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To cite this article: S Khademi and A Ayoubi 2008 *J. Phys.: Conf. Ser.* **128** 012033

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# Dual symmetry in Born-Infeld theory

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**Abstract.** Born-Infeld theory is a non-linear formalism which has many applications in string and electromagnetic theories. Although, the existence of magnetic monopoles and dyons are suggested by Born-Infeld theory, but this theory is not invariant under the dual transformations. In this theory electric fields for point charged particles are not singular at origin ( $r = 0$ ), but magnetic fields and vector potentials are still singular. In this paper we show that the vanishing of dual symmetry is responsible for these singularities. Furthermore, we present the dual symmetric Born-Infeld theory, by a symmetric definition of electromagnetic fields in terms of new scalar and vector potentials, as well as the ordinary ones. All singularities of vector potential and magnetic field are removed as an immediate consequence of this symmetry.

## 1. Introduction

Study of symmetries is very important in all disciplines of science. One of the important symmetries in electrodynamics is dual symmetry. Dual symmetry of Maxwell's equations gives us the existence of magnetic monopoles and quantization of electromagnetic charges [1]. In this paper we investigate the dual symmetry of Born-Infeld (B-I) theory. This theory has many applications in string and electrodynamics theories in presence of magnetic monopoles or dyons which are interacted with electromagnetic fields [2]. In contrast to the ordinary electrodynamics, B-I theory is a non-linear formalism. In this formalism of electrodynamics, electric fields are expressed not only in terms of electric charge but also in terms of magnetic monopole charges. Then the singularity of electric field for a point charge at the origin is removed. But magnetic field for a point magnetic charge at the origin and vector potential are still singular. In this non-linear theory one has non-symmetric definitions for electric and magnetic fields in terms of electromagnetic potentials [3]. Thus dual symmetry between electric and magnetic fields is broken. We show symmetry breaking is a consequence of non-symmetric definitions of electromagnetic fields in terms of vector and scalar potentials. So, the treatment of electric fields, in the non-linear B-I theory, depends on the existence/non-existence of magnetic fields but magnetic field is just explained in terms of the magnetic charges [2, 3]. Thus the treatment of electric and magnetic fields are different. This result is a consequence of dual symmetry breaking of B-I theory. We introduce two additional (new) scalar and vector potentials to have new symmetric definitions of electromagnetic fields. Then we obtain the dual symmetry invariance of B-I theory, by symmetrically explained electromagnetic fields in terms of new scalar and vector potentials as well as the ordinary ones. As an immediate consequence of dual symmetry of B-I theory and new electromagnetic potentials, the singularities in vector potentials and magnetic fields are removed.

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In the next section a brief review of dual symmetry in electrodynamics is presented. Section three belongs to the description of B-I theory and in section four dual symmetry invariance of B-I theory is presented. Section five is devoted to the conclusions.

## 2. Dual symmetry in electrodynamics

Dual symmetry of Maxwell's equations predicts the existence of magnetic monopole, although it didn't get still any experimental confirmation. Dual transformations are defined by [4]

$$\begin{aligned}\mathbf{E} &= \mathbf{B}\sin\theta + \mathbf{E}\cos\theta, \\ \mathbf{B} &= -\mathbf{E}\sin\theta + \mathbf{B}\cos\theta, \\ \rho_e &= \rho_m\sin\theta + \rho_e\cos\theta, \\ \rho_m &= \rho_m\cos\theta - \rho_e\sin\theta, \\ \mathbf{J}_e &= \mathbf{J}_m\sin\theta + \mathbf{J}_e\cos\theta, \\ \mathbf{J}_m &= \mathbf{J}_m\cos\theta - \mathbf{J}_e\sin\theta.\end{aligned}\tag{1}$$

Symmetrized Maxwell's equations

$$\begin{aligned}\nabla \cdot \mathbf{E} &= \rho_e, \\ \nabla \cdot \mathbf{B} &= \rho_m, \\ \nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} &= -\mathbf{J}_m, \\ \nabla \times \mathbf{B} - \frac{\partial \mathbf{E}}{\partial t} &= \mathbf{J}_e,\end{aligned}\tag{2}$$

are invariant under the dual transformation (1). In symmetric Maxwell's equations (2),  $\rho_m$  and  $\mathbf{J}_m$  are magnetic charge and current density, respectively. The electric and magnetic fields of static point charges are obtained from Maxwell's equations (2), as

$$\mathbf{E} = \frac{q}{4\pi r^2} \hat{\mathbf{r}},\tag{3}$$

$$\mathbf{B} = \frac{g}{4\pi r^2} \hat{\mathbf{r}}.\tag{4}$$

These electric and magnetic fields are symmetric and proportional to  $r^{-2}$  and are singular at  $r = 0$ . Electromagnetic fields in Maxwell's equations are explained in terms of electromagnetic scalar and vector potentials

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} - \nabla \phi,\tag{5}$$

$$\mathbf{B} = \nabla \times \mathbf{A}.\tag{6}$$

Clearly, the electric and magnetic fields (5), (6) are not symmetric (dual symmetry) in terms of potentials. Thus, Maxwell's equations are not invariant under dual transformation (1) in the potential levels, although, they are invariant in terms of electromagnetic fields. Therefore substituting, Eqs. (5) and (6) into Maxwell's equations (2), will break dual symmetry.

To remove this inconsistency and keep dual symmetric Maxwell's equations in potential levels, we introduce two additional (new) scalar and vector potentials ( $\psi, \mathbf{G}$ ). Symmetric electromagnetic fields (5), (6) are explained symmetrically in terms of four electromagnetic (two scalar and two vector) potentials

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} - \nabla \phi + \nabla \times \mathbf{G},\tag{7}$$

$$\mathbf{B} = \frac{\partial \mathbf{G}}{\partial t} - \nabla \psi + \nabla \times \mathbf{A}.\tag{8}$$

Dual transformations for electromagnetic potentials are

$$\begin{aligned}\psi &\rightarrow \phi \\ \phi &\rightarrow -\psi \\ \mathbf{G} &\rightarrow \mathbf{A} \\ \mathbf{A} &\rightarrow -\mathbf{G}.\end{aligned}\tag{9}$$

Now we have a dual symmetric invariant Maxwell's equations in both potential and electromagnetic field levels. To obtain the electric and magnetic fields of a moving magnetic point charge, a singular vector potential is used [5]. But if one applies the new definition of electric and magnetic fields (7) and (8), the singular potentials is unnecessary. One obtains electric and magnetic fields for moving magnetic point charges by a well defined new scalar and vector potentials. Substituting Eq. (8) in Maxwell's equations give us

$$\nabla \cdot \mathbf{B} = \rho_m \rightarrow \nabla \cdot \left( \frac{\partial \mathbf{G}}{\partial t} + \nabla \psi + \nabla \times \mathbf{A} \right) = \rho_m.\tag{10}$$

By a (Coulomb like) gauge freedom for new vector potential  $\nabla \cdot \mathbf{G} = 0$ , Eq. (10) reduces to  $\nabla^2 \psi = \rho_m$ , which give us Eq. (6), for a static point magnetic charge, with  $\psi = g/4\pi r$ . These new scalar and vector potentials are known and sometimes used by other authors. In reference[?], the new scalar potential is called "magnetic potential" and denoted by  $\phi^*$ . Recently, new vector potential is utilized by P. Drummond to obtain the dual symmetric Lagrangian in quantum electrodynamics [7]. Here we use new electromagnetic potentials in classical electrodynamics to obtain the equation of motions for classical electric and magnetic charges interacting with electromagnetic fields, by the well defined, and nonsingular, electromagnetic potentials. In section four these new potentials and dual symmetry are investigated for B-I theory.

### 3. A brief review of B-I theory

Born-Infeld theory is a non-linear method and has many applications in electromagnetic formalism, where magnetic monopoles or dyons are assumed to be existed. We apply B-I theory to study the electrodynamics of dyons and magnetic monopoles which are interacting with external electromagnetic fields. Dyons are particles which have electric and magnetic charges, simultaneously. Dynamical equations in B-I theory are obtained from a nonlinear Lagrangian

$$L_{BI} = \beta^2 \left[ 1 - \sqrt{1 + \frac{1}{2\beta^2} F_{\mu\nu} F^{\mu\nu} - \frac{1}{16\beta^4} (F_{\mu\nu} \tilde{F}^{\mu\nu})} \right] + J^\mu A_\mu,\tag{11}$$

where  $F_{\mu\nu}$ ,  $J^\mu$  and  $A_\mu$  are conventional electromagnetic field strength tensor, four-current density and four-vector potential. If parameter  $\beta$  goes to infinity, B-I Lagrangian (11) reduce to a Lagrangian for the ordinary classical electrodynamics. Dynamical equations in B-I theory are obtained by applying Euler-Lagrange equation,  $\partial_\mu \frac{\partial L_{BI}}{\partial A_{\nu,\mu}} - \frac{\partial L_{BI}}{\partial A_\nu} = 0$  as

$$\begin{aligned}\nabla \cdot \frac{1}{R} \left[ \mathbf{E} + \frac{1}{\beta^2} (\mathbf{E} \cdot \mathbf{B}) \mathbf{B} \right] &= \rho_e, \\ \nabla \times \frac{1}{R} \left[ \mathbf{B} - \frac{1}{\beta^2} (\mathbf{E} \cdot \mathbf{B}) \mathbf{E} \right] - \frac{\partial}{\partial t} \frac{1}{R} \left[ \mathbf{E} + \frac{1}{\beta^2} (\mathbf{E} \cdot \mathbf{B}) \mathbf{B} \right] &= \mathbf{J}_e, \\ \nabla \cdot \mathbf{B} &= \rho_m, \\ \nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} &= \mathbf{J}_m,\end{aligned}\tag{12}$$

where

$$R = \sqrt{1 - \frac{1}{\beta^2}(\mathbf{E}^2 - \mathbf{B}^2) - \frac{1}{\beta^4}(\mathbf{E} \cdot \mathbf{B})^2}. \quad (13)$$

Equations (12) are Maxwell like equations corresponding to B-I theory. Hereafter these equations are called B-I equations. First part of B-I equations are obtained from Euler-Lagrange equation while the second one are borrowed from dual symmetric Maxwell's equations. Electric field for a point charge, in the absence of magnetic fields, is obtained from B-I equations as

$$\mathbf{E} = \frac{q}{4\pi r^2} \left(1 + \frac{1}{r^4} \frac{q^2}{(4\pi\beta)^2}\right)^{1/2} \hat{\mathbf{r}}, \quad \mathbf{B} = 0. \quad (14)$$

Clearly, electric field is not singular at  $r = 0$ . In the presence of magnetic fields the electric field is

$$\mathbf{E} = \frac{q}{4\pi r^2} \left(1 + \frac{1}{r^4} \frac{q^2 + g^2}{(4\pi\beta)^2}\right)^{1/2} \hat{\mathbf{r}}, \quad \mathbf{B} \neq 0, \quad (15)$$

which describes a dyon. The effect of magnetic charge on electric field is a consequence of nonlinearity in B-I equations. Thus the existence of magnetic fields affects the magnitude of electric field. Magnetic field in Born-Infeld equations is equivalent to the magnetic field in Maxwell's equations (4), either in the presence or in the absence of electric fields. Although, the singularity of electric field is removed in B-I theory, but the magnetic field of a magnetic point charge is still singular. Therefore, the magnetic and electric fields in B-I equations are not invariant under dual transformations. In the next section we use the new electromagnetic potentials to have a dual symmetric B-I equations and remove the singularity of magnetic field and vector potential.

#### 4. Dual symmetry invariant B-I theory

As we explained in the previous section, B-I electromagnetic fields are not invariant under dual transformations. Existence of singularity in magnetic field is due to the dual symmetry breaking. In this section we apply a new definition of field strength tensor

$$F'_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} (\partial_\alpha G_\beta - \partial_\beta G_\alpha), \quad (16)$$

in B-I Lagrangian, where  $G_\alpha \equiv (\psi, -\mathbf{G})$  and  $J_m^\mu$  are new four-potential and four-magnetic current density, respectively. This field strength tensor (16) is symmetric under dual transformations (1) and (9). Thus, nonlinear and dual symmetric Lagrangian of B-I theory is

$$L'_{BI} = \beta^2 \left[ 1 - \sqrt{1 + \frac{1}{2\beta^2} F'_{\mu\nu} F'^{\mu\nu} - \frac{1}{16\beta^4} (F'_{\mu\nu} \tilde{F}'^{\mu\nu})^2} \right] + J_e^\mu A_\mu + J_m^\mu G_\mu. \quad (17)$$

By applying the Euler-Lagrange equations,  $\partial_\mu \frac{\partial L'_{BI}}{\partial G_{\nu,\mu}} - \frac{\partial L'_{BI}}{\partial G_\nu} = 0$  and  $\partial_\mu \frac{\partial L'_{BI}}{\partial A_{\nu,\mu}} - \frac{\partial L'_{BI}}{\partial A_\nu} = 0$ , one obtains the symmetric B-I equations

$$\begin{aligned} \nabla \cdot \frac{1}{R} \left[ \mathbf{E} + \frac{1}{\beta^2} (\mathbf{E} \cdot \mathbf{B}) \mathbf{B} \right] &= \rho_e, \\ \nabla \times \frac{1}{R} \left[ \mathbf{B} - \frac{1}{\beta^2} (\mathbf{E} \cdot \mathbf{B}) \mathbf{E} \right] - \frac{\partial}{\partial t} \frac{1}{R} \left[ \mathbf{E} + \frac{1}{\beta^2} (\mathbf{E} \cdot \mathbf{B}) \mathbf{B} \right] &= \mathbf{J}_e, \\ \nabla \cdot \frac{1}{R} \left[ \mathbf{B} - \frac{1}{\beta^2} (\mathbf{E} \cdot \mathbf{B}) \mathbf{E} \right] &= \rho_m, \\ \nabla \times \frac{1}{R} \left[ \mathbf{E} + \frac{1}{\beta^2} (\mathbf{E} \cdot \mathbf{B}) \mathbf{B} \right] + \frac{\partial}{\partial t} \frac{1}{R} \left[ \mathbf{B} - \frac{1}{\beta^2} (\mathbf{E} \cdot \mathbf{B}) \mathbf{E} \right] &= -\mathbf{J}_m. \end{aligned} \quad (18)$$

The new Born-Infeld equations (18) are invariant under dual transformations, (1) and (9) where  $\beta \rightarrow i\beta$ . Electric and magnetic fields for the electric and magnetic point charge (or dyons) are obtained from symmetric B-I equations (18) as:

$$\mathbf{E} = \frac{q}{(4\pi r^2)} \left[ \frac{1 - \frac{1}{r^4} \frac{q^2}{(4\pi\beta)^2}}{1 + \frac{1}{r^4} \frac{q^2}{(4\pi\beta)^2}} \right] \hat{\mathbf{r}}, \quad (19)$$

and

$$\mathbf{B} = \frac{g}{(4\pi r^2)} \left[ \frac{1 + \frac{1}{r^4} \frac{q^2}{(4\pi\beta)^2}}{1 - \frac{1}{r^4} \frac{g^2}{(4\pi\beta)^2}} \right] \hat{\mathbf{r}}. \quad (20)$$

These symmetric electric and magnetic fields describe the electromagnetic fields of dyons. As an important consequence of nonlinearity in B-I theory, the electric (magnetic) field is not independent of magnetic (electric) point charge. Electric field (15) will be obtained by expansion of (19) in terms of powers of  $q$  and  $g$ , up to  $q^2 g^2$ ,

$$\frac{1 + \frac{1}{r^4} \frac{q^2}{(4\pi\beta)^2}}{1 - \frac{1}{r^4} \frac{g^2}{(4\pi\beta)^2}} = \left[ 1 + \frac{1}{r^4} \frac{q^2}{(4\pi\beta)^2} \right] \left[ 1 - \frac{1}{r^4} \frac{g^2}{(4\pi\beta)^2} \right]^{-1} \left[ 1 + \frac{1}{r^4} \frac{q^2}{(4\pi\beta)^2} + \frac{1}{r^4} \frac{g^2}{(4\pi\beta)^2} + O(> q^2 p^2) \right]. \quad (21)$$

Ordinary electromagnetic fields for point charges (3),(4), and Maxwell's equations (2) are obtained from symmetric electromagnetic fields (19) and (20) and symmetric B-I equations (18) while  $\beta$  approaches to infinity.

## 5. Conclusions

In ordinary electrodynamics theory, to describe magnetic monopole, one may use a singular vector potential. In the other hand, ordinary electric and magnetic fields, for electric and magnetic point charges, are also singular at the origin. We found that the origin of the singularities is the absence of new scalar and vector potentials. Using these new electromagnetic potentials, one may remove these singularities not only in vector potential of ordinary Maxwell's equations, but also in electromagnetic fields of dyons in B-I theory. We obtained symmetric electric and magnetic fields as well as symmetric Maxwell's equations (B-I dynamical equations) by introducing the new scalar and vector potentials. They reduced to the ordinary electromagnetic fields and Maxwell's equations, by setting. Although, in this theory we have two scalar and two vector potentials, however, we have just one kind of electric and magnetic fields, which are determined by these electromagnetic potentials.

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