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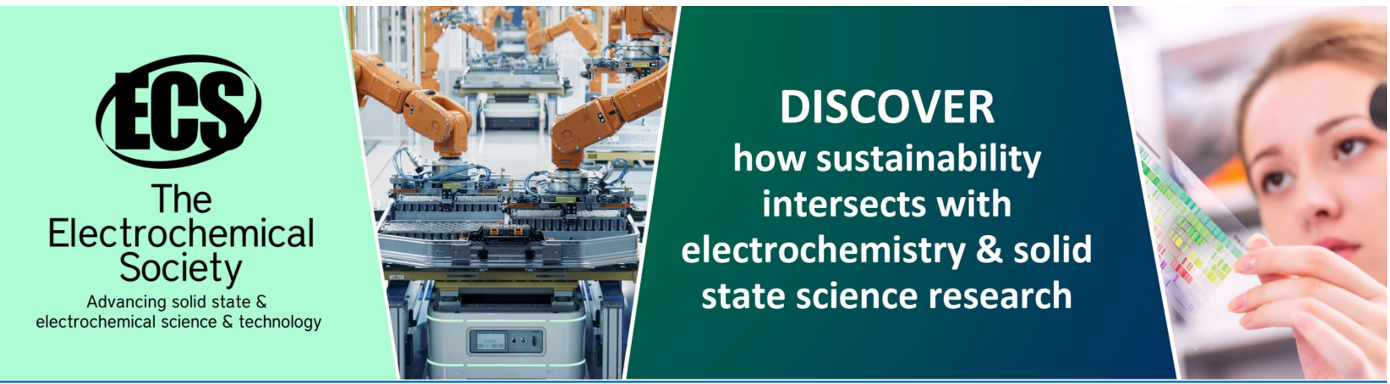

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Nonlinear completion of massive gravity of the Fierz-Pauli type

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Abstract. A possible nonlinear completion of massive gravity of the Fierz-Pauli type is proposed. The theory describes a system consisting of a massive tensor field of the Fierz-Pauli type and an additional massive vector field. Massless limit as well as flat-spacetime limit can be taken smoothly. Constructing a nonlinear version of the physical-state condition which drives an extra scalar ghost from physical states is still unsettled.

1. Introduction

In a series of papers [1, 2, 3, 4, 5], attempts to construct the theory of massive gravity with smooth massless limit were made.

We studied infrared regularization of linearized massive tensor fields in [1, 2, 3, 4]. Two model theories were considered: one is of the pure-tensor (PT) type, which describes an ordinary massive tensor field of five degrees of freedom; the other is of the additional-scalar-ghost (ASG) type, which contains a scalar ghost in addition to the pure tensor. The ASG model shows second-order massless singularities in two-point functions, whereas the PT model contains fourth-order singularities. It turns out that two procedures, the BRS one and the Nakanishi one, are effective in regularizing such singularities. The BRS procedure produces transparent structures to the resulting theories, as compared with the Nakanishi one. So we studied the former in detail. In order to drive away the second-order infrared singularities in the ASG model, we introduce an auxiliary vector-like field, and promote the original theory to the one that is invariant under the vector BRS transformation. On the other hand, to carry out infrared regularization of the fourth-order singularities in the PT model, we need to introduce an auxiliary scalar field in addition to the vector-like one, and make the resulting theory invariant under the scalar BRS transformation as well as the vector one.

When we try to perform nonlinear completion, the ASG model is easier to deal with than the PT model. This is because only the vector BRS transformation is involved there. The nonlinear form of this transformation is simply the quantum version of the general coordinate transformation. The scalar BRS, on the other hand, has no classical counterpart. Constructing its proper nonlinear generalization is not an easy task. A possible nonlinear completion of the BRS model of the ASG-type massive tensor was proposed in [5] (See also [6].). We also pointed out there that ghost condensation mechanism may work well for making innocuous the additional scalar ghost.

The purpose of the present paper is to put into practice nonlinear completion of the infrared-regularized PT model. In order to avoid introducing scalar BRS, we ask for the help of the

Nakanishi procedure. Then it is found straightforward to construct nonlinear Lagrangian for that BRS+Nakanishi model of the PT-type massive tensor.

However, this is not the end of the story. In the present formulation, there occurs a new trouble concerning physical-state condition of the Nakanishi-type. Finding a nonlinear version of such condition requires further studies.

In section 2, we review the case of Abelian vector field. This is to see how the BRS and the Nakanishi procedures work for regularizing massless singularities contained in the original massive theory. Stress is put on the fact that choosing massive gauge in the BRS procedure gives simple pole structure to two-point functions and makes it easy to investigate particle contents of physical states. In section 3, we treat linear theories of massive tensor field. Second-order massless singularities in the ASG model are regularized by the BRS procedure, whereas fourth-order singularities in the PT model are regularized by the use of both the BRS *and* the Nakanishi procedures. Emphasis is laid also here on the usefulness of adopting massive gauge. Section 4 treats nonlinear completion of the BRS+Nakanishi model of the PT-type massive tensor. After introducing nonlinear BRS transformation and basic BRS invariants, we propose possible nonlinear forms for the Lagrangian. Difficulties of finding a nonlinear version of the Nakanishi-type physical-state condition are also pointed out. Summary and discussion are given in section 5.

2. Massive vector

2.1. Massless vector

Let us begin with massless vector. The Lagrangian is given by[1]

$$L_0 = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + L^{\alpha}_{\mathrm{GF+FP}}, \tag{1}$$

where $L^{\alpha}_{\mathrm{GF+FP}}$ is the gauge-fixing and Faddeev-Popov (GF+FP) Lagrangian

$$\begin{aligned} L^{\alpha}_{\mathrm{GF+FP}} &= b\left(\partial_\mu A^\mu + \frac{\alpha}{2}b\right) + \mathrm{i}\bar{c}\,\Box\, c \\ &= -\mathrm{i}\delta\left[\bar{c}\left(\partial_\mu A^\mu + \frac{\alpha}{2}b\right)\right] \end{aligned} \tag{2}$$

with the gauge parameter α. The theory is invariant under the BRS transformation

$$\delta A_\mu = \partial_\mu c, \qquad \delta\bar{c} = \mathrm{i}b. \tag{3}$$

For $\alpha = 1$, two-point functions take simple forms:[2]

$$\langle A^\mu A^\nu\rangle = \frac{\eta^{\mu\nu}}{\Box}\delta, \quad \langle A^\mu b\rangle = \frac{\partial^\mu}{\Box}\delta, \quad \langle bb\rangle = 0, \quad \langle c\bar{c}\rangle = -\mathrm{i}\frac{1}{\Box}\delta. \tag{4}$$

Physical states are defined by the use of the conserved BRS charge Q_{B}:

$$Q_{\mathrm{B}}|\mathrm{phys}\rangle = 0. \tag{5}$$

In order to clarify the particle contents of the physical states, we expand any field $\Phi_A(x)$ as

$$\Phi_A(x) = \frac{1}{(2\pi)^{3/2}}\int \mathrm{d}^4p\,\theta(p^0)\left[\Phi_A(p)\mathrm{e}^{\mathrm{i}px} + \Phi^\dagger_A(p)\mathrm{e}^{-\mathrm{i}px}\right]. \tag{6}$$

[1] The flat spacetime metric used in the present paper is $\eta_{\mu\nu} = (-1, +1, +1, +1)$.
[2] Here and hereafter the spacetime coordinates are omitted in the field variables as well as in the δ-functions.

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In the axial coordinate $(p^1 = p^2 = 0,\ p^3 > 0)$, let us define as

$$\varphi^1(p) \stackrel{\mathrm{d}}{\equiv} A^1(p), \qquad \varphi^2(p) \stackrel{\mathrm{d}}{\equiv} A^2(p), \qquad \chi(p) \stackrel{\mathrm{d}}{\equiv} \frac{1}{p^3} A^3(p). \tag{7}$$

Then we find that $\{\varphi^i(p)\,(i = 1,2)\}$ are BRS singlets and physical, whereas $\{\chi(p), b(p), c(p), \bar{c}(p)\}$ constitute a BRS quartet.

2.2. Massive vector: naive model

Mass is introduced through the Proca Lagrangian

$$L_m\left[A^\mu\right] = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{m^2}{2} A_\mu A^\mu. \tag{8}$$

This naive model of massive vector field gives two-point functions with the second-order massless singularities like

$$\langle A^\mu A^\nu \rangle = \frac{1}{\Box - m^2} \left(\eta^{\mu\nu} - \frac{\partial^\mu \partial^\nu}{m^2} \right) \delta. \tag{9}$$

The field equations

$$\begin{cases} (\Box - m^2) A^\mu = 0, \\ \partial_\mu A^\mu = 0 \end{cases} \tag{10}$$

ensure that the physical degrees of freedom count three, $\{\varphi^i(p)\ (i = 1,2),\ \chi(p)\}$, in this case.

2.3. Massive vector: Nakanishi model

In order to remove the massless singularities involved in the naive model, Nakanishi [7] proposed the following type of Lagrangian:

$$L_{\mathrm{N}} = L_m\left[A^\mu\right] + L^{\alpha}_{\mathrm{GF}'}, \qquad L^{\alpha}_{\mathrm{GF}'} = b \left(\partial_\mu A^\mu + \frac{\alpha}{2} b \right), \tag{11}$$

where $L_m\left[A^\mu\right]$ is the Proca Lagrangian and $L^{\alpha}_{\mathrm{GF}'}$ is the gauge-fixing-like one.[3] For $\alpha = 1$, two-point functions are then

$$\langle A^\mu A^\nu \rangle = \frac{\eta^{\mu\nu}}{\Box - m^2} \delta, \qquad \langle A^\mu b \rangle = \frac{\partial^\mu}{\Box - m^2} \delta, \qquad \langle bb \rangle = -\frac{m^2}{\Box - m^2} \delta, \tag{12}$$

which show the massless singularities have disappeared. Note that the Nakanishi-Lautrup field $b(x)$ is a ghost for $m \neq 0$. Physical states are picked out by the condition

$$b^{(+)}(x) |\mathrm{phys}\rangle = 0, \tag{13}$$

where $b^{(+)}(x)$ denotes the positive frequency part of $b(x)$. In the massive case, we can introduce the field

$$\tilde{\chi}(p) \stackrel{\mathrm{d}}{\equiv} \chi(p) + \mathrm{i} \frac{1}{m^2} b(p), \tag{14}$$

and find that the set of the fields $\{\varphi^i(p)\ (i = 1,2),\ \tilde{\chi}(p)\}$ is physical and the ghost $b(p)$ is unphysical. In the massless case, on the other hand, the set of the fields $\{\varphi^i(p)\ (i = 1,2),\ b(p)\}$ becomes physical and the field $\chi(p)$ becomes unphysical. Note that the field b is zero-normed in this case. That means b is not observable, there remaining only two observable degrees of freedom.

[3] The Proca Lagrangian does not show any gauge invariance. Therefore, adding the term $L^{\alpha}_{\mathrm{GF}'}$ has nothing to do with gauge-fixing procedure.

2.4. Massive vector: BRS model

The BRS model Lagrangian is constructed as follows: introduce an auxiliary scalar field θ, promote the Proca Lagrangian (8) to a gauge invariant one by replacing A^μ with $A^\mu - \frac{1}{m}\partial^\mu\theta$, and append a certain GF+FP Lagrangian. The total Lagrangian is given as

$$\begin{aligned} L_{\mathrm{BRS}} &= L_m\left[A^\mu - \frac{1}{m}\partial^\mu\theta\right] + L^{m\alpha}_{\mathrm{GF+FP}} \\ &= L_m\left[A^\mu\right] - m\theta\partial_\mu A^\mu - \frac{1}{2}\partial_\mu\theta\partial^\mu\theta + L^{m\alpha}_{\mathrm{GF+FP}}. \end{aligned} \tag{15}$$

For the GF+FP Lagrangian, we adopt here the following massive type instead of the massless one (2):

$$\begin{aligned} L^{m\alpha}_{\mathrm{GF+FP}} &= b\left(\partial_\mu A^\mu - m\theta + \frac{\alpha}{2}b\right) + \mathrm{i}\bar{c}\left(\Box - m^2\right)c \\ &= -\mathrm{i}\delta\left[\bar{c}\left(\partial_\mu A^\mu - m\theta + \frac{\alpha}{2}b\right)\right]. \end{aligned} \tag{16}$$

The choice of (16) gives simple pole structure to two-point functions and makes it easy to investigate particle contents of physical states. The BRS transformation which keeps the theory invariant is

$$\delta A_\mu = \partial_\mu c, \qquad \delta\theta = mc, \qquad \delta\bar{c} = \mathrm{i}b. \tag{17}$$

For $\alpha = 1$, two-point functions are calculated to give

$$\begin{aligned} \langle A^\mu A^\nu\rangle = \frac{\eta^{\mu\nu}}{\Box - m^2}\delta, \quad \langle A^\mu b\rangle = \frac{\partial^\mu}{\Box - m^2}\delta, \quad \langle bb\rangle = 0, \quad \langle c\bar{c}\rangle = -\mathrm{i}\frac{1}{\Box - m^2}\delta, \\ \langle A^\mu\theta\rangle = 0, \quad \langle b\theta\rangle = \frac{m}{\Box - m^2}\delta, \quad \langle\theta\theta\rangle = \frac{1}{\Box - m^2}\delta. \end{aligned} \tag{18}$$

Physical states are defined as

$$Q_{\mathrm{B}}|\mathrm{phys}\rangle = 0. \tag{19}$$

If we introduce the field

$$\tilde{\theta}(p) \overset{\mathrm{d}}{\equiv} \theta(p) + \mathrm{i}m\chi(p) + \frac{m}{(p^3)^2}b(p), \tag{20}$$

we can find the following particle contents: $\{\varphi^i(p)\ (i = 1, 2),\ \tilde{\theta}(p)\}$ are BRS singlets and physical; and $\{\chi(p), b(p), c(p), \bar{c}(p)\}$ make up a BRS quartet.

3. Massive tensor: linear theories

3.1. Massless tensor

A massless tensor field is described by the Lagrangian

$$L_0 = \frac{1}{2}h^{\mu\nu}\Lambda_{\mu\nu,\rho\sigma}h^{\rho\sigma} + L^{\alpha}_{\mathrm{GF+FP}} \tag{21}$$

with

$$\Lambda_{\mu\nu,\rho\sigma} \overset{\mathrm{d}}{\equiv} \left(\eta_{\mu\rho}\eta_{\nu\sigma} - \eta_{\mu\nu}\eta_{\rho\sigma}\right)\Box - \left(\eta_{\mu\rho}\partial_\nu\partial_\sigma + \eta_{\nu\sigma}\partial_\mu\partial_\rho\right) + \left(\eta_{\rho\sigma}\partial_\mu\partial_\nu + \eta_{\mu\nu}\partial_\rho\partial_\sigma\right). \tag{22}$$

For the GF+FP Lagrangian, we choose

$$\begin{aligned} L^{\alpha}_{\mathrm{GF+FP}} &= b_\mu\left(\partial_\nu h^{\mu\nu} - \frac{1}{2}\partial^\mu h + \frac{\alpha}{2}b^\mu\right) + \mathrm{i}\bar{c}_\mu\Box\, c^\mu \\ &= -\mathrm{i}\delta\left[\bar{c}_\mu\left(\partial_\nu h^{\mu\nu} - \frac{1}{2}\partial^\mu h + \frac{\alpha}{2}b^\mu\right)\right], \end{aligned} \tag{23}$$

where α is the gauge parameter and h is defined by $h \overset{\mathrm{d}}{\equiv} h^{\mu}_{\mu}$. The theory is invariant under the BRS transformation

$$\delta h^{\mu\nu} = \partial^{\mu} c^{\nu} + \partial^{\nu} c^{\mu}, \qquad \delta \bar{c}_{\mu} = \mathrm{i} b_{\mu}. \tag{24}$$

For $\alpha = \frac{1}{2}$, we have simple forms of two-point functions:

$$\begin{aligned} \langle h^{\mu\nu} h^{\rho\sigma} \rangle &= \frac{1}{\Box} \frac{1}{2} \left(\eta^{\mu\rho} \eta^{\nu\sigma} + \eta^{\mu\sigma} \eta^{\nu\rho} - \eta^{\mu\nu} \eta^{\rho\sigma} \right) \delta, \\ \langle h^{\mu\nu} b^{\rho} \rangle &= \frac{1}{\Box} \left(\eta^{\mu\rho} \partial^{\nu} + \eta^{\nu\rho} \partial^{\mu} \right) \delta, \qquad \langle b^{\mu} b^{\nu} \rangle = 0, \qquad \langle c^{\mu} \bar{c}_{\nu} \rangle = -\mathrm{i} \frac{1}{\Box} \delta^{\mu}_{\nu} \delta. \end{aligned} \tag{25}$$

Physical states are defined by the condition

$$Q_{\mathrm{B}} |\mathrm{phys}\rangle = 0. \tag{26}$$

In the axial coordinate ($p^1 = p^2 = 0,\ p^3 > 0$) we define

$$\begin{aligned} &\phi^1(p) \overset{\mathrm{d}}{\equiv} \frac{1}{2} \left[h^{11}(p) - h^{22}(p) \right], \qquad \phi^2(p) \overset{\mathrm{d}}{\equiv} h^{12}(p), \\ &\chi^0(p) \overset{\mathrm{d}}{\equiv} \frac{1}{2p^0} h^{00}(p), \quad \chi^1(p) \overset{\mathrm{d}}{\equiv} \frac{1}{p^0} h^{01}(p), \quad \chi^2(p) \overset{\mathrm{d}}{\equiv} \frac{1}{p^0} h^{02}(p), \quad \chi^3(p) \overset{\mathrm{d}}{\equiv} \frac{1}{2p^3} h^{33}(p). \end{aligned} \tag{27}$$

Particle contents are then as follows: $\{\phi^i(p)\ (i = 1, 2)\}$ are BRS singlets and physical; $\{\chi^{\mu}(p), b_{\mu}(p), c^{\mu}(p), \bar{c}_{\mu}(p)\}$ form BRS quartets.

3.2. Massive tensor: naive model

Naive introduction of mass is carried out through the Lagrangian

$$L^a_m \left[h^{\mu\nu} \right] = \frac{1}{2} h^{\mu\nu} \Lambda_{\mu\nu,\rho\sigma} h^{\rho\sigma} - \frac{m^2}{2} \left(h^{\mu\nu} h_{\mu\nu} - a h^2 \right). \tag{28}$$

The parameter a has two choices of interest: $a = 1$ and $a = \frac{1}{2}$, corresponding to the PT model and the ASG one respectively. In the case of $a = 1$, the Lagrangian has the Fierz-Pauli type mass term, and gives the field equations

$$\begin{cases} \left(\Box - m^2 \right) h^{\mu\nu} = 0, \\ \partial_{\nu} h^{\mu\nu} = 0, \\ h = 0. \end{cases} \tag{29}$$

Therefore, this model does describe an ordinary massive tensor field of five degrees of freedom. The two-point functions

$$\begin{aligned} \langle h^{\mu\nu} h^{\rho\sigma} \rangle = \frac{1}{\Box - m^2} \Bigg\{ & \frac{1}{2} \left(\eta^{\mu\rho} \eta^{\nu\sigma} + \eta^{\mu\sigma} \eta^{\nu\rho} - \eta^{\mu\nu} \eta^{\rho\sigma} \right) \\ & - \frac{1}{2m^2} \left(\eta^{\mu\rho} \partial^{\nu} \partial^{\sigma} + \eta^{\mu\sigma} \partial^{\nu} \partial^{\rho} + \eta^{\nu\rho} \partial^{\mu} \partial^{\sigma} + \eta^{\nu\sigma} \partial^{\mu} \partial^{\rho} \right) \\ & + \frac{2}{3} \left(\frac{1}{2} \eta^{\mu\nu} + \frac{\partial^{\mu} \partial^{\nu}}{m^2} \right) \left(\frac{1}{2} \eta^{\rho\sigma} + \frac{\partial^{\rho} \partial^{\sigma}}{m^2} \right) \Bigg\} \delta \end{aligned} \tag{30}$$

show the fourth-order massless singularities. In the case of $a = \frac{1}{2}$, on the other hand, field equations reduce to

$$\begin{cases} \left(\Box - m^2 \right) h^{\mu\nu} = 0, \\ \partial_{\nu} h^{\mu\nu} - \frac{1}{2} \partial^{\mu} h = 0. \end{cases} \tag{31}$$

The number of physical degrees of freedom of this model is six; five coresoponds to a massive tensor, and one is to an additional scalar ghost field. The two-point functions

$$\langle h^{\mu\nu} h^{\rho\sigma}\rangle = \frac{1}{\Box - m^2}\left\{\frac{1}{2}\left(\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho} - \eta^{\mu\nu}\eta^{\rho\sigma}\right) - \frac{1}{2m^2}\left(\eta^{\mu\rho}\partial^\nu\partial^\sigma + \eta^{\mu\sigma}\partial^\nu\partial^\rho + \eta^{\nu\rho}\partial^\mu\partial^\sigma + \eta^{\nu\sigma}\partial^\mu\partial^\rho\right)\right\}\delta \tag{32}$$

contain only the second-order massless singularities in this case.

3.3. ASG-type massive tensor: BRS model

In order to promote the ASG model to a BRS invariant one, introduce an auxiliary vector field θ^μ, replace $h^{\mu\nu}$ with the combination $h^{\mu\nu} - \frac{1}{m}(\partial^\mu\theta^\nu - \partial^\nu\theta^\mu)$ in the Lagrangian (28) with $a = \frac{1}{2}$, and append a certain GF+FP Lagrangian. The total Lagrangian is

$$\begin{aligned} L_{\mathrm{BRS}}^{a=\frac{1}{2}} &= L_m^{a=\frac{1}{2}}\left[h^{\mu\nu} - \frac{1}{m}\left(\partial^\mu\theta^\nu + \partial^\nu\theta^\mu\right)\right] + L_{\mathrm{GF+FP}}^{m\alpha} \\ &= L_m^{a=\frac{1}{2}}\left[h^{\mu\nu}\right] - 2m\theta_\mu\left(\partial_\nu h^{\mu\nu} - \frac{1}{2}\partial^\mu h\right) - \partial_\mu\theta_\nu\partial^\mu\theta^\nu + L_{\mathrm{GF+FP}}^{m\alpha}. \end{aligned} \tag{33}$$

Following the BRS procedure for the massive vector in section 2.4, we adopt the following GF+FP Lagrangian of massive type:

$$\begin{aligned} L_{\mathrm{GF+FP}}^{m\alpha} &= b_\mu\left(\partial_\nu h^{\mu\nu} - \frac{1}{2}\partial^\mu h - m\theta^\mu + \frac{\alpha}{2}b^\mu\right) + \mathrm{i}\bar{c}_\mu\left(\Box - m^2\right)c^\mu \\ &= -\mathrm{i}\delta\left[\bar{c}_\mu\left(\partial_\nu h^{\mu\nu} - \frac{1}{2}\partial^\mu h - m\theta^\mu + \frac{\alpha}{2}b^\mu\right)\right]. \end{aligned} \tag{34}$$

The BRS transformation

$$\delta h^{\mu\nu} = \partial^\mu c^\nu + \partial^\nu c^\mu, \quad \delta\theta^\mu = mc^\mu, \quad \delta\bar{c}_\mu = \mathrm{i}b_\mu \tag{35}$$

keeps the system invariant. For $\alpha = \frac{1}{2}$, two-point functions are calculated as

$$\begin{gathered} \langle h^{\mu\nu}h^{\rho\sigma}\rangle = \frac{1}{\Box - m^2}\,\frac{1}{2}\left(\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho} - \eta^{\mu\nu}\eta^{\rho\sigma}\right)\delta, \\ \langle h^{\mu\nu}b^\rho\rangle = \frac{1}{\Box - m^2}\left(\eta^{\mu\rho}\partial^\nu + \eta^{\nu\rho}\partial^\mu\right)\delta, \quad \langle b^\mu b^\nu\rangle = 0, \quad \langle c^\mu\bar{c}_\nu\rangle = -\mathrm{i}\frac{1}{\Box - m^2}\,\delta^\mu_\nu\delta, \\ \langle h^{\mu\nu}\theta^\rho\rangle = 0, \quad \langle b^\mu\theta^\nu\rangle = \frac{m}{\Box - m^2}\eta^{\mu\nu}\delta, \quad \langle\theta^\mu\theta^\nu\rangle = \frac{1}{2}\,\frac{1}{\Box - m^2}\eta^{\mu\nu}\delta. \end{gathered} \tag{36}$$

Physical states are defined by

$$Q_{\mathrm{B}}|\mathrm{phys}\rangle = 0. \tag{37}$$

Let us introduce a field $\tilde{\theta}^\mu(p)$ as the combination

$$\tilde{\theta}^\mu(p) \overset{\mathrm{d}}{\equiv} \theta^\mu(p) + \mathrm{i}m\chi^\mu(p) + m\omega^{\mu\nu}b_\nu(p) \tag{38}$$

with the matrix $\omega^{\mu\nu}$ having the components

$$\begin{cases} \omega^{00} = \dfrac{1}{8(p^0)^2}, \quad \omega^{03} = \omega^{30} = \dfrac{1}{8p^0p^3}, \quad \omega^{33} = \dfrac{1}{8(p^3)^2}, \\ \omega^{11} = \omega^{22} = -\dfrac{1}{2(p^0)^2}, \quad \text{the others} = 0. \end{cases} \tag{39}$$

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Then we find the following particle contents: $\{\phi^i(p)\ (i=1,2),\ \tilde{\theta}^\mu(p)\}$ are BRS singlets and physical; $\{\chi^\mu(p), b_\mu(p), c^\mu(p), \bar{c}_\mu(p)\}$ make up BRS quartets. Note that there remains a ghost $\tilde{\theta}^0(p)$ in the physical states. Nonlinear completion of this model including a possible mechanism of killing the ghost was reported at QTS-4 [6] (See also [5].).

3.4. PT-type massive tensor: BRS+Nakanishi model

We have seen in section 3.2 that the PT model of massive tensor shows fourth-order massless singularities in two-point functions. Those singularities cannot be removed by such simple application of the BRS procedure as done in section 3.3. So we invoke the Nakanishi procedure in addition to the BRS one. For the Lagrangian, we adopt the following form:

$$\begin{aligned} L^{a=1}_{\mathrm{BRS+N}} &= L^{a=1}_m\left[h^{\mu\nu} - \frac{1}{m}\left(\partial^\mu\theta^\nu + \partial^\nu\theta^\mu\right)\right] + L^{m\alpha}_{\mathrm{GF+FP}} + L^{\beta}_{\mathrm{GF}'} \\ &= L^{a=1}_m\left[h^{\mu\nu}\right] - 2m\theta_\mu\left(\partial_\nu h^{\mu\nu} - \partial^\mu h\right) - \frac{1}{2}\left(\partial^\mu\theta^\nu - \partial^\nu\theta^\mu\right)^2 + L^{m\alpha}_{\mathrm{GF+FP}} + L^{\beta}_{\mathrm{GF}'}. \end{aligned} \tag{40}$$

Here the first and the second terms on the right side of the first line are from the BRS procedure, and the third term $L^{\beta}_{\mathrm{GF}'}$ represents the gauge-fixing-like term in the Nakanishi procedure. For $L^{\beta}_{\mathrm{GF}'}$, we also choose massive type of the following form:

$$L^{\beta}_{\mathrm{GF}'} = b\left(\partial_\mu\theta^\mu - \frac{m}{2}h + \frac{\beta}{2}b\right) \tag{41}$$

with the second parameter β. Assuming the Nakanishi-Lautrup field b is BRS invariant, the total Lagrangian is invariant under the BRS transformation

$$\delta h^{\mu\nu} = \partial^\mu c^\nu + \partial^\nu c^\mu, \quad \delta\theta^\mu = mc^\mu, \quad \delta\bar{c}_\mu = \mathrm{i}b_\mu, \quad \delta b = 0. \tag{42}$$

Two-point functions show simple pole structure for $\alpha = \beta = \frac{1}{2}$ as follows:

$$\begin{gathered} \langle h^{\mu\nu}h^{\rho\sigma}\rangle = \frac{1}{\Box - m^2}\,\frac{1}{2}\left(\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho} - \eta^{\mu\nu}\eta^{\rho\sigma}\right)\delta, \\ \langle h^{\mu\nu}b^\rho\rangle = \frac{1}{\Box - m^2}\left(\eta^{\mu\rho}\partial^\nu + \eta^{\nu\rho}\partial^\mu\right)\delta, \quad \langle b^\mu b^\nu\rangle = 0, \quad \langle c^\mu\bar{c}_\nu\rangle = -\mathrm{i}\frac{1}{\Box - m^2}\,\delta^\mu_\nu\delta, \\ \langle h^{\mu\nu}\theta^\rho\rangle = 0, \quad \langle b^\mu\theta^\nu\rangle = \frac{m}{\Box - m^2}\eta^{\mu\nu}\delta, \quad \langle\theta^\mu\theta^\nu\rangle = \frac{1}{2}\,\frac{1}{\Box - m^2}\eta^{\mu\nu}\delta, \\ \langle h^{\mu\nu}b\rangle = -\frac{m}{\Box - m^2}\eta^{\mu\nu}\delta, \quad \langle\theta^\mu b\rangle = \frac{\partial^\mu}{\Box - m^2}\delta, \quad \langle b^\mu b\rangle = 0, \quad \langle bb\rangle = -\frac{6m^2}{\Box - m^2}\delta. \end{gathered} \tag{43}$$

The fourth-order massless singularities have been driven away indeed. Note that, as seen from the last equation of (43), the field b is a ghost for $m \neq 0$. This is the same situation as in the case of massive vector in section 2.3. Physical states are picked out by two conditions of the BRS type and the Nakanishi type:

$$Q_{\mathrm{B}}|\mathrm{phys}\rangle = 0, \qquad b^{(+)}(x)|\mathrm{phys}\rangle = 0. \tag{44}$$

In order to investigate the particle contents, we introduce the following quantities:

$$\varphi^1(p) \overset{\mathrm{d}}{\equiv} \tilde{\theta}^1(p), \qquad \varphi^2(p) \overset{\mathrm{d}}{\equiv} \tilde{\theta}^2(p), \qquad \chi(p) \overset{\mathrm{d}}{\equiv} \frac{1}{p^3}\tilde{\theta}^3(p). \tag{45}$$

For the massive case, we can introduce the combination

$$\tilde{\chi}(p) \stackrel{\mathrm{d}}{\equiv} \chi(p) + \mathrm{i}\,\frac{1}{6}\left(\frac{1}{m^2} + \frac{1}{2(p^3)^2}\right) b(p). \tag{46}$$

Particle contents are then: $\{\phi^i(p),\ \varphi^i(p)\ (i=1,2),\ \tilde{\chi}(p)\}$ are BRS singlets and physical; $b(p)$ is a BRS singlet but unphysical (*ghost*); $\{\chi^\mu(p), b_\mu(p), c^\mu(p), \bar{c}_\mu(p)\}$ constitute BRS quartets. For the massless case, on the other hand, we cannot define a field like $\tilde{\chi}$. In this case, we find the following particle contents: $\{\phi^i(p),\ \varphi^i(p)\ (i=1,2),\ b(p)\}$ are BRS singlets and physical; $\chi(p)$ is a BRS singlet but unphysical; $\{\chi^\mu(p), b_\mu(p), c^\mu(p), \bar{c}_\mu(p)\}$ form BRS quartets. Note again that in the massless case, b is physical but unobservable because it is zero-normed. From now on, we focus on the model described by the Lagrangian (40) with $\alpha = \beta = \frac{1}{2}$.

4. Massive tensor: nonlinear completion

4.1. Nonlinear BRS transformation

To study nonlinear theories we introduce the metric $g_{\mu\nu}$ and the tetrad $e_k{}^\mu$ through

$$g_{\mu\nu} \stackrel{\mathrm{d}}{\equiv} \eta_{\mu\nu} - \kappa h_{\mu\nu}, \qquad e_k{}^\mu e^{k\nu} = g^{\mu\nu} \tag{47}$$

with the gravitational constant κ. The linear BRS transformation (42) is extended to its nonlinear form:

$$\begin{cases} \delta e_k{}^\mu = \kappa\left(\partial_\rho c^\mu \cdot e_k{}^\rho - c^\rho \partial_\rho e_k{}^\mu\right), \\ \delta\theta^\mu = mc^\mu - \kappa c^\rho \partial_\rho \theta^\mu, \\ \delta c^\mu = -\kappa c^\rho \partial_\rho c^\mu, \\ \delta \bar{c}_\mu = \mathrm{i} b_\mu, \\ \delta b = -\kappa c^\rho \partial_\rho b. \end{cases} \tag{48}$$

Basic quantities invariant under the nonlinear BRS transformation can be constructed as

$$E_k{}^\mu \stackrel{\mathrm{d}}{\equiv} e_k{}^\mu - \frac{\kappa}{m} e_k{}^\rho \partial_\rho \theta^\mu, \tag{49}$$

$$G^{\mu\nu} \stackrel{\mathrm{d}}{\equiv} E_k{}^\mu E^{k\nu} = g^{\mu\nu} - \frac{\kappa}{m}\left(g^{\rho\mu}\partial_\rho\theta^\nu + g^{\rho\nu}\partial_\rho\theta^\mu\right) + \left(\frac{\kappa}{m}\right)^2 g^{\rho\sigma}\partial_\rho\theta^\mu\partial_\sigma\theta^\nu. \tag{50}$$

In fact they behave as scalars under the transformation (48):

$$\delta E_k{}^\mu = -\kappa c^\rho \partial_\rho E_k{}^\mu, \qquad \delta G^{\mu\nu} = -\kappa c^\rho \partial_\rho G^{\mu\nu}. \tag{51}$$

Possible Lagrangians are therefore of the form

$$L = \sqrt{-g}\, F\left(E_k{}^\mu, b\right), \tag{52}$$

where F is an arbitrary function. The action is indeed invariant, because such Lagrangian as (52) is transformed as

$$\delta L = -\kappa \partial_\mu \left(c^\mu L\right). \tag{53}$$

4.2. Nonlinear Lagrangian

We require for the Lagrangian to be at most quadratic in ${E_k}^\mu$ *and* to reduce to $L^{a=1}_{\mathrm{BRS+N}}$ in the flat-spacetime limit ($\kappa \to 0$). Then we have the following form consisting of four terms:

$$L = \tilde{L}_m + \gamma \tilde{L}_\mathrm{R} + \tilde{L}^\alpha_{\mathrm{GF+FP}} + \tilde{L}^\beta_{\mathrm{`GF'}}, \tag{54}$$

with an arbitrary real number γ. These terms are given by

$$\tilde{L}_m = \frac{1}{2\kappa^2}\sqrt{-g}\left\{R + \frac{m^2}{2}\left[6 - G^{\mu\nu}\eta_{\mu\nu} - \left({E_k}^\mu \delta^k_\mu\right)^2 + 2{E_k}^\mu \delta^l_\mu {E_l}^\nu \delta^k_\nu\right]\right\}, \tag{55}$$

$$\tilde{L}_\mathrm{R} = \frac{m^2}{\kappa^2}\sqrt{-g}\left[-3 + 2{E_k}^\mu \delta^k_\mu - \frac{1}{2}\left({E_k}^\mu \delta^k_\mu\right)^2 + \frac{1}{2}{E_k}^\mu \delta^l_\mu {E_l}^\nu \delta^k_\nu\right], \tag{56}$$

$$\begin{aligned}\tilde{L}^\alpha_{\mathrm{GF+FP}} &= -\mathrm{i}\delta\left[\bar{c}_\mu\left(\frac{1}{\kappa}\partial_\nu \tilde{g}^{\mu\nu} - m\theta^\mu + \frac{\alpha}{2}\eta^{\mu\nu}b_\nu\right)\right] \\ &= b_\mu\left(\frac{1}{\kappa}\partial_\nu \tilde{g}^{\mu\nu} - m\theta^\mu + \frac{\alpha}{2}\eta^{\mu\nu}b_\nu\right) + \mathrm{i}\bar{c}_\mu\left(\partial_\nu {D^{\mu\nu}}_\rho - m^2\delta^\mu_\rho\right)c^\rho + \mathrm{i}\kappa m \bar{c}_\mu c^\rho \partial_\rho \theta^\mu,\end{aligned} \tag{57}$$

and

$$\tilde{L}^\beta_{\mathrm{`GF'}} = \sqrt{-g}\, b\left[\frac{m}{\kappa}\left(\delta^\mu_k - {E_k}^\mu\right)\delta^k_\mu + \frac{\beta}{2}b\right], \tag{58}$$

where we have used the definitions

$$\tilde{g}^{\mu\nu} \stackrel{\mathrm{d}}{\equiv} \sqrt{-g}\, g^{\mu\nu}, \tag{59}$$

$${D^{\mu\nu}}_\rho \stackrel{\mathrm{d}}{\equiv} \tilde{g}^{\mu\sigma}\delta^\nu_\rho \partial_\sigma + \tilde{g}^{\nu\sigma}\delta^\mu_\rho \partial_\sigma - \tilde{g}^{\mu\nu}\partial_\rho - \left(\partial_\rho \tilde{g}^{\mu\nu}\right). \tag{60}$$

We can easily verify that the main part of the Lagrangian $\tilde{L}_m + \tilde{L}^\alpha_{\mathrm{GF+FP}} + \tilde{L}^\beta_{\mathrm{`GF'}}$ goes to $L^{a=1}_{\mathrm{BRS+N}}$ and the redundant part $\tilde{L}_\mathrm{R}$ becomes null in the flat spacetime limit, $\kappa \to 0$.

4.3. Physical states

In the linear theory, physical states are picked out by the two conditions, the BRS-type one $Q_\mathrm{B}|\mathrm{phys}\rangle = 0$ and the Nakanishi-type one $b^{(+)}(x)|\mathrm{phys}\rangle = 0$, as stated in section 3.4. Going to the nonlinear theory, the BRS-type condition takes over the same form:

$$Q_\mathrm{B}|\mathrm{phys}\rangle = 0. \tag{61}$$

However, it is not an easy task to find a nonlinear version of the Nakanishi-type condition:

$$\text{“}b^{(+)}(x)\text{”}|\mathrm{phys}\rangle = 0. \tag{62}$$

The problem is how to define “$b^{(+)}(x)$” in the nonlinear case. In the linear case, $b(x)$ satisfies the free field equation $\left(\Box - m^2\right)b(x) = 0$. This fact allows to impose the physical-state condition of the Nakanishi type $b^{(+)}(x)|\mathrm{phys}\rangle = 0$. In the nonlinear case, however, $b(x)$ obeys some nonlinear equation. Setting up an auxiliary condition consistently in that case is still unsolved.

5. Summary and discussion

We have presented a possible nonlinear completion of massive gravity of the Fierz-Pauli type. Physical implications of this model are under study.

This model has the smooth massless ($m \to 0$) as well as the smooth flat-spacetime ($\kappa \to 0$) limits. In the flat-spacetime limit, it reduces to the BRS+Nakanishi extension of the PT (Fierz-Pauli) model.

Finding a nonlinear version of the Nakanishi-type physical-state condition is still unsettled.

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