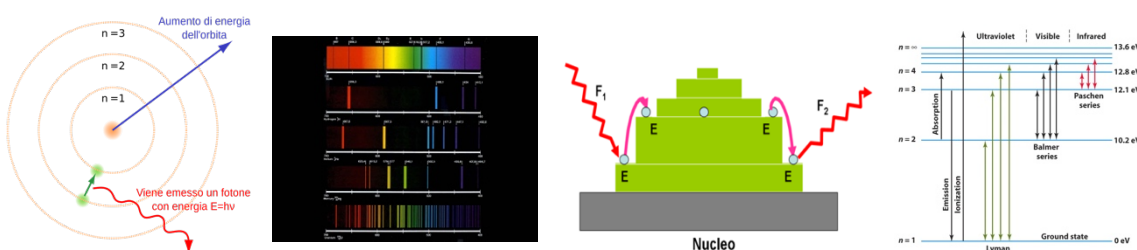


Figure 2. The four representations of Bohr's atomic model used in the questionnaire, from left to right: 2.A [16], 2.B, 2.C [18], 2.D [19].

In the wake of the literature, students' preferences and open answers highlighted two main expectations:

- The *language* of the representations has to be *immediate*: students seem to expect representations to be immediate in the sense of *time of comprehension*, and to be immediate in the sense of *without the need of any mediation*, being it of knowledge or of an analogical structure. For the photoelectric effect, in fact, the preferred representations are the 1.A and the 1.B, and some of the explanations are that “*it is all clear at a glance*”, “*it's clear and immediate*”, “*it is the most complete*”, or “*comprehensible for its essentiality*”. The least scored option for representing the photoelectric effect is quite unanimously the 1.C, because “*it doesn't represent the process*”, “*it requires a knowledge grounding to be interpreted*”, and to someone “*it's less interesting, because it's a graph*”. Concerning Bohr's atomic model, the most chosen representation is 2.A, on the basis of explanations like “*it's intuitive and detailed, it's easy to be visually remembered*”, “*it's immediate*” or “*it is the most concrete representation*”.
- The *content* of the representations has to be focused on *physical objects*: students seem to look at the *objects* involved in the phenomena, more than at the processes and at the experimental data, by searching for their *positions, movements, and shapes*. For example, someone describes her/his choice of not choosing the metaphorical representation 2.C as follows: “*it is not effective, because it doesn't recall the shape of an atom*”. This is a well-known issue in the literature [20] and we think it to be decisive also for what concerns the photoelectric effect, and for all those physical phenomena that require a high degree of abstraction.

The research literature shows that representations can be powerful tools, but these results, together with the other cited above, claim for a special attention in using them for educational purposes. Furthermore, some research has shown that understanding physical phenomena can be enhanced by improving students understanding of representations [11], developing what diSessa and Sherin [21] term *meta-representational competences*. Based on these results we fleshed out the third design principle:

DP3: to compare the many *representational forms* (pictures, drawings, schemes, applets, verbal descriptions, graphs) so as to lead students to develop meta-representational competences and to individuate the main constituting elements upon which the representation are built and the hidden models implicitly accounted.

3. The general structure of the lab activities

The teaching activities have been designed for a private foundation [22] promoting science culture and practice, in cooperation with schools. The activities are targeted to groups of about 20 volunteer upper secondary school students, grade 13 (18-19 years old). The students are usually encouraged to attend these extra-school activities by their teachers. The activities are complementary to regular school teaching since the necessary equipment is not usually present in the schools. They represent for most of the students the first encounter with quantum physics.

Each class was divided into 6 groups, and the activity last 4 hours in total, 2 for the *Photoelectric Effect* and 2 for the *Franck-Hertz experiment*. The requirement for participating to the lab was only a basic knowledge about electromagnetism and atomic models.

From a disciplinary point of view the laboratory was built on the main results of physics education research about photoelectric effect and Franck-Hertz experiment [10, 20, 23–28]; the structure and the didactical choices implement the Design Principles (DPs) introduced in the sections above, as described in what follows.

The experimental activities on the photoelectric effect and on the Franck-Hertz experiment are symmetrical in their structure, and comprise:

- an *alignment of students' knowledge*, that through a historical introduction aims at contextualizing the phenomenon and creating a common framing for the whole group of

students. The reflections are built on the line of DP1 and DP2, so as to underline the modeling nature of scientific research development, and use words that focus the attention on the *interaction processes*, more than on the models of physical *objects*.

- a *qualitative investigation* concerning the phenomenon, and an analysis of the evidence coming from the experiment's realization. This choice, in coherence with DP1, aims at setting the variables of the *physical system model*, building up an accessible imaginary upon which the quantitative part of the module can be modeled discussing with students.
- an explicit *modeling activity*, again following DP1, is carried out through a collective discussion that let students themselves model physical systems and measurement tools to make a quantitative analysis of the phenomenon.
- a *quantitative investigation* of the data obtained with the new experimental setup. The results of this part are systematically commented by recalling the imaginary the qualitative observations.
- a *meta-reflection* from different perspectives (historical, physical, mathematical experimental and technological) to interpret the results, summarize the whole activity and open new other questions. In this part, following DP2 and DP3, a final reflection is proposed about the difference between the representations that can be used for presenting and describing the phenomenon, with a special focus on the implicit physical models used to build the latter, and that cannot be given for granted.

4. The activity on the photoelectric effect

In this section, we describe in detail the design of the lab activity on the *photoelectric effect*, in order to give an idea of how the given DPs have been translated into practice. We note that the other activity, concerning the Franck-Hertz experiment, although having some different learning tasks, has been built with the same structure: the qualitative part is realized by observing the behavior of light spots in the Neon tube, and the quantitative one by wiring an oscilloscope to the tube. We do not describe it in detail due to space considerations and because it would not add critical elements to the design structure.

4.1. Knowledge alignment – historical/theoretical introduction

The module starts with a review of the models of light and a meta-reflection about what we can infer from observations of geometrical and physical optics phenomena. Following DP2 the discussion is held so that students can reflect on the difference between the questions “*what do you think light is?*” and “*what do you know about the behavior of light?*”, explicitly highlighting to them the role of modeling in physics.

After a brief historical introduction, the principal features of electromagnetic waves are recalled, supporting the students to take confidence with the parameters that identify a propagating plane wave – amplitude and frequency; playing with dimmable lights and lasers of different colors; this distinction is known to be not always resolved in students' conceptions [23]. The proportionality of the energy transfer to $|A|^2$ is recalled, pointing out that this is what the classical electromagnetic theory requires; attention is paid to avoid talking about light with high frequencies as ‘more energetic’, as this is a linguistic inference coming from quantum physics.

Finally, Hertz' experiment on the detection of electromagnetic waves, and his posthumously published observations about the effects of interposing plates of different materials are briefly introduced to build a historical-like bridge between the electromagnetic theory and the first encountering with the phenomenon next recognized as the *photoelectric effect*.

4.2. Qualitative investigation – Hallwachs' experiment

Hallwachs' experiments, performed with a leaf-electroscope, a zinc plate and different light sources (dimmable white lights, and a UV-C neon tube), serve to provide students with an accessible imaginary upon which the quantitative part of the module can be modeled and grounded, in coherence with DP1. The simplicity of the experimental setup allows the discussion to be moved onto an interpretive plane, and serves to recognize the main variables and parameters of the *physical system model* (i.e. the classical

electromagnetic model of light), upon which the modeling activity is based; the main aim, in fact, is to individuate the critical points in which the photoelectric effect puts in crisis the electromagnetic theory, problematizing the common belief that the existence of the photoelectric *per se* requires the development of a new theory [25].

The *qualitative* observations that can be made are that (1) variation of white light intensity does not affect the discharge of the electroscope leaves, (2) variation of the frequency can affect the discharge of the electroscope leaves, probably with a threshold frequency, (3) when negatively charged, the electroscope discharge is almost immediate and (4) when positively charged it discharges more slowly, so probably what is *emitted* in the interaction with light are negative charges (this last observation would be clear if performed in vacuum conditions, where the positively charged leaves would be opening at a greater angle). Generally, students associate the charges with electrons by themselves, but we point out with them that at the time the model of the electron was still developing.

All these observations are crucial to understand the photoelectric effect, and will be re-stressed in the quantitative part of the lab; with this experimental setup, the only ‘invisible’ phenomenon is the dependence of the discharge to the light intensity variation when the frequency is greater than the threshold.

4.3. Modeling the physical system - discussion on experimental improvements

This intermediate part aims at figuring out with students an experimental setup to investigate quantitatively the photoelectric effect encountered with the Hallwachs’ experiment; this *modeling activity* reveals to be crucial also for the theoretical understanding of the Lenard experiment results, and explicitly requires students to model both the *physical system* and the *measurement process*.

The discussion is conducted so as to let students think freely about what more could be experimentally investigated, by asking questions like “*What would you like to investigate better about this phenomenon?*”. This is the part concerning the physical system modeling and students tend to individuate the *number*, the *velocity* and the *direction* of the emitted charges as investigable quantities; when the hypothesis that electrons may have a velocity is introduced, we chose to write down together an equation as general as possible that can hold for the entire lab:

$$E_{\text{light}} = W_{\text{bond}} + (\text{eventual}) E_{\text{kinetic}} \quad (1)$$

where the symbol “=” stands for *has to be equal to*, so that it is clear that what we want to investigate is the energy of light. We notice that by doing so we are implicitly taking Einstein’s perspective about the necessity of a revision of the physical laws describing the process of light energy transfer; historically, this is not the only reasonable perspective [10], but at this stage we chose to take it with students to let them enter slowly into Einstein’s proposal (only in the last part of the activity is this point stressed again).

The design task now turns to the *measurement tools*, by finding out an experimental setup that could allow study and measurement of the effect of varying light intensity and frequency on the direction, number and velocity of the emitted electrons. The crucial point is to associate the number of electrons (in time) with their knowledge of electrical currents, so as to spread the way for introducing circuits; with a circuit we could measure the current with a simple ammeter, and it would also provide a way to collect the electrons emitted in all the directions. The Lenard experiment is hence introduced.

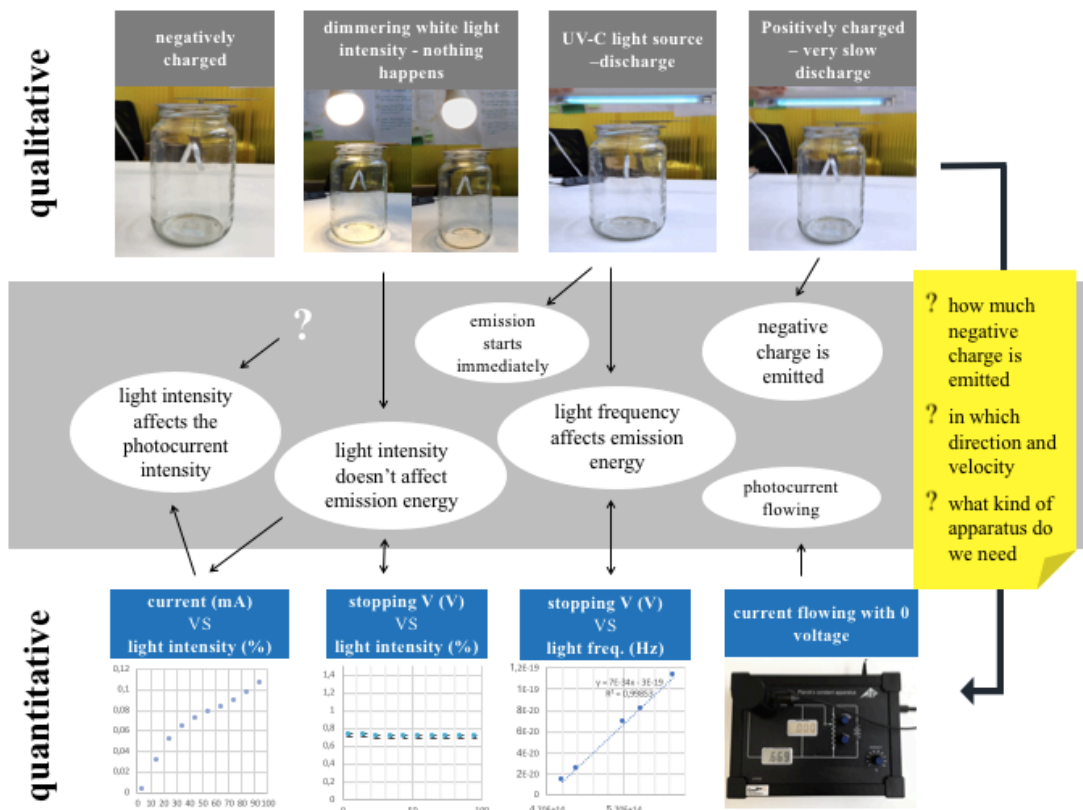


Figure 3. Schematic image of the conceptual passage from Hallwachs' experiment to Lenard's experiment.

4.4. Quantitative investigation – Lenard's experiment

The quantitative investigation is structured to follow Hallwachs' experiment's conceptual steps, and the results are systematically commented by recalling its imagery, as schematically summarised in Fig. 3. The experiments are performed with a didactical apparatus built with a Cs photocathode, that physically focuses the attention on the role of the variables of light intensity, and control voltage.

Firstly, the aims of the experiment are recalled to set down the *sense* of the procedures: to measure the *current* (the *number* of emitted electrons, indirectly) with an ammeter, and the *kinetic energy* of the emitted electrons, varying the intensity and the frequency of light. By inspecting the kinetic energy, in fact, we have direct information about the energy of light, thanks to the equation (1). The procedure to measure the kinetic energy is not trivial to invent, so it is introduced following some steps: (1) setting the light intensity to be $\neq 0$, and $V = 0$, current is flowing even though we would be tempted to use Ohm's law $V=RI$, thus the cathode is emitting electrons due to the photoelectric effect, with a certain kinetic energy, and (2) setting the potential as the measured current is $= 0$ (*stopping potential*) we can deduce the *kinetic energy*, by the expression

$$E_{kin} = eV_{stop} \quad (2)$$

Next, the students follow a 2-step tutorial to take the experimental data, investigating the observations accounted with the Hallwachs' experiment: (1) does the light intensity affect the current and the energy of the emitted electrons? This is an important issue, as the literature shows that students

acquire the concept that the intensity does not affect the energy, but it comes to be extended to the idea that it doesn't affect anything at all [28]. (2) How (*i.e.* with which type of dependence) does the frequency of light affect the energy of the emitted electrons? Physically speaking, and again accounting implicitly Einstein's point of view, what is E_{light} a function of? The quantitative results and graphs are shown schematically in Fig. 3.

4.5. Conclusive part – meta-reflections and representations

In this last part, the experimental results are commented on from different perspectives: historical, physical, mathematical, experimental and technological. The comments and the discussion have been built following DP2 and DP3; in particular, a conclusive reflection about some representations of the photoelectric effect is proposed.

In more detail, the relationship between the kinetic energy and the frequency is identified with a straight line with a certain slope and a discussion is held about the physical meanings of the *intercept with the frequency-axis* (threshold frequency) and of the *slope* (Planck's constant). The results are commented by pointing out all their incompatibilities with the electromagnetic theory. Finally, Einstein's 1905 article is introduced by reading the whole introduction to give an expressive idea of his point of view, and by summarizing his proposal for explaining Lenard's results.

Next, a discussion is presented about some representations concerning the photoelectric effect, reasoning with students about the different elements constituting them to individuate the hidden implicit models and discuss their plausibility, potentialities, possible risks, and limitations. For example, the representation of Lenard's experiment in Fig. 4 is discussed in order to recognize the *elements of experimental reality*, as the ammeter and the generator, the representations of *models of physical objects*, as the little balls for the electrons and the light spots with waves inside for the photons, and *models of physical processes*, as the arrows indicating the emission direction.

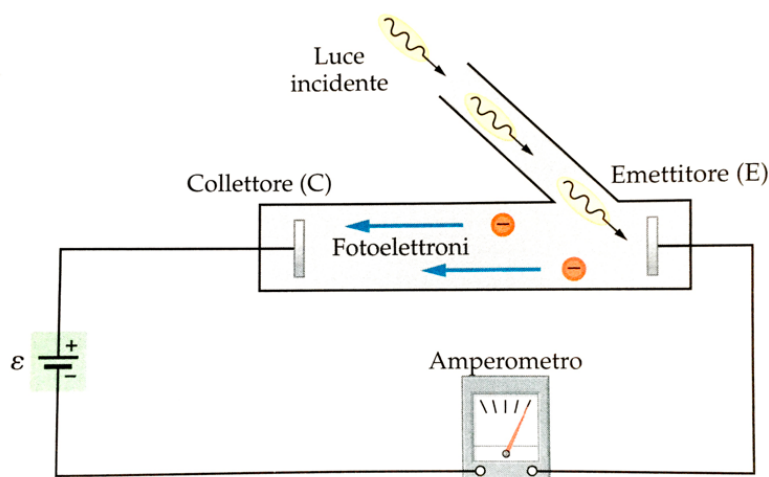


Figure 4. schematic figure representing Lenard's experiment, from a high-school physics textbook [17].

This representation is then discussed to stress in what sense it hides some implicit assumptions, and at least one important simplification. For instance, the spatial division of photons, represented as oscillating spots, is not a principle upon which the quantum model of photon is built, but only an inference to visualize their characteristic of transferring discrete amounts energy; furthermore, it is also not required by the experimental results obtained in the quantitative part of the module. Even historically, in fact, Einstein's proposal about the nature of light was not the only reasonable one for explaining the photoelectric effect, and for many years (until after Millikan's results in 1916 [10]) it was

not fully accepted. Moreover, the straight-oriented direction of the emitted electrons and the equal length of the two arrows are an implicit and dangerous simplification in the representation of the phenomenon; these details hide the assumption that all electrons are emitted with the same energy and in the same direction, which is in contrast with the experimental data obtained plotting the current I in function of the potential V [28].

This discussion, carried out interactively, can improve *meta-representational competences* [21], turning the cited request of *immediacy* (see Section II) into a more sophisticated request of sense; individuate the different models, with their specific limitations, and fix the distinction between model principles and model inferences. Moreover, it helps also to stress the idea that what we experimentally observe does not immediately imply a certain *object ontology*, as the *photon* for example, but forces us to look at and to model the interaction processes (DP2). A very similar discussion can be carried out on spatial representations of Bohr's atomic model, that can lead students to implicitly consider the atomic spatial structure as a planetary system with electrons moving on circular orbits, although the experimental results and the theoretic model do not require that inference.

Finally, some of the actual applications of the photoelectric effect are mentioned.

5. Remarks and future directions

In this paper, the design of a teaching module on experiments belonging to the *Old Quantum Physics* has been described, built according to three design principles pointed out from the research literature about laboratories, learning/teaching quantum physics, and representations in physics education. The most innovative aspects of the proposal are the choice to stress the model of the measurement processes, besides the models of physical systems, and the attention paid to the different forms of representations of physical phenomena. Some possible future directions are:

- to find a reliable tool to monitor and test the efficacy of the lab and of the design principles in dialoguing with students' understanding of quantum physics;
- to monitor the influence of the teaching module, and in particular of the choice to focus the attention on the processes and not on the objects, when students deal with modern quantum physics;
- to find out a theoretical framework to categorize images and representations in force of precise criteria, so as to be able to individuate which elements, and how, activate or challenge students' understanding processes;
- to build up a vocabulary, based on the framework, to be used by teachers to better recognize the implicit elements of a representation, and to find out some criteria for using them in a productive manner.

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