PAPER • OPEN ACCESS

MHD fluid flow with the effect of mixed convection passing on a magnetic sphere in newtonian nano fluid

To cite this article: Basuki Widodo *et al* 2019 *J. Phys.: Conf. Ser.* **1218** 012057

View the article online for updates and enhancements.

MHD fluid flow with the effect of mixed convection passing on a magnetic sphere in newtonian nano fluid

Basuki Widodo[1,a]**, Yolanda Norasia**[2]**, Dieky Adzkiya**[3]

[1,2,3]Department of Mathematics, Faculty of Mathematics, Computing, and Data Sciences, Institut Teknologi Sepuluh Nopember, Surabaya 60111, Indonesia

E-mail: [a]b_widodo@matematika.its.ac.id

Abstract. This paper considers magneto-hydrodynamics (MHD) fluid flow with the effect of mixed convection passing on a magnetic sphere in nano fluid. Titanium dioxide (TiO_2) is chosen as solid particle in nano fluid. The dimensional governing equations are developed from continuity equation, momentum equation, and energy equation. Furthermore, the dimensional governing equations are transformed into the non-dimensional governing equation by using non-dimensional parameters and variables. Then, the non-dimensional governing equations are transformed into similarity equations using stream function and then solved numerically using implicit finite difference method. The results of numerical solution are obtained using variation of non-dimensional parameters which consist of magnetic parameter, mixed convective parameter, and volume fraction. The numerical results show that the velocity and temperature decrease when magnetic parameter increases. The velocity and temperature increase when mixed convective parameter also increases. Finally, the velocity decreases and temperature increases when volume fraction parameters increase.

1. Introduction

Nano fluid is a mixed fluid consists of solid particle and fluid base with nano particles solid scale 1 to 100 nanometers (nm) [1]. These types of fluids are used in industrial area that needs to control heat flow. In this paper, titanium dioxide (TiO_2) is chosen as solid particle and water is chosen as the base fluid. Nano particle TiO_2 has high thermal conductivities and can increase heat transfer coefficient [2]. TiO_2 with water can conduct an electric current. Therefore, TiO_2-water has MHD characteristics fluid.

The study of MHD nano fluid flows have attracted many researchers. Akbar et al [3] have investigated unsteady MHD nanofluid flow through a channel with moving porous walls and medium. Mahat et al [4] also have observed mixed convection boundary layer flow past a horizontal circular cylinder in visco-elastic nano fluid with constant wall temperature and solved numerically by using the Keller-Box method. Juliyanto [5] also has solved numerically of the problem of the effect of heat generation on mixed convection in nano fluids over a horizontal circular cylinder. In all the previous researches, MHD TiO_2-water nano fluid flow passing on a magnetic sphere have not been investigated. Therefore, we are interested to develop the problem of the MHD TiO_2-water nano fluid on boundary layer that flow passing on a magnetic sphere effected by mixed convection on unsteady and incompressible condition. For solving this problem, we develop a mathematical model that is constructed from continuity equation, momentum equation, and energy equation. The influence of magnetic parameter (M), mixed

convective parameter (λ), and volume fraction (χ) on the velocity profiles and the temperature profiles are investigated and analyzed.

2. Mathematical Formulation

The unsteady MHD fluid flow with the effect of mixed convection passing on a magnetic sphere in nano fluid is considered. **Figure 1** illustrates the physical model of the problem and the coordinate system used to develop the mathematical model. The flow of nano fluid is assumed to be incompressible. Therefore the magnetic Reynolds number is assumed to be very small.
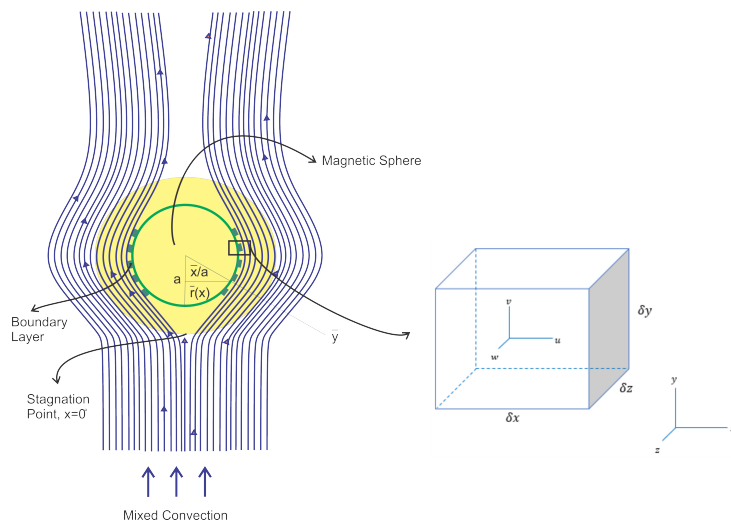
Figure 1. Physical model and coordinate system.

Based on the physical model and coordinate system, the MHD fluid flow with the effect of mixed convection passing on a magnetic sphere in nano fluid is illustrated in **Figure 1**. The 2D dimensional governing equations are developed from continuity equation, momentum equation, and energy equation, which can be written as follows:

Continuity Equation:

$$\frac{\partial(\bar{r}\bar{u})}{\partial\bar{x}} + \frac{\partial(\bar{r}\bar{v})}{\partial\bar{y}} = 0 \tag{1}$$

Momentum Equation:
in x axis

$$\rho_{fn}\left(\frac{\partial\bar{u}}{\partial\bar{t}} + \bar{u}\frac{\partial\bar{u}}{\partial\bar{x}} + \bar{v}\frac{\partial\bar{u}}{\partial\bar{y}}\right) = -\nabla p + \mu_{fn}\nabla^2\mathbf{V} + \sigma(B_0)^2\bar{u} - \rho_{fn}\beta$$
$$(\bar{T} - T_\infty)g_{\bar{x}}$$

in y axis

$$\rho_{fn}\left(\frac{\partial\bar{v}}{\partial\bar{t}} + \bar{u}\frac{\partial\bar{v}}{\partial\bar{x}} + \bar{v}\frac{\partial\bar{v}}{\partial\bar{y}}\right) = -\nabla p + \mu_{fn}\nabla^2\mathbf{V} + \sigma(B_0)^2\bar{v} - \rho_{fn}\beta$$
$$(\bar{T} - T_\infty)g_{\bar{y}} \tag{2}$$

Energy Equation:

$$\left(\frac{\partial\bar{T}}{\partial\bar{t}} + \bar{u}\frac{\partial\bar{T}}{\partial\bar{x}} + \bar{v}\frac{\partial\bar{T}}{\partial\bar{y}}\right) = \alpha_{fn}\left(\frac{\partial^2\bar{T}}{\partial\bar{x}^2} + \frac{\partial^2\bar{T}}{\partial\bar{y}^2}\right) \tag{3}$$

With the initial and boundary conditions are follows

$\bar{t} = 0 : \bar{u} = \bar{v} = 0, \bar{T} = T_\infty$ for every $\bar{x}, \bar{y}$

$\bar{t} > 0 : \bar{u} = \bar{v} = 0, \bar{T} = T_w$ for $\bar{y} = 0$

$\bar{u} = \bar{u}_e(\bar{x}), \bar{u} = \bar{v}, \bar{T} = T_\infty$ as $\bar{y} \to \infty$

where ρ_{fn} is density of nano fluid, μ_{fn} is dynamic viscosity of nano fluid, g is the gravitational acceleration, and α_{fn} is thermal diffusivity of nano fluid.

Further, the dimensional governing equations (**1-3**) are changed into non-dimensional equations by using both non-dimensional parameters and variables. We propose the following non-dimensional variables, as in [6]:

$$x = \frac{\bar{x}}{a}; \ y = Re^{\frac{1}{2}}\frac{\bar{y}}{a}; \ t = \frac{U_\infty \bar{t}}{a}; \ u = \frac{\bar{u}}{U_\infty}$$

$$v = Re^{\frac{1}{2}}\frac{\bar{v}}{U_\infty}; \ r(x) = \frac{\bar{r}(\bar{x})}{a}$$

for $g_{\bar{x}}$, and $g_{\bar{y}}$ are defined as in [7]

$$g_{\bar{x}} = -g \ sin(\frac{\bar{x}}{a})$$
$$g_{\bar{y}} = g \ cos(\frac{\bar{x}}{a})$$

and with the value of r is defined as

$$\bar{r}(\bar{x}) = a \ sin(\frac{\bar{x}}{a})$$

Substituting these non-dimensional variables into (**1-3**) leads to the following non-dimensional equations

Continuity Equation

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial y} = 0 \tag{4}$$

Momentum Equation
in x axis

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{v_{fn}}{Re v_f}\frac{\partial^2 u}{\partial x^2} + \frac{v_{fn}}{v_f}\frac{\partial^2 u}{\partial y^2} + Mu$$
$$+ \ \lambda T sinx$$

in y axis

$$\frac{1}{Re}\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \frac{v_{fn}}{v_f}\frac{1}{Re^2}\frac{\partial^2 v}{\partial x^2} + \frac{v_{fn}}{v_f}\frac{1}{Re}\frac{\partial^2 v}{\partial y^2} + \frac{M}{Re}v$$
$$- \ \frac{\lambda}{Re^{1/2}}T cosx \tag{5}$$

Energy Equation

$$\frac{\partial T}{\partial t} + u\left(\frac{\partial T}{\partial x}\right) + v\left(\frac{\partial T}{\partial y}\right) = \frac{1}{Re}\frac{1}{Pr}\frac{\alpha_{fn}}{\alpha_f}\frac{\partial^2 T}{\partial x^2} + \frac{1}{Pr}\frac{\alpha_{fn}}{\alpha_f}\frac{\partial^2 T}{\partial y^2} \tag{6}$$

Boundary layer theory is further [8] applied to non-dimensional equations, we obtain the following results

Continuity Equation

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial y} = 0 \tag{7}$$

Momentum Equation
in x axis

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{v_{fn}}{v_f}\frac{\partial^2 u}{\partial y^2} + Mu + \lambda T\ sinx$$

in y axis

$$-\frac{\partial p}{\partial y} = 0 \tag{8}$$

Energy Equation

$$\frac{\partial T}{\partial t} + u\left(\frac{\partial T}{\partial x}\right) + v\left(\frac{\partial T}{\partial y}\right) = \frac{1}{Pr}\frac{\alpha_{fn}}{\alpha_f}\frac{\partial^2 T}{\partial y^2} \tag{9}$$

where these nano fluid constants are defined by [9],
Density of nano fluid

$$\rho_{fn} = (1 - \chi)\rho_f + \chi\rho_s$$

Dynamic viscosity

$$\mu_{fn} = \frac{\mu_f}{(1-\chi)^{2.5}}$$

Heat Specific

$$(\rho C_p)_{fn} = (1 - \chi)(\rho C_p)_f + \chi(\rho C_p)_s$$

Heat conductivity

$$\frac{k_{fn}}{k_f} = \frac{(k_s + 2k_f) - 2\chi(k_f - k_s)}{(k_s + 2k_f) + \chi(k_f - k_s)}$$

The thermo-physical properties of nano particles TiO_2 and base fluid water is given in **Table 1** [10].

Table 1. Thermo-physical properties

Properties	Water	TiO_2
density	997.1	4250
specific heat of constant pressure	4179	686.2
thermal conductivity	0.613	8.9538

We substitute those the nano fluid constant into **(7)** and **(8)**, we further obtain
Momentum Equation

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{1}{(1-\chi)^{2.5}\left((1-\chi)+\chi\left(\frac{\rho_s}{\rho_f}\right)\right)}\frac{\partial^2 u}{\partial y^2}$$
$$+ Mu + \lambda T sinx \tag{10}$$

Energy Equation

$$\frac{\partial T}{\partial t} + u\left(\frac{\partial T}{\partial x}\right) + v\left(\frac{\partial T}{\partial y}\right)$$
$$= \frac{1}{Pr}\left(\frac{(k_s+2k_f)-2\chi(k_f-k_s)}{((k_s+2k_f)+\chi(k_f-k_s))\left((1-\chi)+\left(\frac{\chi(\rho C_p)_s}{(\rho c_p)_f}\right)\right)}\right)\frac{\partial^2 T}{\partial y^2} \tag{11}$$

by converting **(10)** and **(11)** into similarity equations using stream function, which is given as follows [11]

$$u = \frac{1}{r}\frac{\partial \psi}{\partial y}$$

$$v = -\frac{1}{r}\frac{\partial \psi}{\partial x}$$

With

$$\Psi = t^{1/2}u_e(x)r(x)f(x,\eta,t)$$

$$\eta = \frac{y}{t^{1/2}}$$

$$T = s(x,\eta,t)$$

The equation **(10)** and **(11)** can be written respectively as follows:
Momentum Equation

$$\left[\frac{1}{(1-\chi)^{2.5}\left((1-\chi)+\chi\left(\frac{\rho_s}{\rho_f}\right)\right)}\right]\frac{\partial^3 f}{\partial \eta^3} + \frac{\eta}{2}\frac{\partial^2 f}{\partial \eta^2} + t\frac{\partial u_e}{\partial x}\left(1-\left(\frac{\partial f}{\partial \eta}\right)^2 + f\frac{\partial^2 f}{\partial \eta^2}\right)$$
$$= t\frac{\partial^2 f}{\partial t\partial \eta} + tu_e\left(\frac{\partial f}{\partial \eta}\frac{\partial^2 f}{\partial x\partial \eta} - \frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial \eta^2} - \frac{1}{r}\frac{\partial r}{\partial x}f\frac{\partial^2 f}{\partial \eta^2}\right) + Mt\left(1-\frac{\partial f}{\partial \eta}\right) - \frac{\lambda st}{u_e}sinx \tag{12}$$

Energy Equation

$$\left(\frac{(k_s + 2k_f) - 2\chi(k_f - k_s)}{((k_s + 2k_f) + \chi(k_f - k_s))\left((1-\chi) + \left(\frac{\chi(\rho C_p)_s}{(\rho c_p)_f}\right)\right)}\right)\frac{\partial^2 s}{\partial \eta^2} + Pr\frac{\eta}{2}\frac{\partial s}{\partial \eta}$$
$$+ \quad Prt\frac{\partial u_e}{\partial x}f\frac{\partial s}{\partial \eta} = Prt\left[\frac{\partial s}{\partial t} + u_e\left(\frac{\partial f}{\partial \eta}\frac{\partial s}{\partial x} - \frac{\partial f}{\partial x}\frac{\partial s}{\partial \eta} - \frac{1}{r}\frac{\partial r}{\partial x}f\frac{\partial s}{\partial \eta}\right)\right] \tag{13}$$

With the initial and boundary condition are as follows :

$t = 0 : f = \frac{\partial f}{\partial \eta} = s = 0$ for every x, η

$t > 0 : f = \frac{\partial f}{\partial \eta} = 0, s = 1$ for $\eta = 0$

$\frac{\partial f}{\partial \eta} = 1, s = 0$ as $\eta \to \infty$

By substituting local free stream for sphere case [12], $u_e = \frac{3}{2}sin\ x$ into **(12)** and **(13)** respectively, further we obtain

Momentum Equation

$$\left[\frac{1}{(1-\chi)^{2.5}\left((1-\chi) + \chi\left(\frac{\rho_s}{\rho_f}\right)\right)}\right]\frac{\partial^3 f}{\partial \eta^3} + \frac{\eta}{2}\frac{\partial^2 f}{\partial \eta^2} + \frac{3}{2}\ tcos\ x$$
$$\left(1 - \left(\frac{\partial f}{\partial \eta}\right)^2 + 2f\frac{\partial^2 f}{\partial \eta^2}\right) = t\frac{\partial^2 f}{\partial t\partial \eta} + \frac{3}{2}\ tsin\ x\left(\frac{\partial f}{\partial \eta}\frac{\partial^2 f}{\partial x\partial \eta} - \frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial \eta^2}\right)$$
$$+ \quad Mt\left(1 - \frac{\partial f}{\partial \eta}\right) - \frac{2}{3}\lambda st \tag{14}$$

Energy Equation

$$\left(\frac{(k_s + 2k_f) - 2\chi(k_f - k_s)}{((k_s + 2k_f) + \chi(k_f - k_s))\left((1-\chi) + \left(\frac{\chi(\rho C_p)_s}{(\rho c_p)_f}\right)\right)}\right)\frac{\partial^2 s}{\partial \eta^2}$$
$$+ \quad Pr\frac{\eta}{2}\frac{\partial s}{\partial \eta} + 3\ cos\ xPrtf\frac{\partial s}{\partial \eta} = Prt\frac{\partial s}{\partial t} + Prt\frac{3}{2}sin\ x$$
$$\left(\frac{\partial f}{\partial \eta}\frac{\partial s}{\partial x} - \frac{\partial f}{\partial s}\frac{\partial s}{\partial \eta}\right) \tag{15}$$

3. Results and Discussions

We further investigated the influence of various parameters such as magnetic parameter (M) , mixed convective parameter (λ), and volume fraction (χ) towards velocity and temperature in front of the lower stagnation point of the sphere. We consider the value of the Prandtl number for water (Pr) of 6.2 [13]. TiO_2-water is chosen as Newtonian nano fluid. The value of volume fraction for Newtonian Fluid varies from 0 to 0.2.

The numerical results of the velocity and temperature with respect to the position in front of the lower stagnation at the point $x = 0°$ with various value of magnetic parameters $M = 0$, 2, 4, and 8 are illustrated in **Figure 2** and **Figure 3** respectively. **Figure 2** shows the velocity profiles of the MHD mixed convection TiO_2-water nano fluid passing on a magnetic sphere at various M when mixed convective parameter $\lambda = 1$ and volume fraction $\chi = 0.1$. The results
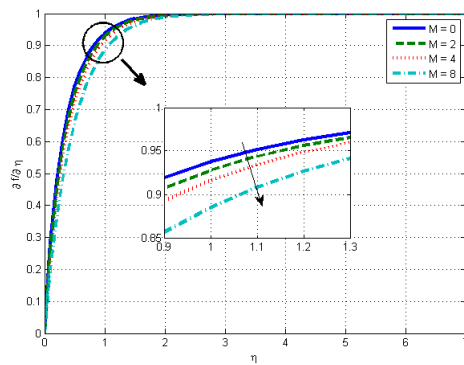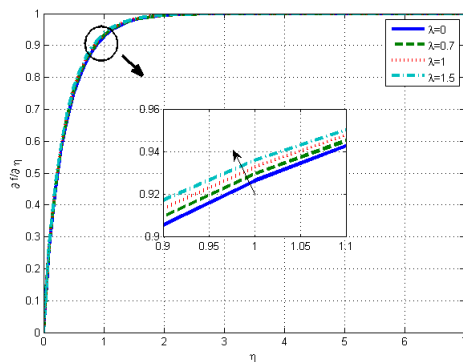
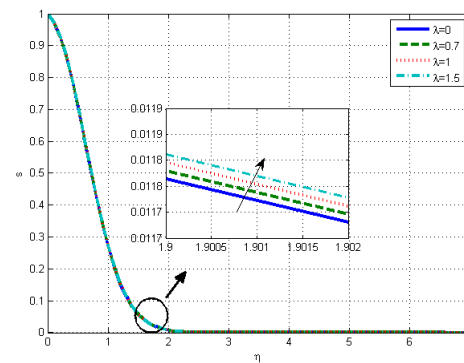**Figure 2.** Velocity profile for various M
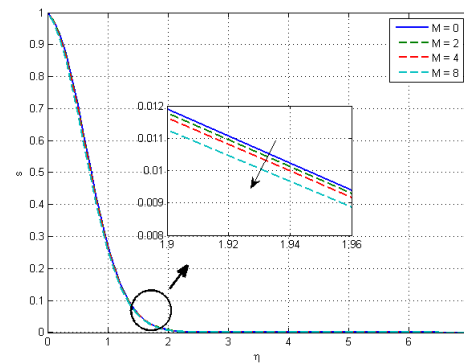
Figure 3. Temperature profile for various M

show that velocity profiles in **Figure 2** decrease when magnetic parameter increases. **Figure 3** shows temperature profile decreases when magnetic parameter increases. This is caused by the presence of Lorentz force from magnetic sphere.

Figure 4 and **Figure 5** illustrate the velocity profiles and temperature profiles of the MHD fluid flow with the effect of mixed convection TiO_2-water nano fluid passing on a magnetic sphere at various mixed convective λ. These numerical results are obtained at fixed values of $M = 1$, $Pr = 6.2$, and $\chi = 0.1$. The velocity profiles in **Figure 4** increase when mixed convective parameter increases. Also, in **Figure 5** when mixed convective parameter increases then the temperature profiles increase.
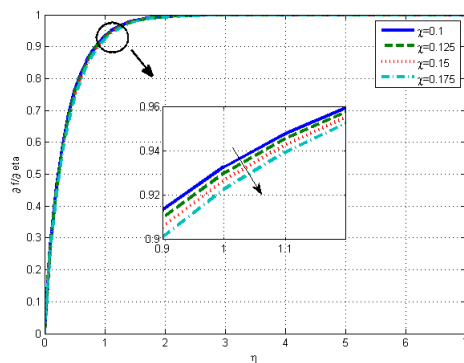

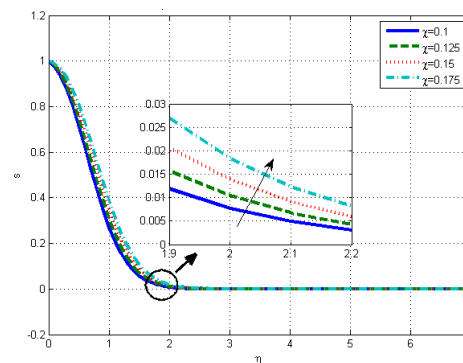**Figure 4.** Velocity profile for various λ

Figure 5. Temperature profile for various λ

The velocity profiles and temperature profiles of the MHD fluid flow with the effect of mixed convection TiO_2-water nano fluid passing on a magnetic sphere with various volume fraction of Newtonian nano fluid $\chi = 0.1$, 0.125, 0.15, and 0.175 are illustrated in **Figure 6** and **Figure 7** respectively. These numerical results are obtained at fixed values of $M = 1, \lambda = 1$, and $Pr = 6.2$. The results show that the velocity profiles in **Figure 6** decrease when volume fraction nano fluid increases. The effect of the increase of volume fraction is the increase of friction between particles. If the friction between particles is higher, then velocity profiles are lower. Also, in **Figure 7** when volume fraction nano fluid increases then the temperature profile increases. This is caused by nano particle TiO_2 that has high conductivity.

Figure 6. Temperature profile for various χ

Figure 7. Temperature profile for various χ

4. Conclusions

MHD fluid flow with the effect of mixed convection passing on a magnetic sphere in Newtonian nano fluid is investigated numerically by using implicit finite difference method. We have considered TiO_2 as solid particle and water as base fluid. We further obtain numerical results that when effects of magnetic parameter, mixed convective parameter, and volume fraction are included, the velocity and temperature profiles change. It is concluded that the velocity and the temperature profiles decrease when the magnetic parameter increases. Also, the velocity and the temperature profiles increase as a consequence of the increasing of mixed convective parameter. However, the velocity profiles decrease and the temperature profiles increase when the volume fraction parameter increases.

Acknowledgments

References

[1] Ding Y, Chen Y, He Y, Lapkin A, Yeganeh M, Siller L, Butenko Y V 2007 *Advanced Powder Technol.* **18** 813-24
[2] Das S, Chakraborty S, Jana R N, Makinde O D 2015 *Appl. Math. Mech.* **36** 1593-610
[3] Akbar M Z, Ashraf M, Iqbal M F, Ali K 2016 *AIP Conf Proc* **6** 045222
[4] Mahat R, Rawi N A, Kasim, A R M, Shafie S 2017 *Malaysian Journal of Fundamental and Applied Sciences* 310-4
[5] Juliyanto B, Widodo B, Imron C 2018 *IOP Conf. Series* **1008** 012001
[6] Widodo B, Anggriani I, Imron C 2016 *International Journal of Advances in Science Engineering and Technology* **4** 2321-9009.
[7] Karimipour A, Nezhad A H, D'Orazio A, Shirani E 2013 *Journal of Theoretical and Applied Mechanics* **51** 447-62.
[8] Zaib A, Kasim A R M, Mohammad N F, Shafie S 2013 *Heat Transfer Research* **44** 603-20.
[9] Widodo B, Arif D K, Aryany D, Asiyah N, Widjajati F A Kamiran 2017 *AIP Conference Proceedings* **1867** 020069
[10] Freidoonimehr N, Rashidi M M, Mahmud S 2015 *International Journal of Thermal Sciences* **87** 136-45
[11] Nursalim R, Widodo B, Imron C 2017 *IOP Conf. Series: Journal of Physics* **893** 012044
[12] Tham L and Nazar R 2013 *AIP Conf Proc* **1557** 291-5.
[13] Oztop H and Abu-Nada E 2008 *Int. J. Heat Fluid Flow* **29** 1326-36.