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Fermion-fermion and boson-boson amplitudes: surprising similarities

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Abstract. Amplitudes for fermion-fermion, boson-boson and fermion-boson interactions are calculated in the second order of perturbation theory in the Lobachevsky space. An essential ingredient of the model is the Weinberg's $2(2j+1)-$ component formalism for describing a particle of spin j. The boson-boson amplitude is then compared with the two-fermion amplitude obtained long ago by Skachkov on the basis of the Hamiltonian formulation of quantum field theory on the mass hyperboloid, $p_0^2 - \mathbf{p}^2 = M^2$, proposed by Kadyshevsky. The parametrization of the amplitudes by means of the momentum transfer in the Lobachevsky space leads to same spin structures in the expressions of $T-$ matrices for the fermion case and the boson case. However, certain differences are found. Possible physical applications are discussed.

1. Introduction

The problem of correct description of quarkonium (the bound state of quark and antiquark – the mesons) and of barions were hot topics since long ago. However, we are now faced at the correct description of a bound state of spin-1 particles (and higher spins). Instead of developing other methods we suggest to think about the general methods of describing higher-spin particles (and their bound states) on an equal footing with fermions. The pioneer ideas have been proposed in refs. [8, 10]. The amplitudes in the Lobachevsky space are calculated in the next Section.

2. Amplitudes

The scattering amplitude for the two-fermion interaction had been obtained in the 3-momentum Lobachevsky space [1] in the second order of perturbation theory long ago [2a,Eq.(31)]:

$$T_V^{(2)}(\mathbf{k}(-)\mathbf{p},\mathbf{p}) = -g_v^2\frac{4m^2}{\mu^2+4æ^2} - 4g_v^2\frac{(\sigma_\mathbf{1}\mathbf{æ})(\sigma_\mathbf{2}\mathbf{æ})-(\sigma_\mathbf{1}\sigma_\mathbf{2})\mathbf{æ^2}}{\mu^2+4æ^2} -$$
$$- \quad \frac{8g_v^2 p_0 æ_0}{m^2}\,\frac{i\sigma_\mathbf{1}[\mathbf{p}\times\mathbf{æ}]+\mathbf{i}\sigma_\mathbf{2}[\mathbf{p}\times\mathbf{æ}]}{\mu^2+4æ^2} - \frac{8g_v^2}{m^2}\,\frac{p_0^2æ_0^2+2p_0æ_0(\mathbf{p}\cdot\mathbf{æ})-m^4}{\mu^2+4æ^2} -$$
$$- \quad \frac{8g_v^2}{m^2}\,\frac{(\sigma_\mathbf{1}\mathbf{p})(\sigma_\mathbf{1}\mathbf{æ})(\sigma_\mathbf{2}\mathbf{p})(\sigma_\mathbf{2}\mathbf{æ})}{\mu^2+4æ^2}\quad, \tag{1}$$

g_v is the coupling constant. The additional term (the last one) has usually *not* been taken into account in the earlier Breit-like calculations of two-fermion interactions. This consideration is based on use of the formalism of separation of the Wigner rotations and parametrization of currents by means of the Pauli-Lubanski vector, developed long ago [3]. The quantities

$$æ_0 = \sqrt{\frac{m(\Delta_0 + m)}{2}} \quad , \quad \mathbf{æ} = \mathbf{n}_\Delta \sqrt{\frac{m(\Delta_0 - m)}{2}}$$

are the components of the 4-vector of a momentum half-transfer. This concept is closely connected with a notion of the half-velocity of a particle [4]. The 4-vector Δ_μ:

$$\mathbf{\Delta} = \mathbf{\Lambda_p^{-1} k} = \mathbf{k}(-)\mathbf{p} = \mathbf{k} - \frac{\mathbf{p}}{\mathbf{m}}(\mathbf{k_0} - \frac{\mathbf{k}\cdot\mathbf{p}}{\mathbf{p_0}+\mathbf{m}}) , \tag{2}$$

$$\Delta_0 = (\Lambda_p^{-1} k)_0 = (k_0 p_0 - \mathbf{k}\cdot\mathbf{p})/m = \sqrt{m^2 + \mathbf{\Delta^2}} \tag{3}$$

can be regarded as the momentum transfer vector in the Lobachevsky space instead of the vector $\mathbf{q} = \mathbf{k} - \mathbf{p}$ in the Euclidean space.[1] This amplitude had been used for physical applications in the framework of the Kadyshevsky's version of the quasipotential approach [1, 2].

On the other hand, in ref. [8] an attractive $2(2j+1)$ component formalism for describing particles of higher spins has been proposed. As opposed to the Proca 4-vector potentials which transform according to the $(\frac{1}{2}, \frac{1}{2})$ representation of the Lorentz group, the $2(2j+1)$ component functions are constructed via the representation $(j, 0) \oplus (0, j)$ in the Weinberg formalism. This description of higher spin particles is on an equal footing to the description of the Dirac spinor particle, whose field function transforms according to the $(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})$ representation. The $2(2j+1)$- component analogues of the Dirac functions in the momentum space are

$$\mathcal{U}(\mathbf{p}) = \sqrt{\frac{M}{2}} \begin{pmatrix} D^J\left(\alpha(\mathbf{p})\right)\xi_\sigma \\ D^J\left(\alpha^{-1\,\dagger}(\mathbf{p})\right)\xi_\sigma \end{pmatrix} \quad , \tag{4}$$

for the positive-energy states; and[2]

$$\mathcal{V}(\mathbf{p}) = \sqrt{\frac{M}{2}} \begin{pmatrix} D^J\left(\alpha(\mathbf{p})\Theta_{[1/2]}\right)\xi^*_\sigma \\ D^J\left(\alpha^{-1\,\dagger}(\mathbf{p})\Theta_{[1/2]}\right)(-1)^{2J}\xi^*_\sigma \end{pmatrix} \quad , \tag{5}$$

for the negative-energy states, ref. [5, p.107], with the following notations being used:

$$\alpha(\mathbf{p}) = \frac{p_0 + M + (\sigma\cdot\mathbf{p})}{\sqrt{2M(p_0+M)}}, \quad \Theta_{[1/2]} = -i\sigma_2 \quad . \tag{6}$$

[1] I keep a notation and a terminology of ref. [2]. In such an approach all particles (even in the intermediate states) are on the mass shell (but, spurious particles present). The technique of construction of the Wigner matrices $D^J(A)$ can be found in ref. [5, p.51,70,English edition]. In general, for each particle in interaction one should understand under 4-momenta p_i^μ and k_i^μ $(i = 1, 2)$ their covariant generalizations, $\breve{p}_i^\mu$, $\breve{k}_i^\mu$, *e.g.*, refs. [3, 6, 7]:

$$\breve{\mathbf{k}} = (\mathbf{\Lambda}_{\mathcal{P}}^{-1}\mathbf{k}) = \mathbf{k} - \frac{\mathcal{P}}{\sqrt{\mathcal{P}^2}}\left(\mathbf{k_0} - \frac{\mathcal{P}\cdot\mathbf{k}}{\mathcal{P}_0 + \sqrt{\mathcal{P}^2}}\right) \quad ,$$

$$\breve{k}_0 = (\Lambda_{\mathcal{P}}^{-1}k)_0 = \sqrt{m^2 + \breve{\mathbf{k}}^{\,2}},$$

with $\mathcal{P} = p_1 + p_2$, $\Lambda_{\mathcal{P}}^{-1}\mathcal{P} = (\mathcal{M}, \mathbf{0})$. However, we omit the circles above the momenta in the following, because in the case under consideration we do not miss physical information if we use the corresponding quantities in c.m.s., $\mathbf{p}_1 = -\mathbf{p}_2 = \mathbf{p}$ and $\mathbf{k}_1 = -\mathbf{k}_2 = \mathbf{k}$.

[2] When setting $\mathcal{V}(\mathbf{p}) = S^c_{[1]}\mathcal{U}(\mathbf{p}) \equiv \mathcal{C}_{[1]}\mathcal{K}\mathcal{U}(\mathbf{p}) \sim \gamma_5\mathcal{U}(\mathbf{p})$, like the Dirac $j = 1/2$ case we have other type of theories [9, 10, 11]. $S^c_{[1]}$ is the charge conjugation operator for $j = 1$. $\mathcal{K}$ is the operation of complex conjugation.

These functions obey the orthonormalization equations, $\mathcal{U}^\dagger(\mathbf{p})\gamma_{00}\,\mathcal{U}(\mathbf{p}) = M$, M is the mass of the $2(2j+1)-$ particle. The similar normalization condition exists for $\mathcal{V}(\mathbf{p})$, the functions of "negative-energy states".

For instance, in the case of spin $j=1$, one has

$$D^1\left(\alpha(\mathbf{p})\right) = 1 + \frac{(\mathbf{J}\cdot\mathbf{p})}{M} + \frac{(\mathbf{J}\cdot\mathbf{p})^2}{M(p_0+M)} \quad , \tag{7}$$

$$D^1\left(\alpha^{-1\,\dagger}(\mathbf{p})\right) = 1 - \frac{(\mathbf{J}\cdot\mathbf{p})}{M} + \frac{(\mathbf{J}\cdot\mathbf{p})^2}{M(p_0+M)} \quad , \tag{8}$$

$$D^1\left(\alpha(\mathbf{p})\Theta_{[1/2]}\right) = \left[1 + \frac{(\mathbf{J}\cdot\mathbf{p})}{M} + \frac{(\mathbf{J}\cdot\mathbf{p})^2}{M(p_0+M)}\right]\Theta_{[1]} \quad , \tag{9}$$

$$D^1\left(\alpha^{-1\,\dagger}(\mathbf{p})\Theta_{[1/2]}\right) = \left[1 - \frac{(\mathbf{J}\cdot\mathbf{p})}{M} + \frac{(\mathbf{J}\cdot\mathbf{p})^2}{M(p_0+M)}\right]\Theta_{[1]} \quad , \tag{10}$$

($\Theta_{[1/2]}, \Theta_{[1]}$ are the Wigner operators for spin 1/2 and 1, respectively). Recently, much attention has been paid to this formalism [12].

In refs. [5, 8, 13, 14, 15] the Feynman diagram technique was discussed in the above-mentioned six-component formalism for particles of spin $j=1$. The Lagrangian is the following one:[3]

$$\begin{aligned}\mathcal{L} = {} & \nabla_\mu\overline{\Psi}(x)\Gamma_{\mu\nu}\nabla_\nu\Psi(x) - M^2\overline{\Psi}(x)\Psi(x) - \frac{1}{4}F_{\mu\nu}F_{\mu\nu} + \\ + {} & \frac{e\lambda}{12}F_{\mu\nu}\overline{\Psi}(x)\gamma_{5,\mu\nu}\Psi(x) + \frac{e\kappa}{12M^2}\partial_\alpha F_{\mu\nu}\overline{\Psi}(x)\gamma_{6,\mu\nu,\alpha\beta}\nabla_\beta\Psi(x)\,.\end{aligned} \tag{11}$$

In the above formula we have $\nabla_\mu = -i\partial_\mu \mp eA_\mu$; $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field tensor; A_μ is the 4-vector of electromagnetic field; $\overline{\Psi}, \Psi$ are the six-component field functions of the massive $j=1$ Weinberg particle. The following expression has been obtained for the interaction vertex of the particle with the vector potential, ref. [13, 14]:

$$-e\Gamma_{\alpha\beta}(p+k)_\beta - \frac{ie\lambda}{6}\gamma_{5,\alpha\beta}q_\beta + \frac{e\kappa}{6M^2}\gamma_{6,\alpha\beta,\mu\nu}q_\beta q_\mu(p+k)_\nu \quad , \tag{12}$$

where $\Gamma_{\alpha\beta} = \gamma_{\alpha\beta} + \delta_{\alpha\beta}$; $\gamma_{\alpha\beta}$; $\gamma_{5,\alpha\beta}$; $\gamma_{6,\alpha\beta,\mu\nu}$ are the $6\otimes 6$-matrices which have been described in ref. [16, 8]:

$$\gamma_{ij} = \begin{pmatrix} 0 & \delta_{ij}\mathbb{1} - J_iJ_j - J_jJ_i \\ \delta_{ij}\mathbb{1} - J_iJ_j - J_jJ_i & 0\end{pmatrix} \quad , \tag{13}$$

$$\gamma_{i4} = \gamma_{4i} = \begin{pmatrix} 0 & iJ_i \\ -iJ_i & 0\end{pmatrix} \quad , \quad \gamma_{44} = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0\end{pmatrix} \quad , \tag{14}$$

and

$$\gamma_{5,\alpha\beta} = i[\gamma_{\alpha\mu},\gamma_{\beta\mu}]_- \quad , \tag{15}$$

$$\gamma_{6,\alpha\beta,\mu\nu} = [\gamma_{\alpha\mu},\gamma_{\beta\nu}]_+ + 2\delta_{\alpha\mu}\delta_{\beta\nu} - [\gamma_{\beta\mu},\gamma_{\alpha\nu}]_+ - 2\delta_{\beta\mu}\delta_{\alpha\nu} \quad . \tag{16}$$

J_i are the spin matrices for a $j=1$ particle, e is the electron charge, λ and κ correspond to the magnetic dipole moment and the electric quadrupole moment, respectively.

[3] In the following I prefer to use the Euclidean metric because this metric got application in a lot of papers on the $2(2j+1)$ formalism.

In order to obtain the 4-vector current for the interaction of a boson with the external field one can use the known formulas of refs. [2, 3], which are valid for any spin:

$$\mathcal{U}^\sigma(\mathbf{p}) = \mathbf{S_p}\mathcal{U}^\sigma(\mathbf{0}) \quad , \quad \mathbf{S_p^{-1}S_k} = \mathbf{S_{k(-)p}}\cdot\mathbf{I}\otimes\mathbf{D^1}\left\{\mathbf{V^{-1}}(\mathbf{\Lambda_p},\mathbf{k})\right\} \quad , \tag{17}$$

$$W_\mu(\mathbf{p})\cdot D\left\{V^{-1}(\Lambda_p,k)\right\} = D\left\{V^{-1}(\Lambda_p,k)\right\}\cdot\left[W_\mu(\mathbf{k}) + \frac{p_\mu+k_\mu}{M(\Delta_0+M)}p_\nu W_\nu(\mathbf{k})\right], \tag{18}$$

$$k_\mu W_\mu(\mathbf{p})\cdot D\left\{V^{-1}(\Lambda_p,k)\right\} = -D\left\{V^{-1}(\Lambda_p,k)\right\}\cdot p_\mu W_\mu(\mathbf{k}) \quad . \tag{19}$$

W_μ is the Pauli-Lubanski 4-vector of relativistic spin.[4] The matrix $D^{(j=1)}\left\{V^{-1}(\Lambda_p,k)\right\}$ is for spin 1:

$$\begin{aligned} D^{(j=1)}\left\{V^{-1}(\Lambda_p,k)\right\} &= \frac{1}{2M(p_0+M)(k_0+M)(\Delta_0+M)}\Big\{[\mathbf{p}\times\mathbf{k}]^2 + \\ &+ \quad [(p_0+M)(k_0+M)-\mathbf{k}\cdot\mathbf{p}]^2 + 2i\left[(p_0+M)(k_0+M)-\mathbf{k}\cdot\mathbf{p}\right]\{\mathbf{J}\cdot[\mathbf{p}\times\mathbf{k}]\} - \\ &- \quad 2\{\mathbf{J}\cdot[\mathbf{p}\times\mathbf{k}]\}^2\Big\} \quad . \end{aligned} \tag{23}$$

The formulas have been obtained in ref. [15]:

$$\mathbf{S_p^{-1}}\gamma_{\mu\nu}\mathbf{S_p} = \gamma_{44}\left\{\delta_{\mu\nu} - \frac{1}{M^2}\chi_{[\mu\nu]}(\mathbf{p})\otimes\gamma_5 - \frac{2}{M^2}\Sigma_{[\mu\nu]}(\mathbf{p})\right\} \quad , \tag{24}$$

$$\mathbf{S_p^{-1}}\gamma_{\mathbf{5},\mu\nu}\mathbf{S_p} = 6i\left\{-\frac{1}{M^2}\chi_{(\mu\nu)}(\mathbf{p})\otimes\gamma_5 + \frac{2}{M^2}\Sigma_{(\mu\nu)}(\mathbf{p})\right\} \quad , \tag{25}$$

where

$$\chi_{[\mu\nu]}(\mathbf{p}) = p_\mu W_\nu(\mathbf{p}) + p_\nu W_\mu(\mathbf{p}) \quad , \tag{26}$$

$$\chi_{(\mu\nu)}(\mathbf{p}) = p_\mu W_\nu(\mathbf{p}) - p_\nu W_\mu(\mathbf{p}) \quad , \tag{27}$$

$$\Sigma_{[\mu\nu]}(\mathbf{p}) = \frac{1}{2}\left\{W_\mu(\mathbf{p})W_\nu(\mathbf{p}) + W_\nu(\mathbf{p})W_\mu(\mathbf{p})\right\} \quad , \tag{28}$$

$$\Sigma_{(\mu\nu)}(\mathbf{p}) = \frac{1}{2}\left\{W_\mu(\mathbf{p})W_\nu(\mathbf{p}) - W_\nu(\mathbf{p})W_\mu(\mathbf{p})\right\} \quad , \tag{29}$$

lead to the 4- current of a $j=1$ Weinberg particle more directly:[5]

$$j_\mu^{\sigma_p\nu_p}(\mathbf{p},\mathbf{k}) = j_{\mu\,(S)}^{\sigma_p\nu_p}(\mathbf{p},\mathbf{k}) + j_{\mu\,(V)}^{\sigma_p\nu_p}(\mathbf{p},\mathbf{k}) + j_{\mu\,(T)}^{\sigma_p\nu_p}(\mathbf{p},\mathbf{k}) \quad , \tag{35}$$

[4] It is usually introduced because the usual commutation relation for spin is not covariant in the relativistic domain. The Pauli-Lubanski 4-vector is defined as

$$W_\mu(\mathbf{p}) = (\Lambda_\mathbf{p})_\mu^\nu W_\nu(\mathbf{0})\,, \tag{20}$$

where $W_0(\mathbf{0}) = 0$, $\mathbf{W}(\mathbf{0}) = M\sigma/\mathbf{2}$. The properties are:

$$p^\mu W_\mu(\mathbf{p}) = 0\,, \quad W^\mu(\mathbf{p})W_\mu(\mathbf{p}) = -M^2 j(j+1)\,. \tag{21}$$

The explicit form is

$$W_0(\mathbf{p}) = (\mathbf{S}\cdot\mathbf{p})\,, \quad \mathbf{W}(\mathbf{p}) = M\mathbf{S} + \frac{\mathbf{p}(\mathbf{S}\cdot\mathbf{p})}{p_0+M}\,. \tag{22}$$

[5] *Cf.* with a $j=1/2$ case:

$$\mathbf{S_p^{-1}}\gamma_\mu\mathbf{S_k} = \mathbf{S_p^{-1}}\gamma_\mu\mathbf{S_p}\mathbf{S_{k(-)p}}\mathbf{I}\otimes\mathbf{D^{1/2}}\{\mathbf{V^{-1}}(\mathbf{\Lambda_p},\mathbf{k})\}\,, \tag{30}$$

$$j^{\sigma_p\nu_p}_{\mu\,(S)}(\mathbf{p},\mathbf{k}) = -g_S\xi^\dagger_{\sigma_p}\left\{(p+k)_\mu\left(1+\frac{(\mathbf{J}\cdot\mathbf{\Delta})^2}{M(\Delta_0+M)}\right)\right\}\xi_{\nu_p} \quad , \tag{36}$$

$$j^{\sigma_p\nu_p}_{\mu\,(V)}(\mathbf{p},\mathbf{k}) = -g_V\xi^\dagger_{\sigma_p}\left\{(p+k)_\mu+\frac{1}{M}W_\mu(\mathbf{p})(\mathbf{J}\cdot\mathbf{\Delta})-\frac{\mathbf{1}}{\mathbf{M}}(\mathbf{J}\cdot\mathbf{\Delta})\mathbf{W}_\mu(\mathbf{p})\right\}\xi_{\nu_p} \quad , \tag{37}$$

$$j^{\sigma_p\nu_p}_{\mu\,(T)}(\mathbf{p},\mathbf{k}) = -g_T\xi^\dagger_{\sigma_p}\left\{-(p+k)_\mu\frac{(\mathbf{J}\cdot\mathbf{\Delta})^2}{M(\Delta_0+M)}+\right. \tag{38}$$

$$\left.+\frac{1}{M}W_\mu(\mathbf{p})(\mathbf{J}\cdot\mathbf{\Delta})-\frac{\mathbf{1}}{\mathbf{M}}(\mathbf{J}\cdot\mathbf{\Delta})\mathbf{W}_\mu(\mathbf{p})\right\}\xi_{\nu_p} \quad .$$

Next, let me now present the Feynman matrix element corresponding to the diagram of two-boson interaction, mediated by the particle described by the vector potential, in the form [2, 14] (read the remark in the footnote # 1):

$$
\begin{aligned}
&< p_1,p_2;\,\sigma_1,\sigma_2|\hat{T}^{(2)}|k_1,k_2;\,\nu_1,\nu_2 >= \\
&= \sum_{\sigma_{ip},\nu_{ip},\nu_{ik}=-1}^{1} D^{\dagger\;(j=1)}_{\sigma_1\sigma_{1p}}\left\{V^{-1}(\Lambda_{\mathcal{P}},p_1)\right\} D^{\dagger\;(j=1)}_{\sigma_2\sigma_{2p}}\left\{V^{-1}(\Lambda_{\mathcal{P}},p_2)\right\}\times \\
&\times\; T^{\nu_{1p}\nu_{2p}}_{\sigma_{1p}\sigma_{2p}}(\mathbf{k}(-)\mathbf{p},\mathbf{p})D^{(j=1)}_{\nu_{1p}\nu_{1k}}\left\{V^{-1}(\Lambda_{p_1},k_1)\right\}D^{(j=1)}_{\nu_{1k}\nu_1}\left\{V^{-1}(\Lambda_{\mathcal{P}},k_1)\right\}\times \\
&\times\; D^{(j=1)}_{\nu_{2p}\nu_{2k}}\left\{V^{-1}(\Lambda_{p_2},k_2)\right\}D^{(j=1)}_{\nu_{2k}\nu_2}\left\{V^{-1}(\Lambda_{\mathcal{P}},k_2)\right\} \quad ,
\end{aligned} \tag{39}
$$

where

$$T^{\nu_{1p}\nu_{2p}}_{\sigma_{1p}\sigma_{2p}}(\mathbf{k}(-)\mathbf{p},\mathbf{p}) = \xi^\dagger_{\sigma_{1p}}\xi^\dagger_{\sigma_{2p}}T^{(2)}(\mathbf{k}(-)\mathbf{p},\,\mathbf{p})\,\xi_{\nu_{1p}}\xi_{\nu_{2p}} \quad , \tag{40}$$

$\xi^\dagger$, ξ are the 3-analogues of 2-spinors. The calculation of the amplitude (40) yields ($p_0 = -ip_4$, $\Delta_0 = -i\Delta_4$):

$$
\begin{aligned}
\hat{T}^{(2)}(\mathbf{k}(-)\mathbf{p},\mathbf{p}) \;&=\; g^2\left\{\frac{[p_0(\Delta_0+M)+(\mathbf{p}\cdot\mathbf{\Delta})]^2-M^3(\Delta_0+M)}{M^3(\Delta_0-M)}+\right. \\
+\quad &\frac{i(\mathbf{J}_1+\mathbf{J}_2)\cdot[\mathbf{p}\times\mathbf{\Delta}]}{\Delta_0-M}\left[\frac{p_0(\Delta_0+M)+\mathbf{p}\cdot\mathbf{\Delta}}{M^3}\right]+\frac{(\mathbf{J_1}\cdot\mathbf{\Delta})(\mathbf{J_2}\cdot\mathbf{\Delta})-(\mathbf{J_1}\cdot\mathbf{J_2})\mathbf{\Delta^2}}{2M(\Delta_0-M)}- \\
-\quad &\left.\frac{1}{M^3}\frac{\mathbf{J}_1\cdot[\mathbf{p}\times\mathbf{\Delta}]\;\mathbf{J}_2\cdot[\mathbf{p}\times\mathbf{\Delta}]}{\Delta_0-M}\right\} \quad .
\end{aligned} \tag{41}
$$

$$\mathbf{S_p^{-1}}\gamma_\mu\mathbf{S_p} = \frac{\mathbf{1}}{\mathbf{m}}\gamma_\mathbf{0}\left\{\mathbb{1}\otimes\mathbf{p}_\mu+\mathbf{2}\gamma_\mathbf{5}\otimes\mathbf{W}_\mu(\mathbf{p})\right\}, \tag{31}$$

$$\mathbf{S_p^{-1}}\sigma_{\mu\nu}\mathbf{S_p} = -\frac{\mathbf{4}}{\mathbf{m^2}}\mathbb{1}\otimes\mathbf{\Sigma}_{(\mu\nu)}(\mathbf{p})+\frac{\mathbf{2}}{\mathbf{m^2}}\gamma_\mathbf{5}\otimes\chi_{(\mu\nu)}(\mathbf{p})\,. \tag{32}$$

Of course, the product of two Lorentz boosts is *not* a pure Lorentz transformation. It contains the rotation, which describes the Thomas spin precession (the Wigner rotation $V(\Lambda_\mathbf{p},\mathbf{k})\in SU(2)$)). And, then,

$$j^{\sigma_p\nu_p}_\mu(\mathbf{k}(-)\mathbf{p},\mathbf{p}) = \frac{1}{m}\xi^\dagger_{\sigma_p}\left\{2g_v æ_0 p_\mu+f_v æ_0 q_\mu+4g_{\mathcal{M}}W_\mu(\mathbf{p})(\sigma\cdot\mathbf{æ})\right\}\xi_{\nu_p} \quad , \quad (g_{\mathcal{M}}=g_v+f_v)\,. \tag{33}$$

The indices $\mathbf{p}$ indicate that the Wigner rotations have been separated out and, thus, all spin indices have been "resetted" on the momentum $\mathbf{p}$. One can re-write [2b] the electromagnetic current (33):

$$j^{\sigma_p\nu_p}_\mu(\mathbf{k},\mathbf{p}) = -\frac{e\,m}{æ_0}\xi^\dagger_{\sigma_p}\left\{g_{\mathcal{E}}(q^2)\,(p+k)^\mu+g_{\mathcal{M}}(q^2)\,\left[\frac{1}{m}W_\mu(\mathbf{p})(\sigma\cdot\mathbf{\Delta})-\frac{\mathbf{1}}{\mathbf{m}}(\sigma\cdot\mathbf{\Delta})\mathbf{W}_\mu(\mathbf{p})\right]\right\}\xi_{\nu_p}\,. \tag{34}$$

$g_{\mathcal{E}}$ and $g_{\mathcal{M}}$ are the analogues of the Sachs electric and magnetic form factors. Thus, if we regard $g_{S,T,V}$ as effective coupling constants depending on the momentum transfer one can ensure ourselves that the forms of the currents for a spinor particle and those for a $j=1$ boson are the same (with the Wigner rotations separated out).

We have assumed $g_S = g_V = g_T$ above. The expression (41) reveals the advantages of the $2(2j+1)$- formalism, since it looks like the amplitude for the interaction of two spinor particles with the substitutions

$$\frac{1}{2M(\Delta_0 - M)} \Rightarrow \frac{1}{\mathbf{\Delta^2}} \quad \text{and} \quad \mathbf{J} \Rightarrow \sigma \quad .$$

The calculations hint that many analytical results produced for a Dirac fermion could be applicable to describing a $2(2j+1)$ particle. Nevertheless, an adequate explanation is required for the obtained difference. You may see that

$$\frac{1}{\mathbf{\Delta}^2} = \frac{1}{2M(\Delta_0 - M)} - \frac{1}{2M(\Delta_0 + M)} \tag{42}$$

and

$$(p+k)_\mu (p+k)^\mu = 2M(\Delta_0 + M)\,. \tag{43}$$

Hence, if we add an additional diagramm of another channel ($\mathbf{k} \to -\mathbf{k}$), we can obtain the *full* coincidence in the T-matrices of the fermion-fermion interaction and the boson-boson interaction. But, of course, one should take into account that there is no the Pauli principle for bosons, and additional sign " $-$ " would be related to the indefinite metric.

3. Conclusions

The conclusions are: The main result of this paper is the boson-boson amplitude calculated in the framework of the $2(2j+1)-$ component theory. The separation of the Wigner rotations permits us to reveal certain similarities with the $j = 1/2$ case. Thus, this result provides a ground for the conclusion: if we would accept the description of higher spin particles on using the Weinberg $2(2j+1)-$ scheme many calculations produced earlier for fermion-fermion interactions mediated by the vector potential can be applicable to processes involving higher-spin particles. Moreover, the main result of the paper gives a certain hope at a possibility of the unified description of fermions and bosons. One should realize that all the above-mentioned is not surprising. The principal features of describing a particle on the basis of relativistic quantum field theory are *not* in some special representation of the group representation, $(1/2,0)\oplus(0,1/2)$, or $(1,0)\oplus(0,1)$, or $(1/2,1/2)$, but in the Lorentz group itself. However, certain differences between denominators of the amplitudes are still not explained in full.

Several works dealing with phenomenological description of hadrons in the $(j,0) \oplus (0,j)$ framework have been published [17, 18, 19].

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