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Synchronized optimization of last trains' timetables in mass rail transit networks based on extra dwell time

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Abstract. Synchronized optimization of last trains' timetables is purposed to improve the connecting effects of last trains by coordinating the arrival and departure times of trains at transfer stations. Due to the great complexities of transfer relationships, it is more and more difficult to solve this problem. In this paper, in order to reduce the number of unable transfer passengers, an optimization model of last trains' timetables is proposed by determining the departure time and extra dwell time. And we minimize the difference between last trains' actual departure time and the optimized ones for the convenience of rolling stocks daily maintenance. Possible optimized solution is investigated through a case study in Beijing subway by using the tool of CPLEX. The results indicate that the volume of unable transfer passengers is reduced by 15.0%, and the number of coordinated trains is improved by 8.9%.

1. Introduction

Large-scale mass rail transit (MRT) networks have built up in China. More and more passengers need to use transfer stations to other lines because of the independent operation mode of each line. Synchronizing and coordinating trains of different lines is one of the main problems of MRT network operation, especially for first and last trains. Once last trains of different lines at transfer stations are not synchronized, which means that the transfer passengers are disabled to reach their final destinations. Therefore, synchronized optimization of last trains' timetables becomes a more practical and significant issue.

Synchronized optimization of last trains' timetables is purposed to improve the connecting effects of trains in last service period by coordinating the arrival and departure times of trains at transfer stations. However, it is more and more difficult to solve this problem because of the great complexities of transfer relationships, which are concentrated on in a large-scale network, massive and associated transfer directions of different lines, and decentralized passenger flow, etc. At present, foreign research mainly focuses on the synchronization and coordination optimization of timetable for public bus transport, and integration of public transport and MRT in non-last periods [1-4]. There is little research on synchronizing last trains of different lines in MRT network. But there are some achievements in China. For example, a method was put forward to quantify the importance of the train connection relationship of different lines based on the graph theory and the minimum spanning tree Kruskal algorithm [5]. In order to minimize passengers' transfer waiting time, the optimization strategy and hierarchical algorithm were proposed, and the departure time domain of last trains was studied [6]. A pick-and-filter algorithm was proposed to determine the principle of trains' connection

relations at transfer stations in MRT network [7]. And some optimization models of the first or the last train timetable were proposed by maximizing the number of connection trains [8-9] or minimizing the inaccessible passenger volume [10]. However, some of them were unconsidered the distribution characteristics of passenger flow, or the flexibility and operability of the last trains operation.

In fact, the last trains' running time in every section is stably fixed after repeated measure and debugging in trial operation. In addition, it is stable when trains run in every section of the rail line based on automatic train operation (ATO). These show that adjusting dwell time is more flexible and operational for this problem. Thus, in order to provide more smooth transfer service for passengers, we introduce the extra dwell time as another decision variable to optimize the last trains' timetable, based on the operation characteristics (such as less transfer passenger demand, sufficient transportation capacity, no follow-up train, and stable running time of line section).

The structure of this paper has 5 sections. Section 2 analyzes the connection relationship of last trains at transfer stations in the whole of network. Section 3 constructs the optimization model of last trains' departure time and dwell time to minimize the volume of unable transfer passengers. Section 4 describes the case study of Beijing subway. Finally, the conclusions are made in section 5.

2. Analysis of last trains synchronized connection at transfer stations

2.1. The parameters and variables

In this paper, in order to enhance the applicability of the model in the large-scale network, the process of trains stopping at intermediate stations is simplified. Therefore, the simplified transfer network consists of origin/terminal stations, transfer stations, and the line's segments. The up and down directions in the same line are identified with two different lines, which are shown in Figure 1. And symbols related to synchronized optimization of last trains' timetables are defined as followings:
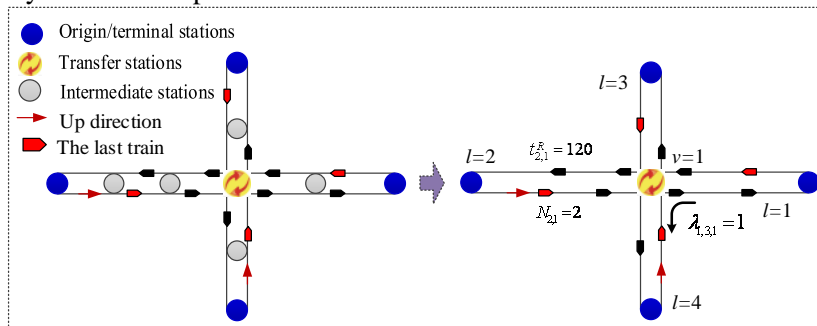

Figure 1. Example of MRT transfer network

L : the set of lines in a MRT network, $l \in L$.

V : the set of transfer stations, $v \in V$.

V_l : the transfer station sequence in line l .

$A_{l,v}$: the last train's arrival time of line l at transfer station v .

$D_{l,v}$: the last train's departure time of line l left from transfer station v .

$D_l^0, D_l'^0$: the last train's departure time of line l at origin station for actual and optimized schemes.

h_l : the headway of line l in the last service period.

$s_{l,v}$: the last train's actual dwell time of line l at station v .

$R_{l,v}$: the last train's total running time of line l from the origin station to transfer station v (including sections' running time and the actual dwell time, except the dwell time of transfer station v).

$E_{l,v}$: the last train's generalized extra dwell time of the line l at transfer station v . Due to the simplified network, the generalized extra dwell time is the sum of the extra dwell time at the stations located in the rear adjacent segment from transfer station v to its previous transfer station v' , as shown in 3.2 for details. It is the same to the generalized extra dwell time.

$E_{l,v}^{\min}$: the last train's minimal generalized extra dwell time of line l at transfer station v .

$E_{l,v}^{\max}$: the last train's maximal generalized extra dwell time of line l at transfer station v .

$N_{l,v}$: the number of intermediate stations involved in the rear adjacent segment from transfer station v to its previous transfer station v' .

$C_{l,k,v}$: the connection time of last trains from the feeder train in line l to connecting train in line k at transfer station v .

$W_{l,k,v}$: the passengers' waiting time from line l to line k at transfer station v .

$T_{l,k,v}$: the average walking time of transfer passengers from the last train in line l to a train in line k at transfer station v .

$T_l^{\min}$, $T_l^{\max}$: respectively the earliest and latest service time of line l .

$\chi_{l,k,v}$: 0-1 variable, used to describe the relationships of transfer directions. Where $\chi_{l,k,v}=1$, it exists the connectivity from line l to line k at transfer station v ; else if $\chi_{l,k,v}=0$, passengers have no chance to transfer in this transfer direction.

2.2. Connection relationship of last trains analysis

2.2.1. Two last trains' connection process in a transfer station. If there is a connection relationship of two last trains from line l to line k at transfer station v , the connection time of last trains in the transfer direction is shown in equation (1). And the connection relationship of the last trains is determined by the connection time, which is described in Figure 2.

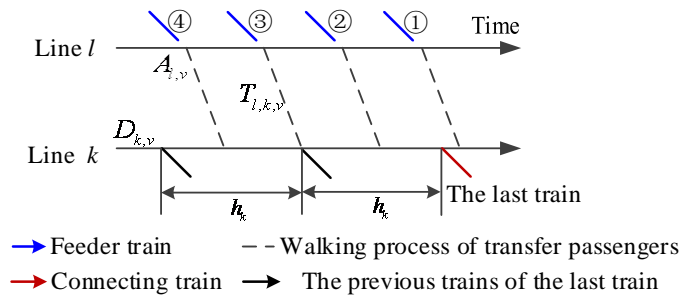$$C_{l,k,v} = D_{k,v} - A_{l,v} - T_{l,k,v} \tag{1}$$

Figure 2. Example of last train connection in a single direction

1) If $C_{l,k,v} \le 0$, the relationship is invalid, which means passengers cannot transfer from the last train in line l to a train in line k , the passengers' waiting time equals infinity.

2) If $0 < C_{l,k,v} < h_k$, the relationship is valid, which means passengers can transfer from the last train in line l to a train in line k . The passengers waiting time is equal to last trains' connection time.

3) If $t_{l,k,v}^C = h_k$, the relationship is valid, which means passengers just miss the previous train of the last train in line k , but they can wait for the last train. So this case is similar to the second one.

4) If $C_{l,k,v} > h_k$, the relationship is valid, which means passengers take the pre-trains of the last train in line k . At this time, the passengers' waiting time is equal to the remainder of last trains' connection time divided by headway of line k .

In summary, it can be seen that when the connection time of last trains is more than 0, then the transfer direction of trains can achieve a valid connection, so the transfer direction is namely by the "coordinated direction", the valid connections of the two trains are named by a pair of "coordinated trains". Otherwise, the connection relationship is invalid; passengers will travel off and cannot reach their destination.

2.2.2. The connection process of a line's last train connected by other lines' last trains at different transfer stations. Due to the network connectivity of MRT, the last train of a line will be connected by other lines' last trains at different transfer stations. Meanwhile, trains running time of every section and dwell time of stations are fundamentally fixed. Taking the scene of Figure 3 as an example to illustrate, once the departure time of the origin station is determined, the arrival time at other stations is also basically determined. So that last trains' departure times of different lines at the same station have interaction effects when the timetables are optimized based on coordinated and synchronized connection. The effects are more pronounced for multiple transfer stations of complex network structures.
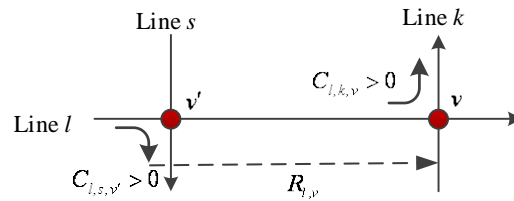

Figure 3. Interaction effects of last trains at neighboring transfer stations

3. Optimization model of last trains' dwell time and departure time

3.1. Model objectives

The optimization goals are minimizing the volume of unable transfer passengers and minimizing the difference between the last trains' actual departure times at the origin station and the optimized ones. The model objective is shown as equation (2):

$$\min \quad Z = \alpha \sum_{l=1}^{m} \sum_{k=1}^{m} \sum_{v=1}^{n} \chi_{l,k,v}(1-\eta_{l,k,v})p_{l,k,v} + \beta \sum_{l=1}^{m} |D_l^0 - D_l'^0| \tag{2}$$

Where α, β are coefficients; $p_{l,k,v}$ is the volume of transfer passengers from the last train in line l to the train in line k at transfer station v; $\eta_{l,k,v}$ is an 0-1 variable described connection relationship of last trains, if $\eta_{l,k,v}=1$, passengers can successfully transfer, else if $\eta_{l,k,v}=0$, passengers cannot transfer. And it is determined by last trains' connection time, as shown in formula (3):

$$\eta_{l,k,v} = \begin{cases} 1, & D_{k,v} - A_{l,v} - T_{l,k,v} > 0 \\ 0, & D_{k,v} - A_{l,v} - T_{l,k,v} \le 0 \end{cases} \tag{3}$$

In order to have a linear programming model, the formula (3) can be transformed as follows:

$$\eta_{l,k,v}M >= (D_{k,v} - A_{l,v} - T_{l,k,v}) - \xi \tag{4}$$

$$(1-\eta_{l,k,v})M - \xi >= -D_{k,v} + A_{l,v} + T_{l,k,v} \tag{5}$$

Where M is a large enough positive; ξ is a small enough positive.

3.2. Model constraints

3.2.1. Train running time. Because the arrival time of trains must meet the constraints of the train running time, the last trains' arrival and departure times can be respectively expressed as equation (6) and equation (7):

$$A_{l,v} = D_l^0 + R_{l,v} + \sum_{i \in b_{l,v}} E_{l,i} - \frac{E_{l,v}}{N_{l,v} + 1} \tag{6}$$

$$D_{l,v} = D_l^0 + R_{l,v} + \sum_{i \in b_{l,v}} E_{l,i} + s_{l,v} \tag{7}$$

Where $b_{l,v}$ is a set of transfer stations located back in transfer station v of line l (including transfer station v).

3.2.2. Generalized dwell time. After simplifying network structure, dwell time of trains in the section $v' \rightarrow v$ can be converted to the dwell time of transfer station v as shown in Figure 4. In this paper, we define the generalized dwell time of trains, which is the total dwell time at their intermediate stations and the transfer station v (not including the transfer station v'). And these stations are contained in its rear adjacent segment started from transfer station v to its previous transfer station v' (or its origin station). Then we define $S_{l,v}$ as the generalized dwell time of line l at transfer station v , which equals the sum of actual dwell time at the stations located in the rear adjacent segment from transfer station v to its previous transfer station v' .
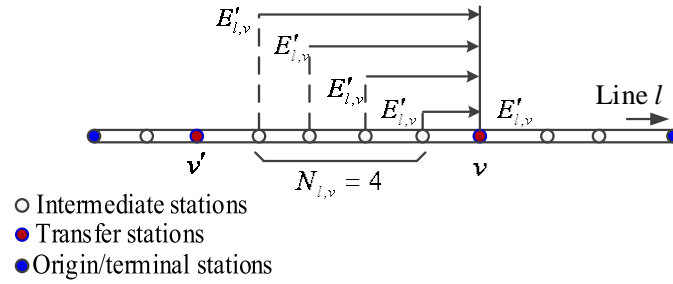

Figure 4. The dwell time transformation within neighboring transfer stations

Meanwhile, if the number of intermediate stations is $N_{l,v}$, the average of extra dwell time is shown in equation (8).

$$E'_{l,v} = E_{l,v} / (N_{l,v} + 1) \tag{8}$$

In actual transportation organization process, train dwell time is generally controlled in a certain range. So the generalized extra dwell time should also meet a specified constraint.

$$E_{l,v}^{\min} \leq E_{l,v} \leq E_{l,v}^{\max}, \forall l \in L, v \in V(l) \tag{9}$$

In order to ensure the rationality of the model parameter, a certain proportion range of extra dwell time is set based on the actual dwell time. Firstly, we define $T_l^{S,base}$ as basic dwell time of line l . And α is the adjustment ratio of extra dwell time, it is taken by 0.5 here. Finally, the minimum and maximum extra dwell times of the segment in line are calculated by equation (10) and equation (11).

$$E_{l,v}^{\min} = \max\{(N_{l,v} + 1)s_{\min} - S_{l,v}, -\alpha S_{l,v}\} \tag{10}$$

$$E_{l,v}^{\max} = \min\{(N_{l,v} + 1)s_{\max} - S_{l,v}, \alpha S_{l,v}\} \tag{11}$$

Where $s_{\min}$ is the minimum dwell time by considering switching time of train door and passengers getting off and getting up time, generally it is taken by 25 seconds; $s_{\max}$ is passengers' acceptable maximum dwell time. Generally, it is not more than 90 seconds.

3.2.3. Train services time. Taking into account the line maintenance, vehicle maintenance and other factors, operating enterprise often set the earliest and latest end of the train operation time. The constraint is as shown in formula (12):

$$T_l^{\min} \leq D_l^0 \leq T_l^{\max}, \forall l \in L, v \in V(l) \tag{12}$$

In summary, the formulas (2, 4-7, 9-12) are formed the optimization model. Because the synchronized coordination timetable problem is N-P hard, and the efficiency of the algorithm directly determines the solution effect of the model on the large-scale MRT network. By rationally simplifying the network for improving the applicability of the model, the proposed model can be quickly solved by CPLEX tool.

4. Case study

4.1. Basic information

Beijing MRT system is the world's busiest in annual ridership, with averaging 10.35 million per day. We take Beijing MRT network as the object of a case study. And the case basic information is as follows:

(1) The study period is the last service period of 22: 00-24: 00 on one Wednesday of March 2018.

(2) The simplified transfer network structure of Beijing MRT network is shown in Figure 5, which includes a total of 19 lines (The eastern and western sections of Line 14 are regarded as two lines, and the Airport Line, XiJiao Line and S1 Line are not included by its special transfer mode), 38 rail lines, 54 transfer stations (ShuangJin station has not yet to achieve transfer), and 405 transfer directions.

(3) According to the train operation plans, the running time and actual dwell time (see Table 1) can be known.

(4) The transfer passengers' demand is derived from Auto Fare Collection Clearing Center (ACC). And the total amount of transfer passengers during this period is 144.37 thousand. Assuming that this passenger flow arriving is satisfied the regular distribution of uniform services, the total transfer passengers volume in the end period is 18820.

(5) The passengers' average walking time is obtained by actual investigations.

In addition, combined with the actual operation demand of Beijing MRT, the service periods of lines are limited, which mean that last trains' departure times of city-lines and suburban-lines are respectively not earlier than 22:30 and 22:00, the end service time of city-lines and suburban-lines should not be later than 00:30 and 24:00 respectively.
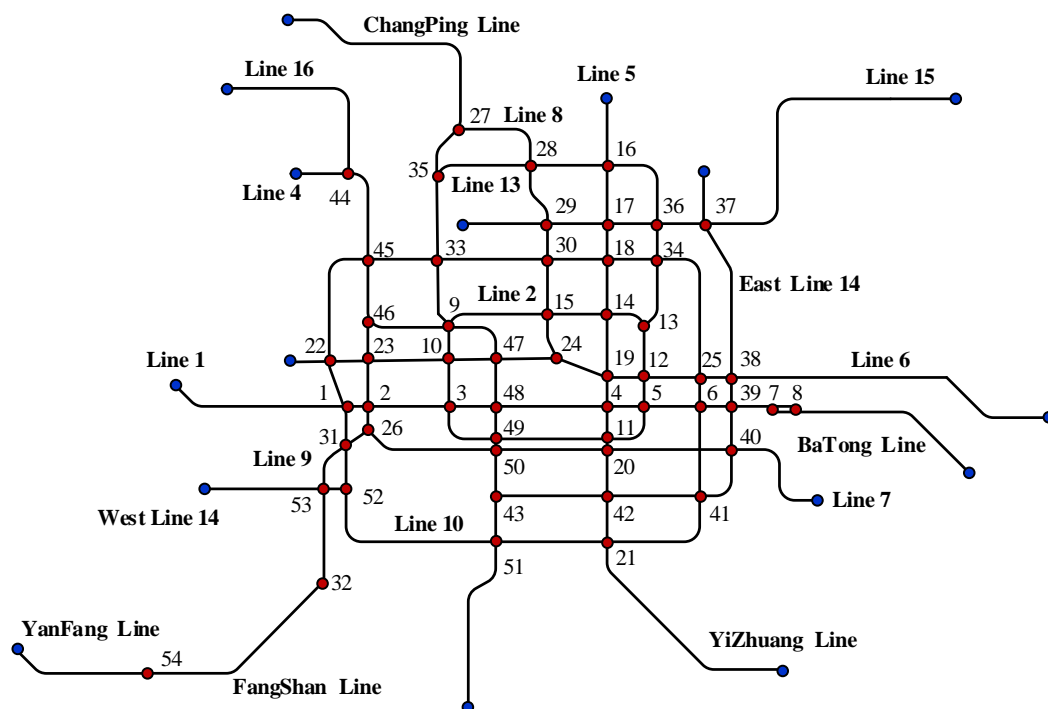

Figure 5. Schematic diagram of Beijing MRT network

Table 1. Trains initial dwell time of different lines

Line Name	Basic dwell time(s)	Line Name	Basic dwell time(s)	Line Name	Basic dwell time(s)
Line 1	26～60	Line 10	30～60	Line 4	30～60
Line 2	30～60	Line 13	30～60	West Line 14	35～60
Line 5	30～60	Line 15	30～60	East Line 14	35～60
Line 6	35～60	Changping Line	30～60	Line 16	40～60
Line 7	30～60	FangShan Line	30～60	YanFang Line	35～60
Line 8	30～60	YiZhuang Line	30～60		
Line 9	30～60	BaTong Line	30～60		

4.2. Analysis of the results

In the Intel i5 processor, 2.20GHz frequency, 8G memory on the computer, we use CPLEX tools to solve the model. The optimized and actual departure times of last trains are shown in Table 2. Compared to actual operation scheme, we can see some conclusions as following:

(1) There are 52% of lines, whose last trains' departure times are basically unchanged, such as the down and up directions of Line 13, Line 16, FangShan Line, YiZhuang Line, and BaTong Line;

(2) The number of last trains with the adjustment range of departure time located in (0, 5], (5, 10] and (10, 15] minutes are respectively eleven, six, and one.

(3) The largest adjustments of departure time appear in the last trains of East Line 14. Compared with the actual departure time at the origin station, the optimized ones in down and up directions of East Line 14 are respectively earlier than 15 minutes and 10 minutes, because the passenger volume of Station 52 and Station 53 transferring to East Line 14 is small. Meanwhile, due to the big passenger flow of the Line 4 in down direction, the optimized departure time is later than 9.9 minutes.

Table 2. The last trains' departure times of the optimized and actual scheme at the origin stations

Line number	Line name	Actual departure time	Optimized departure time
1	Line 1 in down direction	23:31:00	23:32:04
2	Line 2 in up direction	23:30:00	23:30:00
3	Line 2 in down direction	22:56:14	22:56:23
4	Line 2 in up direction	23:00:00	23:00:00
5	Line 5 in down direction	22:49:00	22:47:09
6	Line 5 in up direction	23:19:54	23:19:00
7	Line 6 in down direction	22:25:00	22:25:00
8	Line 6 in up direction	23:14:00	23:11:47
9	Line 7 in down direction	22:07:59	22:05:00
10	Line 7 in up direction	23:00:00	22:58:59
11	**Line 8 in down direction**	**23:12:25**	**23:19:00**
12	Line 8 in up direction	22:40:00	22:40:00
13	Line 9 in down direction	22:32:00	22:32:00
14	Line 9 in up direction	22:42:00	22:38:58
15	Line 10 in down direction	23:45:00	23:45:00
16	Line 10 in up direction	23:45:00	23:48:17
17	Line 13 in down direction	22:11:00	22:11:00
18	Line 13 in up direction	23:15:00	23:15:00

Table 2. cont.

Line number	Line name	Actual departure time	Optimized departure time
19	Line 15 in down direction	23:25:08	23:20:00
20	Line 15 in up direction	22:40:00	22:40:00
21	**ChangPing Line in down direction**	**22:45:14**	**22:53:03**
22	ChangPing Line in up direction	22:38:00	22:34:05
23	FangShan Line in down direction	22:28:00	22:28:00
24	FangShan Line in up direction	22:10:00	22:10:00
25	YiZhuang Line in down direction	22:30:00	22:30:00
26	YiZhuang Line in up direction	22:40:00	22:40:00
27	BaTong Line in down direction	22:30:00	22:30:00
28	BaTong Line in up direction	22:55:00	22:55:00
29	**Line 4 in down direction**	**22:37:31**	**22:47:25**
30	Line 4 in up direction	23:19:00	23:20:00
31	West Line 14 in down direction	22:50:00	22:50:00
32	West Line 14 in up direction	23:35:53	23:40:00
33	***East Line 14 in down direction***	***23:40:00***	***23:25:00***
34	***East Line 14 in up direction***	***22:30:00***	***22:20:00***
35	Line 16 in down direction	23:00:00	23:00:00
36	Line 16 in up direction	23:40:00	23:40:00
37	YanFang Line in down direction	22:30:00	22:30:00
38	YanFang Line in up direction	22:50:00	22:40:50

Table 3 gives the results of actual operation scheme (AOS) and optimized scheme based on extra dwell time (OSEDT), including the volume of unable transfer passengers (VUTP), the proportion of unable transfer passengers to total transfer passengers (PUT), optimized ratio of unable transfer passengers' optimized volume to total transfer passengers (ORUT), coordinated trains pairs (CTP), the proportion of coordinated trains to total transfer directions (PCT), and optimized ratio of coordinated trains' optimized pairs to total transfer directions (ORCT). Some conclusions can be seen by further analyzing:

(1) 65.6% of passengers in the AOS cannot transfer successfully. It suggests that many people are forced to leave the subway system without reaching their final destinations.

(2) Compared OSBEDT with AOS, the decreased volume of unable transfer passengers is 2832, and the optimized ratio of the decreased volume to total transfer passengers is 15.0%. They suggest that the optimized scheme can improve the connecting effect of last trains. And the service level of MRT in latest service period is obviously improved.

(3) From the perspective of the number of coordinated trains, it has 181 coordinated transfer directions for the AOS. Compared with the AOS, it increases by 8.9% for OSBEDT.

Table 3. The results comparison on the optimized scheme to actual operation scheme

Scheme	VUTP (people)	PUT(%)	ORUT (%)	CTP	PCT (%)	ORCT (%)
AOS	12340	65.6	-	181	44.7	-
OSBEDT	9508	50.5	15.0	217	53.6	8.9

Figure 6 shows their extra dwell times. It can be seen that:

(1) The large adjustment range of extra dwell time are in the loop lines, long-distance lines and some suburban-lines, such as Line 3, Line 8, Line 15, Line 16, Line 29, Line 30, Line 23, Line 25, Line 27, Line 25, and Line 31. The average extra dwell time is about 20 seconds.

(2) The largest value of positive extra dwell time is 30 seconds at Station 9, Station 21, Station 52, and Station 54. Those stations are origin station of suburban-lines with 56～90 seconds dwell time. And the biggest dwell time is 90 second at Station 21.

(3) When the last train runs at the terminal station in suburban-lines and intermediate stations in loop lines, and city-lines, the extra dwell times are most negative. For example, the dwell time of Line 36 at Station 44 is reduced to 30 seconds; the dwell time of Line 29 at Station 8 is reduced to 25 seconds. Meanwhile, the dwell time of Line 3 at Station 11 to Station 15 are reduced to 25 seconds.
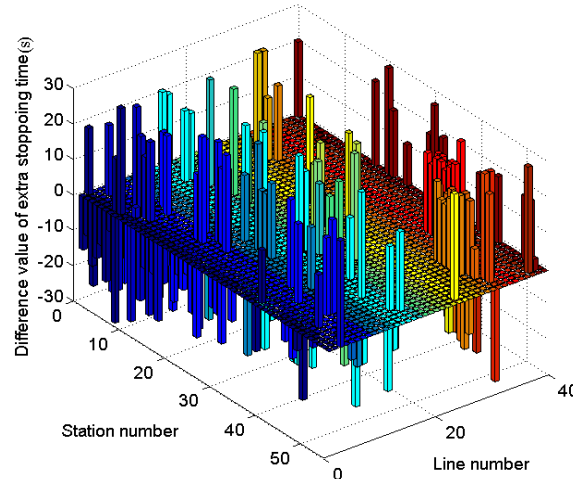

Figure 6. The extra dwell time of last trains at transfer stations

Finally, we make further analysis of coordinated directions of transfer passenger volume as shown in Table 4. Compared with the AOS, the following results can be drawn:

(1) The number of coordination directions is drastically increased by thirty-six, which means that setting extra dwell time can better satisfy the need of larger transfer passengers in last trains' service period.

(2) The effect of connection directions with larger transfer passengers is enhanced by OSEDT. For example, when the volume of transfer passengers is more than ninety, the number of the coordination directions is increased by ten; when the volume of transfer passengers is from sixty to ninety, the number of the coordination directions is increased by eight; when the volume of transfer passengers is from thirty to sixty, the number of the coordination directions is increased by eleven.

Table 4. The number of coordinated directions for different transfer passenger volume

Schemes	The range of enabled transfer passenger volumes				Total of coordinated directions
	[0,30)	[30,60)	[60,90)	≥ 90	
AOS	100	49	15	17	181
OSEDT	107	60	23	27	217

Therefore, the OSEDT not only considered the last trains' reachability requirement in MRT network, but also increases the number of coordination directions with larger transfer passengers and reduced the volume of unable transfer passengers, which finally improved the operation efficiency and transportation service level in the latest service period.

5. Conclusion

The article presented a synchronized optimization model of last trains' timetables in MRT network so as to decrease the number of unable transfer passengers and improve the operation efficiency and transportation service level of last trains. Firstly, we improved the applicability of the model for a large-scale MRT networks by simplifying the network structure. Then based on the adjusting the last trains' departure time and setting the extra dwell time, the model aimed to minimize unable transfer passenger' volume in the whole of network and minimize the difference between the last trains' actual departure times and the optimized ones. To verify the efficiency and accuracy of the proposed model,

we took the Beijing subway as a case study. Finally, the results indicate that the volume of unable transfer passengers is reduced by 15.0%, and the number of coordinated trains is increased by thirty-six (8.9%). The conclusion suggests that our approach can provide the optimized method for last trains' timetable in the large-scale MRT network. Furthermore, future works should consider the surrounding traffic environment, such as the transfer connection between MRT and integration of public transport in the end period of service time.

Acknowledgments

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