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Determining the reliability of network information systems

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V I Sumin[1], A V Dushkin[1], E V Grechishnikov[2] and S V Ivanov[1]

[1]ET Department, Voronezh Institute Federal Penitentiary Service of Russia, 1a Irkutskaya Street, Voronezh, RU
[2]FSO Academy of Russia, Moscow, RUSSIA

E-mail: a_dushkin@mail.ru

Abstract. Mathematical support of reliability analysis of network information systems is developed. As a model, the network information system uses a graph representation based on the formalization of the graph description by parent projections. In this paper, a new operation is introduced on the parent projection of the graph - the operation of removing the vertex from the graph and then contracting the edges incident to the remote vertex. An algorithm for cutting the graph of a network information system is proposed based on the well-known technique of using paths and sections. The state tree of the system, obtained as a result of the implementation of the algorithm for cutting the parent projection of the graph, illustrates the development of failures within the system before the onset of an inoperative state and is a structural function of the original graph. Based on the resulting combination of projections obtained as a result of the implementation of the cutting algorithm, a probabilistic reliability function is constructed.

1. Introduction

The rapid development of information systems and processes has led to the complication of network information systems (NISs). In the process of operation, the NIS inevitably undergoes a number of changes caused by failures of its elements - nodes and communication lines between them. It should be borne in mind that the failure of one element of the NIS may lead to the transition of the system to an inoperative condition. In addition, the failure of the entire system can cause a dangerous situation with the subsequent occurrence of an accident. The question of ensuring the reliability and safety of the functioning of complex NISs is one of the most important in its design and operation [1-5]. The analysis of research works allows drawing a conclusion about some challenges in the construction of systems for monitoring the state of reliability and safety of large NISs.

2. Statement of the problem

In the study of SIS, a graph $G(V,E)$ is used as a mathematical model, the vertices of which $v \in V$ represent nodes of the NIS, and the edges $e \in E$ are the connections between the NIS nodes.

The NIS, which is modeled with a graph, is usually considered inoperable, if in the case of removing vertices or edges the graph will not satisfy at least one condition of operability. In the scientific literature, the conditions of inoperability usually include the following:

- the graph consists of at least two components;
- there are no paths between certain sets of vertices;

- the number of vertices in the largest component of the graph $G(V,E)$ is less than some predetermined number;
- the shortest path exceeds a specified value, etc.

In this paper, the main condition for the inability to work is the absence of paths between two defined vertices of the graph, called the initial and final vertices.

The existing ways of presenting graphs do not provide an opportunity to formalize their analysis and transformation. It is proposed to use the approach based on the formalization of the description of the NIS graph $G(V,E)$ by its projections $P(v_0)$, $v_0 \in V$ [5-8].

In [6], a method of formal description of graphs and their components using a bracket image was first introduced, concepts and definitions were introduced and justified in [7], provisions that formed the theoretical basis of the proposed model were formulated, a technology for constructing a bracket image of graphs was presented, and evidence of the proposed model was given.

We give the basic information about the projections of the graph [8] used in the paper.

Projection $P(v_i)$ of the graph $G(V,E)$ is the description of the graph in the parenthetic form, the top view of which is the vertex $v_i \in V$.

M_i is set of vertices at the *i*-th level of the bracket projection. $E_i = \{u, v : u \in M_{i-1}, v \in M_i\}$ is a set of edges incident to pairs of vertices from adjacent projection levels.

In [8], a description of typical operations on graphs which allow a formal way to make changes to their structures represented by bracket projections is presented. The new operations introduced below will be used in the algorithm for cutting the projection of the graph.

According to [8], the removal of a node from the NIS corresponds to the removal of the associated vertex from the graph. In this case, it is necessary to remove this vertex and the vertices generated by it from the projection [8]; the projection itself removes the vertex and the vertices that are inside the parentheses at the higher levels.

We illustrate what was said by removing vertex 4 from the view projection:

$$P = (1^{3^{(2,4,5)}, 4^{(3)}}, 2^{(3^{(1,4,5)}, 5^{(3)})})$$

Based on the description of the operation, we obtain:

$$P = (1^{3^{(2,5)}}, 2^{(3^{(1,5)}, 5^{(3)})})$$

We introduce a new operation - the operation of removing the vertex from the graph and then compressing the incident vertices of the deleted vertex. This operation reduces to modifying the edge contraction operation known from [5]. The edge contraction operation $(v_i, v_j) \in E$ in the graph $G(V,E)$ consists in removing one of the two incident vertices adjacent to the removed edge $v_i, v_j \in V$ and expansion of the environment of the left vertex due to the environment of the removed one. For the input of the modified operation, assuming the vertex v_i located in the projection at a level lower than the level of the vertex v_j, we consider the vertex v_j to coincide with the removed one. Then the operation of removing the vertex v_j from the graph and then contracting the incident edges reduces to the operation of contracting the set of edges $(v_i, v_j) \in E$ over all vertices v_i of the original projection.

Let us demonstrate the use of the entered operation in the projection

$$P = (1^{3^{(2,4,5)}, 4^{(3)}}, 2^{(3^{(1,4,5)}, 5^{(3)})}),$$

by removing the vertex 3 and then tightening the edges. According to the description of the operation, it is necessary to make all the edges of the type $(v_i, 3)$. There are two such edges in the original projection - (1,3), (2,3). Here is the result of the operation of removing vertex 3:

$$P = (1^{(2,4,5)}, 2^{(1,4,5)})$$

Let us turn to [5]. The method of using paths and sections is a powerful tool for the analysis of complex structures. Conditional diagrams representing network structures are used as the analyzed structures, in which two nodes - input and output - are distinguished. We present a number of fundamental definitions that are the basis of the method under consideration. A path is a set of elements, the working state of which ensures the working state of the system [9]. Applied to the structural diagrams, the path is a set of elements that ensure the existence of a connection between the input and the output of the circuit. A cross-section is a set of elements, failure of which leads to system failure [9]. Removal of the corresponding set of elements of the cross-section from the block diagram leads to a disruption of the connection between the input and the output of the circuit. The minimal path of a structure is a union of the elements in which none of the components can be removed without violating the condition of the system's operability [9]. Thus, an arbitrary structure can be represented as a parallel connection of the minimal paths between the input and output of the conditional circuit.

In [9], the algorithm of cutting the structural representation of the conditional scheme is used to obtain reliability indicators. We implement this approach to obtain a graph of the change in the state of the NIS, using the parenthetic form of the NIS graph representation.

We use the method of representing the NIS in the form of a parenthetic projection of a graph with a vertex that is an input for a conditional scheme. Since such a vertex for any considered NIS of the form of the conditional scheme is unique, in the parenthetic projection we omit it, and we construct the projection starting from the vertices incident to the input one.

Proceeding from [9], the corresponding algorithm for cutting the parenthetic projection of the NIS graph will contain the following steps:

1. For the NIS in question, it is necessary to construct the initial complete parenthetic projection P using the input vertex of the NIS graph as a view. The constructed projection P will contain many simple paths connecting the input and output vertices.

2. It is necessary to determine the degree of incidence of each element of the original projection P.

$$(n_1, n_2, .., n_m) = \{n_i\} \tag{1}$$

3. Find the maximum value among the values of the incidence degrees n_i. If there are several such elements, choose an arbitrary one from them. The corresponding projection element (without loss of generality, we can assume that it will be a node with the number 1) is first considered to be withdrawn (we perform the operation of removing the vertex from the projection), obtaining a projection, and then absolutely resistant to removal (we perform an operation to remove the vertex with subsequent contraction incident to the vertex of the edges), obtaining the projection P_1.

The operations of this step will be called cutting along the node.

4. We determine the initial projection, as the sum of the projections obtained after the completion of step 3:

$$P = 1' P_0 + 1P_1, \tag{2}$$

where 1 is the node on which cutting was carried out.

5. After applying these transformations and simplifying the original projection, the resulting projections P_0 and P_1 may be empty, or each projection site will have an incidence degree equal to 2, or in the projections there will be nodes with an incidence level greater than 2. In the first and second cases, the cutting algorithm for parenthetic projection is considered complete. If there is a third case,

then for the corresponding projection we determine the values $\{n_i\}$ for all nodes and perform the cutting operation on the node with the maximum value n_i. Without loss of generality, we assume that this is a node with number 2. The numbers of nodes with the maximum values n_i in the projections P_0 and P_1 may not coincide.

If the projection P_0 is subjected to cutting, we define the following sum of projections:

$$P_0 = 2' P_{00} + 2P_{01}. \tag{3}$$

If the projection P_1 is subjected to cutting, we define the following sum of projections:

$$P_1 = 2' P_{10} + 2P_{11}. \tag{4}$$

6. The results obtained in the previous step, we use to transform the expression obtained in step 4.

$$P = 1' P_0 + 1P_1 = 1'(2' P_{00} + 2P_{01}) + 1(2' P_{10} + 2P_{11}). \tag{5}$$

7. Over obtained projections P_{00}, P_{01}, P_{10} and P_{11} we perform the actions specified in 5 and 6.

The process considered is not infinite, since cutting across all elements will lead to the obtaining of projections that do not contain vertices.

We illustrate the algorithm for cutting the SIS bracket projection of the, whose graph is shown in figure 1. The vertices of the graph, with the exception of the input and output vertices, are numbered

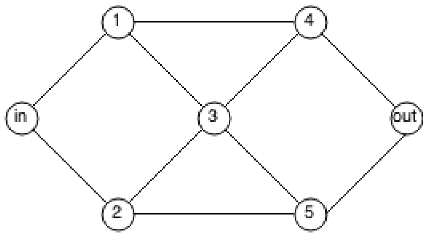

Figure 1. Graph of original SIS.

We construct a bracket projection of the graph of the original network structure, having omitted the input and output vertices. The full projection is as follows:

$$P = (1^{3^{(2,4,5)},4^{(3)}}, 2^{(3^{(1,4,5)},5^{(3)})})$$

We select the element with the maximum degree of incidence. Element 3 is such an element. We carry out the operation of cutting the projection along element 3. Removing element 3 from the projection P, we obtain the projection P_0 of the following form:

$$P_0 = (1^{(4)}, 2^{(5)})$$

The graph corresponding to the projection P_0 is shown in figure 2.

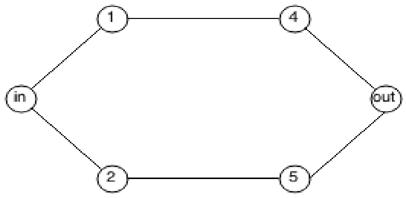

Figure 2. Graph of projection P_0.

Removing the vertex 3 in the projection P and producing the edges incident to the vertex, we obtain the following projection P_1:

$$P_1 = (1^{(2,4,5)},\ 2^{(1,4,5)})$$

The graph corresponding to the projection P_1 is shown in figure 3.

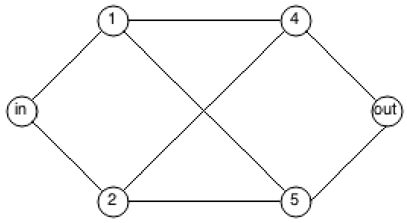

Figure 3. Graph of projection P_1.

In the projection P_0, all nodes have a degree of incidence equal to 2; therefore, no further cutting of this projection is done. We consider the projection P_1. For elements 1, 2, 4 and 5, the degree of incidence is 3. We cut the projection along any of the two elements. We use as the selected element the element with the number 1.

Removing element 1 from the projection P_1, we obtain the following projection P_{10}:

$$P_{10} = (2^{\,(4,5)})$$

The graph corresponding to the projection P_{10} is shown in figure 4.

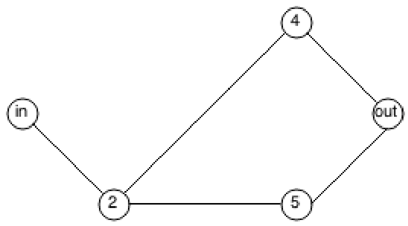

Figure 4. Graph of projection P_{10}.

Removing the vertex 1 from the projection P_1 and producing the edges incident to the vertex, we obtain the projection P_{11} of the following form:

$$P_{11} = (2^{\,(4,5)}, 4)$$

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The graph corresponding to the projection P_{11} is shown in figure 5.

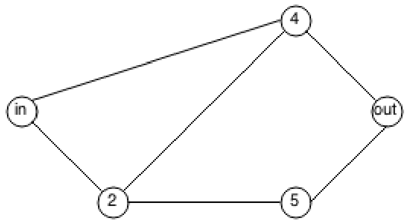

Figure 5. Graph of projection P_{11}.

Projections P_{10} and P_{11} have nodes with a degree of incidence greater than two. In both projections it is, for example, the node with number 2. We cut the projections P_{10} and P_{11} along this vertex.

Removing element 2 from the projection P_{10}, we obtain projection P_{100} of the following form:

$$P_{100} = \varnothing$$

The graph corresponding to the projection P_{100} is shown in figure 6.

Figure 6. Graph of projection P_{100}.

Removing the vertex 2 from the projection P_{10} and producing the edges incident to the vertex, we obtain projection P_{101} of the form:

$$P_{101} = (4,5)$$

As a result of cutting the projection P_{11} on node 2, we get two identical projections of the following form:

$$P_{110} = (4)$$

$$P_{111} = (4,5)$$

The graph corresponding to the projections P_{101} and P_{111} is shown in figure 7.

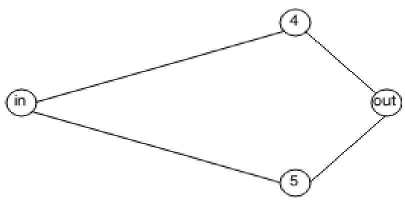

Figure 7. Graph of projections P_{101} and P_{111}.

In the projections $P_{100}, P_{101}, P_{110}$ and P_{111} there are no vertices with a degree of incidence greater than two. Therefore, the algorithm for cutting the bracket projection is considered complete.

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We use the projection cuttings obtained at each step of the algorithm to obtain the resulting expressions:

$$P = 3'P_0 + 3P_1$$
$$P_1 = 1'P_{10} + 1P_{11}$$
$$P_{10} = 2'P_{100} + 2P_{101}$$
$$P_{11} = 2'P_{110} + 2P_{111}$$
$$P = 3'P_0 + 3(1'(2'P_{100} + 2P_{101}) + 1(2'P_{100} + 2P_{111}))$$

The resulting combination of the projection P is shown in the form of a graph, which will be the graph of the state change in the original SIS (figure 8).

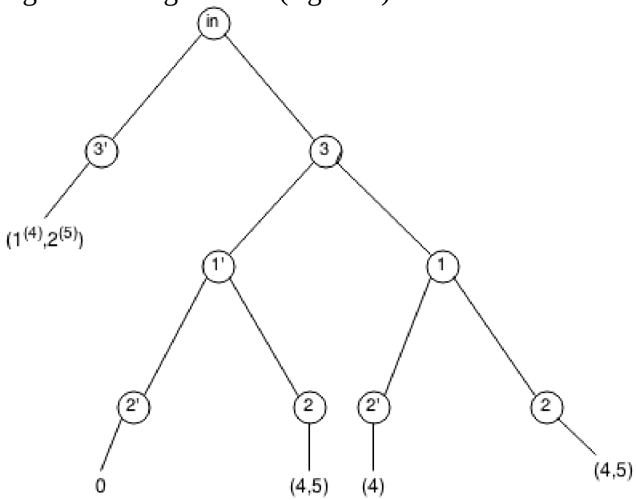

Figure 8. Graph of SIS state change.

We use the proposed algorithm for cutting the bracket projection to construct a fracture graph for the SIS, shown in figure 9.

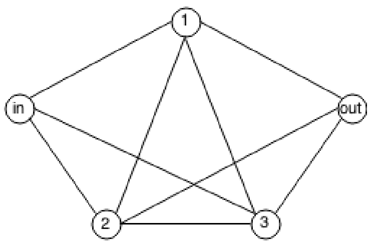

Figure 9. Graph of original SIS.

The projections obtained at each step of the cutting algorithm will give the following resultant expression:

$$P = 1'P_0 + 1P_1$$
$$P_0 = 2'P_{00} + 2P_{01}$$
$$P_1 = 2'P_{10} + 2P_{11}$$
$$P = 1'(2'P_{00} + 2P_{01}) + 1(2'P_{10} + 2P_{11}))$$

The resulting combination of the projection P is shown in the form of a graph, which will be the state change graph of the original SIS (figure 10).

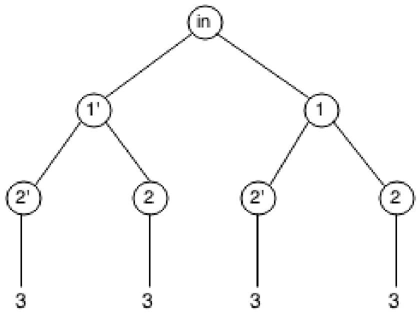

Figure 10. Graphs of SIS state change.

We introduce the following notations: R_c is the reliability of the system, r_i is the reliability of the element. P and p are the probability of failure-free operation of the system and the *i*-th element. Let the *i*-th element be characterized by a random state x_i, and the system has a structure function *y(X).* Then:

$$r_i = [P[x_i = 1]],\\ R_c = [P[y(X)] = 1] \tag{6}$$

We call *h(r)* the reliability function of the system. This is a probability function that is constructed from the resulting combination of projections obtained as a result of cutting the parenthetic projection of the original NIS graph, which is a structural function of *y(X)*, by replacing the elements with their probabilities. Note that if the elements of the system are dependent, the system reliability indicator can be a function not only of *r* [10-12].

We construct the function *h(r)* for the result of cutting the SIS bracket projection, shown in figure 10. We have:

$$P = 1'(2'P_{00} + 2P_{01}) + 1(2'P_{10} + 2P_{11})\\ y(X) = 1'2'P_{00} + 1'2P_{01} + 12'P_{10} + 12P_{11},$$

where $P_{00} = P_{01} = P_{10} = P_{11} = 3$.

By making a replacement in the right-hand part of the equality of logical variables with probabilistic ones, we obtain an expression for the reliability function, bearing in mind that $h(r) = [P[y(X)] = 1]$.

We obtain:

$$h(r) = [P[y(X)] = 1] = (1 - r_1)(1 - r_2)r_3 + (1 - r_1)r_2r_3 + r_1(1 - r_2)r_3 + r_1r_2r_3$$

Substituting $h(r)$ in the values of reliability indexes of elements, we obtain the exact value of the reliability index of the original SIS.

Thus, the state tree of the system, obtained as a result of the implementation of the algorithm for cutting the parenthetic projection of the NIS graph, illustrates the development of failures within the system before the onset of an inoperative state. In the process of monitoring, the system is split into subsystems, the subsystems are divided into nodes that are connected by a network structure. Applying the algorithm of cutting the parenthetic projection of the NIS subsystem graph, the reliability functions $h_i(r)$ for each subsystem are computed. Substituting in the reliability functions $h_i(r)$ the values of the reliability indicators for elements r_i depending on the type of the *j*-th threat, we obtain a set of functions characterizing the overall reliability of the system. The proposed algorithm for cutting the parenthetic projection of the NIS graph is promising for the use in the mathematical support of the NIS reliability and safety. The main advantages of such a system will be accounting for changes in the structure of the system with the possibility of detecting failures in the early stages and reducing the time between the onset of a failure and its prevention.

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