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# **Application of MCC-based Robust High-degree Cubature** Kalman Filter in Integrated Navigation System

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Abstract: As the integrated navigation system is a nonlinear system, in the case of non-gaussian noise, the traditional nonlinear gaussian filtering algorithm has a serious problem of decreasing filtering precision. In this paper, a new robust high-degree Cubature Kalman filtering algorithm is proposed, which takes into account the nonlinearity of the system and non-gaussian noise. The algorithm improves the measurement updating process by using the Maximum correntropy criterion(MCC), and converts the traditional measurement updating problem into the linear regressione quation solving problem. Combines the advantages of Maximum correntropy criterion and Cubature Kalman filter to deal with non-Gaussian and nonlinear systems. The proposed algorithm is applied to the SINS/GPS integrated navigation system, the simulation results show that the proposed algorithm's filtering performance is greatly affected by the kernel width. Under the condition of gaussian mixture noise, the new robust high-degree Cubature Kalman filter based on Maximum correntropy criterion(MCC-HCKF) is more robust and has higher filtering precision than the traditional high-degree Cubature Kalman filter(HCKF).

#### 1.Introduction

Strapdown inertial navigation system (SINS) and global positioning system (GPS) are combined to form SINS/GPS integrated navigation system that combines the advantages of both navigation. In the positioning of the integrated navigation system, the accuracy of the filtering algorithm has a great impact on the positioning accuracy [1-2]. Kalman filtering (KF) is the optimal estimation algorithm for solving linear system problems, but its accuracy is greatly affected for nonlinear systems and systems with noise non-Gaussian conditions. However, the mathematical model of actual physical system is usually non-linear, that is, the system equation or measurement equation is non-linear. At this time, the nonlinear filtering algorithm becomes the key to improve the accuracy of integrated navigation system.

Extended Kalman filter (EKF) expands the Taylor series of the original system and measurement equation and retains linear terms. Truncation error is actually introduced, the filtering precision can only reach the first order, and the calculation amount of Jacobian matrix is also large. Particle filter (PF) is a kind of filtering algorithm which does not need to make an approximation to the state equation, nor to assume the statistical characteristics of noise. However, due to the features of particle degradation and high computational complexity, the filtering accuracy is not high, which is not convenient for practical application. The more commonly used nonlinear filtering method is the gaussian approximate summation filtering method represented by untracked Kalman filtering (UKF)

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and Cubature Kalman filtering (CKF). Different from the linearization idea of EKF, UKF and CKF approximate the density function of state vectors by using a series of selected sample points to approximate the mean and covariance of the gaussian distribution of random variables. The filtering accuracy is high and there is no need to calculate the Jacobian matrix[3-4]. Feng Sun and Lijun Tang studied the filtering accuracy and numerical stability of UKF and CKF, it is pointed out that CKF should be adopted for three-dimensional and above systems, and the numerical stability of CKF algorithm is higher than that of UKF[4]. Jia bin et al. studied the HCKF algorithm of the high cubature rule, and the filtering precision can reach the fifth order[5].

In actual physical systems, measurement information is often interfered, and measurement noise does not satisfy the hypothesis of gaussian noise [6-7]. Therefore, the accuracy of the filtering algorithm based on the gaussian distribution hypothesis of noise will decline or even diverge in practice. Huber proposed M estimation to solve the problem of symmetric interference near gaussian distribution [8-9]. Cheng Chen, Hongxin Jin et al. proposed the robust filtering algorithm based on Huber method and made successful application in target tracking and UAV navigation [10-14]. Maximum Correntropy criterion(MCC) is a robust algorithm proposed in recent years [15]. Badong Chen proposed a robust KF algorithm based on MCC by combining MCC with KF filtering, and verified its feasibility. In this paper, the MCC method is combined with the HCKF algorithm and the integrated navigation system is used for simulation verification. The simulation results show that the algorithm is superior to the traditional non-linear filtering algorithm when the measurement noise is gaussian mixture distribution.

## 2. The Model of SINS/GPS Integrated Navigation System

## 2.1 Integrated navigation system equation of state

The inertial coordinate system is *i*, the earth coordinate system is *e*, and the carrier coordinate system is *b*. The local geographic coordinate system, that is, the "east, north, and sky" coordinate system is selected as the navigation coordinate system, which is *n*, and the calculated navigation coordinate system obtained by SINS is *P*. Because SINS is affected by various error sources, there is a misalignment angle  $\phi = [\phi_x \quad \phi_y \quad \phi_z]^T$  between the *n* and the *P* calculated by the navigation computer. It represents a set of Euler angles from *n* to *P*, and the order of rotation is  $\phi_z, \phi_x, \phi_y$ . So the coordinate transformation matrix from *n* to *P* can be described as

$$\mathbf{C}_{n}^{p} = \begin{bmatrix} \cos\phi_{y} & 0 & -\sin\phi_{y} \\ 0 & 1 & 0 \\ \sin\phi_{y} & 0 & \cos\phi_{y} \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi_{x} & \sin\phi_{x} \\ 0 & -\sin\phi_{x} & \cos\phi_{x} \end{bmatrix} \cdot \begin{bmatrix} \cos\phi_{z} & \sin\phi_{z} & 0 \\ -\sin\phi_{z} & \cos\phi_{z} & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
= \begin{bmatrix} \cos\phi_{y} \cos\phi_{z} + \sin\phi_{y} \sin\phi_{x} \sin\phi_{x} & -\cos\phi_{y} \sin\phi_{x} \cos\phi_{z} & -\sin\phi_{y} \cos\phi_{x} \\ \cos\phi_{x} \sin\phi_{z} & \cos\phi_{x} \cos\phi_{z} & -\sin\phi_{y} \cos\phi_{x} \end{bmatrix} \begin{pmatrix} 1 & 0 & 0 \\ -\sin\phi_{z} & \cos\phi_{z} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
(1)

The GPS/SINS pine integrated navigation system was used as the filtering model, and the error equation of attitude, speed, position, gyro zero bias and accelerometer zero bias of the SINS was used as the state equation of the nonlinear filter model, and the speed and position error between the SINS and GPS were used as the measurement information. Nonlinear error equation of attitude, speed and position of SINS [17] is described as

$$\begin{vmatrix} \dot{\boldsymbol{\phi}} = \boldsymbol{C}_{\phi}^{-1} \left( \left( \boldsymbol{I} - \boldsymbol{C}_{n}^{p} \right) \tilde{\boldsymbol{\omega}}_{in}^{n} + \boldsymbol{C}_{n}^{p} \delta \boldsymbol{\omega}_{in}^{n} - \boldsymbol{C}_{b}^{p} \delta \boldsymbol{\omega}_{ib}^{b} \right) \\ \delta \dot{\boldsymbol{v}}^{n} = \left( \boldsymbol{C}_{n}^{p} - \boldsymbol{I} \right) \boldsymbol{C}_{b}^{n} \boldsymbol{f}^{b} + \boldsymbol{C}_{b}^{p} \delta \boldsymbol{f}^{b} \\ + \left( 2\delta \boldsymbol{\omega}_{ie}^{n} + \delta \boldsymbol{\omega}_{en}^{n} \right) \times \boldsymbol{v}^{n} - \left( 2\boldsymbol{\omega}_{ie}^{n} + \boldsymbol{\omega}_{en}^{n} \right) \times \delta \boldsymbol{v}^{n} \\ \delta \dot{\boldsymbol{L}} = \frac{\delta \boldsymbol{v}_{ie}^{n}}{\tilde{R}_{M} + \tilde{h}} \qquad (2) \\ \delta \dot{\boldsymbol{\lambda}} = \frac{\delta \boldsymbol{v}_{E}^{n}}{\tilde{R}_{N} + \tilde{h}} \operatorname{secL} + \delta \boldsymbol{L} \frac{\boldsymbol{v}_{E}^{p}}{\tilde{R}_{N} + \tilde{h}} \operatorname{tan} \tilde{L} \operatorname{sec} \tilde{\boldsymbol{L}} \\ \delta \dot{\boldsymbol{h}} = \delta \boldsymbol{v}_{U} \\ \dot{\boldsymbol{\varepsilon}}_{bx} = 0, \ \dot{\boldsymbol{\varepsilon}}_{by} = 0, \ \dot{\boldsymbol{\varepsilon}}_{bz} = 0 \\ \dot{\boldsymbol{V}}_{bx} = 0, \ \dot{\boldsymbol{V}}_{by} = 0, \ \dot{\boldsymbol{V}}_{bz} = 0 \end{aligned}$$

The coefficient matrix  $C_{\phi}^{-1}$  in equation (2) is described as

$$\boldsymbol{C}_{\phi}^{-1} = \frac{1}{\cos\phi_{x}} \begin{bmatrix} \cos\phi_{x}\cos\phi_{y} & 0 & \sin\phi_{y}\cos\phi_{x} \\ \sin\phi_{x}\sin\phi_{y} & \cos\phi_{x} & -\sin\phi_{x}\cos\phi_{y} \\ -\sin\phi_{y} & 0 & \cos\phi_{y} \end{bmatrix}$$
(3)

In equation (2),  $C_b^p$  is the coordinate transformation matrix from the system *b* to the computational navigation coordinate system *P*.  $\tilde{\boldsymbol{\omega}}_{ln}^n$  is the projection of the angular velocity of the navigation coordinate system relative to the inertial coordinate system on the navigation coordinate system, and  $\delta \boldsymbol{\omega}_{ln}^p$  is the calculation error of the navigation coordinate system.  $\delta \boldsymbol{\nu}^n$  is the calculation error of the angular velocity of the navigation coordinate system relative to the earth coordinate system on the calculation navigation coordinate system.  $\delta \boldsymbol{\omega}_{en}^n$  is the projection of the angular velocity of the navigation coordinate system relative to the earth coordinate system on the calculation navigation coordinate system.  $\delta \boldsymbol{\omega}_{en}^n$  is the calculation error of  $\boldsymbol{\omega}_{en}^p$ ,  $\boldsymbol{\omega}_{en}^p$  is the projection of the earth's rotation velocity in the calculation navigation coordinate system,  $\delta \boldsymbol{\omega}_{en}^n$  is the calculation error of  $\boldsymbol{\omega}_{en}^p$ .  $\delta \lambda \ \delta h$  are respectively latitude error, longitude error and height error.  $\delta \boldsymbol{\omega}_{bn}^n$  is the measurement error of the gyroscope, consists of gyro constant drift  $\boldsymbol{\varepsilon}_b = \begin{bmatrix} \varepsilon_{bx} & \varepsilon_{by} & \varepsilon_{bz} \end{bmatrix}^T$  and gyro random drift  $\boldsymbol{\omega}_g = \begin{bmatrix} \omega_{gx} & \omega_{gy} & \omega_{gz} \end{bmatrix}^T$ ,  $\boldsymbol{\omega}_g$  is zero mean gaussian white noise.  $\tilde{\boldsymbol{f}}^h$  is the actual measured value of the accelerometer,  $\delta \boldsymbol{f}^h$  is the calculation error of the accelerometer random drift  $\boldsymbol{\omega}_a = \begin{bmatrix} \omega_{ax} & \omega_{ay} & \omega_{ax} \end{bmatrix}^T$ ,  $\boldsymbol{\omega}_a$  is zero mean gaussian white noise.

The 15-dimensional state vector is described as  $X = [\phi_x \quad \phi_y \quad \phi_z \quad \delta v_x \quad \delta v_y$ 

$$\delta v_z \delta L \delta \lambda \delta h \epsilon_{bx} \epsilon_{by} \epsilon_{bz} \nabla_{bx} \nabla_{by} \nabla_{bz}]^T$$

The state equation of the SINS/GPS integrated navigation system can be derived by equation (2)  $\dot{X} = f(X) + w$  (4)

 $f(\bullet)$  is a non-linear function, w is system noise.

#### 2.2 Integrated Navigation System Measuremen-t Equation

The difference of velocity and position information output of SINS and GPS is used as the measurement value of filter

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$$\boldsymbol{Z} = \begin{bmatrix} \boldsymbol{v}_{x,SINS} - \boldsymbol{v}_{x,GPS} \\ \boldsymbol{v}_{y,SINS} - \boldsymbol{v}_{y,GPS} \\ \boldsymbol{v}_{z,SINS} - \boldsymbol{v}_{z,GPS} \\ \boldsymbol{L}_{SINS} - \boldsymbol{L}_{GPS} \\ \boldsymbol{\lambda}_{SINS} - \boldsymbol{\lambda}_{GPS} \\ \boldsymbol{H}_{SINS} - \boldsymbol{H}_{GPS} \end{bmatrix} = \begin{bmatrix} \delta \boldsymbol{v}_{x} \\ \delta \boldsymbol{v}_{y} \\ \delta \boldsymbol{v}_{z} \\ \delta \boldsymbol{L} \\ \delta \boldsymbol{\lambda} \\ \delta \boldsymbol{h} \end{bmatrix}$$
(5)

(6)

the measurement equation of the integrated navigation system is

$$Z = HX + v$$

 $H = [\mathbf{0}_{6\times3} \ I_{6\times6} \ \mathbf{0}_{6\times6}]$  is Measurement matrix, v is Measurement noise.

## 3. Robust High-degree CKF algorithm based on MCC

#### 3.1 Maximum correlation entropy

Suppose two random variables  $x, y \in \mathbf{R}$ , the joint probability density function is  $p_{xy}(x, y)$ , and the correlation entropy is defined as

$$V(x, y) = E[\kappa(x, y)] = \iint \kappa(x, y) p_{xy}(x, y) \cdot dxy \qquad (7)$$

in the equation, E represents expectation,  $\kappa(x, y)$  denotes the gaussian kernel function, the specific expression is

$$\kappa(x,y) = \mathbf{G}_{\sigma}(e) = \exp\left(-\frac{e^2}{2\sigma^2}\right)$$
(8)

Where,  $e=x-y,\sigma$  denotes the scale of kernel and  $\sigma > 0$ . In practical application, only limited data can be obtained and the joint probability density is unknown, and the approximate solution of V(x,y) can be obtained by using a series of sampling points [16]. At this point, equation (8) can be written as

$$\hat{V}(X,Y) = \frac{1}{N} \sum_{i=1}^{N} \boldsymbol{G}_{\sigma}(\boldsymbol{e}(i))$$
(9)

Figure 1 is a schematic diagram of correlation entropy when  $\sigma = 1$ , it can be found that correlation entropy measures the generalized similarity degree between two random variable, and the contribution of error *e* will attenuate in exponential form with the change of kernel width  $\sigma$ , when e=0,  $\kappa(x,y)=1$  takes the maximum value. The cost function based on MCC criterion is defined as



3.2 High-degree Cubature Kalman filtering

1) Initialization of filter

$$\begin{cases} \hat{\boldsymbol{X}}_{0} = \mathbf{E}[\boldsymbol{X}_{0}] \\ \boldsymbol{P}_{0} = \mathbf{E}\left[ \left(\boldsymbol{X}_{0} - \hat{\boldsymbol{X}}_{0}\right) \left(\boldsymbol{X}_{0} - \hat{\boldsymbol{X}}_{0}\right)^{\mathrm{T}} \right] \end{cases}$$
(11)

2) Time update

a) The calculation of cubature points

$$X_{i,k-1|k-1}(i=0,\cdots 2n^2)$$

The singular value decomposition (SVD) with higher numerical stability is used instead of the traditional Cholesky decomposition. This decomposition method c-an better solve the problem of ill-condition of covariance matrix, so that the entire algorithm has higher numerical stability and filtering accuracy. Applying SVD decomposition to covariance matrix  $P_{k-1/k-1}$ , the decomposition result is shown as

$$P_{k-1/k-1} = U_{k-1} S_{k-1} V_{k-1}^{T}$$
(12)  
$$X_{i,k-1|k-1} = U_{k-1|k-1} \sqrt{s_i} \xi_i + \hat{x}_{k-1|k-1}$$
(13)

Where,  $S = diag(s_1, s_2, s_3, ..., s_r)$  denotes singular value of  $P_{k-1/k-1}$ ,  $s_1 \ge s_2 \ge s_3 \ge ... \ge s_r \ge 0$ ,  $U \in \mathbb{R}^{m \times m}$ ,  $V \in \mathbb{R}^{n \times n}$ .  $\xi_i$  denotes integrate point set, when the fifth-order cubature principle is used, the number of cubature point is  $2n^2 + 1$ , specific expression of  $\xi_i$  is written as [5,17]

$$\boldsymbol{\xi}_{i} = \begin{cases} [0, \dots, 0]^{\mathrm{T}} & i = 0 \\ \sqrt{(n+2)} \mathbf{s}_{i}^{+} & i = 1, \dots, n(n-1)/2 \\ -\sqrt{(n+2)} \mathbf{s}_{i-n(n-1)/2}^{+} & i = n(n-1)/2 + 1, \dots, n(n-1) \\ \sqrt{(n+2)} \mathbf{s}_{i-n(n-1)}^{-} & i = n(n-1) + 1, \dots, 3n(n-1)/2 \quad (14) \\ -\sqrt{(n+2)} \mathbf{s}_{i-3n(n-1)/2}^{-} & i = 3n(n-1)/2 + 1, \dots, 2n(n-1) \\ \sqrt{(n+2)} \mathbf{e}_{i-2n(n-1)} & i = 2n(n-1) + 1, \dots, n(2n-1) \\ -\sqrt{(n+2)} \mathbf{e}_{i-n(2n-1)} & i = n(2n-1) + 1, \dots, 2n^{2} \end{cases}$$

Where, *n* denotes the state dimension of system,  $e_i$  denotes *n* dimensional unit vector whose *i* th element

$$\begin{cases} \mathbf{s}_{j}^{+} = \sqrt{\frac{1}{2}}(\mathbf{e}_{l} + \mathbf{e}_{k}) : l < k, l, k = 1, 2, \cdots, n \\ \mathbf{s}_{j}^{-} = \sqrt{\frac{1}{2}}(\mathbf{e}_{l} - \mathbf{e}_{k}) : l < k, l, k = 1, 2, \cdots, n \end{cases}$$
(15)

b) The calculation of cubature point transmitted by state equation

$$X_{i,k/k-1}^{*} = f(X_{i,k-1|k-1})$$
(16)

c) The calculation of one step prediction of state

$$\hat{\mathbf{x}}_{k/k-1} = \sum_{i=0}^{2n^{+}} \omega_i \left( \mathbf{X}_{k/k-1,i}^* \right)$$
(17)

Where,  $\omega_i$  denotes the weight of cubature points, which is shown as follows:

$$\omega_{i} = \begin{cases} \omega_{0} = \frac{2}{n+2} & i = 0\\ \omega_{1} = \frac{1}{(n+2)^{2}} & i = 1, 2, \cdots, 2n(n-1)\\ \omega_{2} = \frac{4-n}{2(n+2)^{2}} & i = 2n(n-1)+1, \cdots, 2n^{2} \end{cases}$$
(18)

d) The calculation of predicted error covariance matrix

$$\boldsymbol{P}_{k/k-1} = \sum_{i=0}^{2n^{2}} \omega_{i} \left( \boldsymbol{X}_{i,k/k-1}^{*} - \hat{\boldsymbol{x}}_{k/k-1} \right) \left( \boldsymbol{X}_{i,k/k-1}^{*} - \hat{\boldsymbol{x}}_{k/k-1} \right)^{\mathrm{T}} + \boldsymbol{Q}_{k-1} (19)$$

- 3) Measurement update
- e) The calculation of updated state cubature point

$$\boldsymbol{X}_{i,k|k-1} = \boldsymbol{S}_{k|k-1} \boldsymbol{\xi}_i + \hat{\boldsymbol{x}}_{k|k-1}$$
(20)

Where,  $P_{k/k-1} = S_{k|k-1} (S_{k|k-1})^{T}$ .

f) The calculation of cubature point transmitted by measurement equation

$$\boldsymbol{Z}_{i,k|k-1} = \boldsymbol{H}_k \boldsymbol{X}_{i,k|k-1} \tag{21}$$

g) The calculation of measurement prediction

$$\hat{z}_{k|k-1} = \sum_{i=0}^{2n^2} \omega_i Z_{i,k|k-1}$$
(22)

h) The calculation of measurement error covariance matrix and predicted cross-correlation covariance matrix

$$P_{zz,k|k-1} = \sum_{i=0}^{2n^{2}} \omega_{i} \left( Z_{i,k|k-1} - \hat{z}_{k|k-1} \right) \left( Z_{i,k|k-1} - \hat{z}_{k|k-1} \right)^{\mathrm{T}} + R_{k} \quad (23)$$

$$P_{xz,k|k-1} = \sum_{i=0}^{2n^{2}} \omega_{i} \left( X_{i,k|k-1} - \hat{x}_{k/k-1} \right) \left( Z_{i,k|k-1} - \hat{z}_{k|k-1} \right)^{\mathrm{T}} \quad (24)$$

4) Filter update

i)The calculation of filter gain matrix, filter state and covariance matrix

$$\boldsymbol{K}_{k} = \boldsymbol{P}_{\boldsymbol{x}\boldsymbol{z},\boldsymbol{k}|\boldsymbol{k}-1} (\boldsymbol{P}_{\boldsymbol{z}\boldsymbol{z},\boldsymbol{k}|\boldsymbol{k}-1})^{-1}$$
(25)

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k \left( z_k - \hat{z}_{k|k-1} \right)$$
(26)

$$\boldsymbol{P}_{k|k} = \boldsymbol{P}_{k|k-1} - \boldsymbol{K}_k \, \boldsymbol{P}_{zz,k|k-1} (\boldsymbol{K}_k)^{\mathrm{T}}$$
(27)

## 3.3 MCC for statistical linear regression

The core of the MCC method is to maximize the defined cost function. By applying MCC to HCKF and changing the way of measurement update, a robust HCKF algorithm based on MCC is obtained. First, the state prediction error is defined as

$$\boldsymbol{\delta}_{k} = \boldsymbol{x}_{k} - \hat{\boldsymbol{x}}_{k|k-l} \quad (28)$$

Where,  $x_k$  denotes the truth value of state at moment k, and  $\hat{x}_{k|k-1}$  denotes the step prediction value at moment k. According to equation (6), the measurement equation of the integrated navigation system is linear. By reference [18], at this point, equations (22), (23) and (24) can be further written as

$$\tilde{\boldsymbol{z}}_{k|k-1} = \boldsymbol{H}_k \tilde{\boldsymbol{x}}_{k|k-1}$$
(29)

$$\boldsymbol{P}_{zz,k|k-1} = \boldsymbol{H}_k \boldsymbol{P}_{k/k-1} \boldsymbol{H}_k^1 + \boldsymbol{R}_k \tag{30}$$

$$\boldsymbol{P}_{\boldsymbol{x}\boldsymbol{z},\boldsymbol{k}|\boldsymbol{k}-1} = \boldsymbol{P}_{\boldsymbol{k}/\boldsymbol{k}-1}\boldsymbol{H}_{\boldsymbol{k}}^{\mathrm{T}}$$
(31)

From the above three equations, linear regression equation can be constructed as

$$\begin{bmatrix} \boldsymbol{z}_k \\ \hat{\boldsymbol{x}}_{k|k-1} \end{bmatrix} = \begin{bmatrix} \boldsymbol{H}_k \\ \boldsymbol{I} \end{bmatrix} \boldsymbol{x}_k + \begin{bmatrix} \boldsymbol{v}_k \\ -\boldsymbol{\delta}_k \end{bmatrix}$$
(32)

Some variables are defined as

$$\mathbf{S}_{k} = \begin{bmatrix} \mathbf{R}_{k} & 0\\ 0 & \mathbf{P}_{k|k-1} \end{bmatrix} = \begin{bmatrix} \mathbf{S}_{u,k} \mathbf{S}_{u,k}^{\mathrm{T}} & 0\\ 0 & \mathbf{S}_{l,k} \mathbf{S}_{l,k}^{\mathrm{T}} \end{bmatrix} \quad (33)$$
$$\mathbf{y}_{k} = \mathbf{S}_{k}^{-1} \begin{bmatrix} \mathbf{z}_{k}\\ \hat{\mathbf{x}}_{k|k-1} \end{bmatrix} \quad (34)$$
$$\mathbf{B}_{k} = \mathbf{S}_{k}^{-1} \begin{bmatrix} \mathbf{H}_{k}\\ \mathbf{I} \end{bmatrix} \quad (35)$$
$$\mathbf{e}_{k} = \mathbf{S}_{k}^{-1} \begin{bmatrix} \mathbf{v}_{k}\\ -\boldsymbol{\delta}_{k} \end{bmatrix} \quad (36)$$

So the expression (32) can be rewritten as

 $\boldsymbol{y}_k = \boldsymbol{B}_k \boldsymbol{x}_k + \boldsymbol{e}_k \tag{37}$ 

The cost function defined as

$$J(e) = \left(1 - \exp\left(-e^2/2\sigma^2\right)\right) / \sqrt{2\pi} \sigma \qquad (38)$$

Equation (38) can be equivalent to the MCC form shown in equation (10)

$$J(e) = \arg \min_{x_k} \sum_{i=1}^{N} J_{MCC}(e_i)$$
  
$$= \arg \min_{x_k} \sum_{i=1}^{N} (1 - \exp(-e^2 / 2\sigma^2)) / \sqrt{2\pi} \sigma$$
  
$$\Leftrightarrow \arg \max_{x_k} \sum_{i=1}^{N} G_{\sigma}(e_{i,k})$$
  
$$= \arg \max_{x_k} \sum_{i=1}^{N} G_{\sigma}(y_{i,k} - B_k x_{i,k})$$
  
(39)

Where, N = n + m,  $e_{i,k}$  denotes the *i*-th element of the residual vector  $e_{i,k} = y_{i,k} - B_k x_{i,k}$ 

The maximum value is obtained by taking the partial derivative of the cost function, specific expression is written as

$$\frac{\partial J(\mathbf{x}_{k})}{\partial \mathbf{x}_{k}} = \frac{\partial \sum_{i=1}^{N} \mathbf{G}_{\sigma}(\mathbf{e}_{i,k})}{\partial \mathbf{e}_{i,k}} \cdot \frac{\partial \mathbf{e}_{i,k}}{\partial \mathbf{x}_{i,k}}$$

$$= \sum_{i=1}^{N} \mathbf{e}_{i,k} \cdot \exp\left(-\mathbf{e}_{i,k}^{2} / 2\sigma^{2}\right) \cdot \frac{\partial \mathbf{e}_{i,k}}{\partial \mathbf{x}_{i,k}} = 0$$
(40)

Where,  $\varphi_k(e_i) = e_{i,k} \cdot G_{\sigma}(e_{i,k})$ , the above expression can be rewritten as

$$\sum_{i=1}^{N} \varphi_{k}\left(e_{i}\right) \cdot \frac{\partial e_{i,k}}{\partial x_{i,k}} = 0$$

$$\tag{41}$$

The weight function is defined as  $C_k = \varphi_k(e_i)/e_i$ , and substitute it into equation (41), we can get

$$\boldsymbol{B}_{k}^{\mathrm{T}}\boldsymbol{C}_{k}\left(\boldsymbol{B}_{k}\boldsymbol{x}_{k}-\boldsymbol{y}_{k}\right)=0 \tag{42}$$

The above formula can be solved by the fixed point iteration method proposed in literature [16]. Meanwhile, Badong Chen analyzed that the iterative solution is always convergent and obtains the iterative solution[17]

$$\boldsymbol{x}_{k} = \left(\boldsymbol{B}_{k}^{\mathrm{T}}\boldsymbol{C}_{k}\boldsymbol{B}_{k}\right)^{-1}\boldsymbol{B}_{k}^{\mathrm{T}}\boldsymbol{C}_{k}\boldsymbol{y}_{k}$$
(43)

$$\boldsymbol{C}_{n,k} = diag\left(\boldsymbol{G}_{\sigma}\left(\boldsymbol{e}_{l,k}\right) \quad \cdots \quad \boldsymbol{G}_{\sigma}\left(\boldsymbol{e}_{n,k}\right)\right) \tag{44}$$

$$\boldsymbol{C}_{m,k} = diag\left(\boldsymbol{G}_{\sigma}\left(\boldsymbol{e}_{n+1,k}\right) \quad \cdots \quad \boldsymbol{G}_{\sigma}\left(\boldsymbol{e}_{n+m,k}\right)\right) \tag{45}$$

The variance obtained after iteration is

$$\boldsymbol{P}_{k|k} = \left(\boldsymbol{I} \cdot \boldsymbol{\overline{K}}_{k} \boldsymbol{H}_{k}\right) \boldsymbol{P}_{k|k-1} \left(\boldsymbol{I} \cdot \boldsymbol{\overline{K}}_{k} \boldsymbol{H}_{k}\right)^{\mathrm{T}} + \boldsymbol{\overline{K}}_{k} \boldsymbol{R}_{k} \boldsymbol{\overline{K}}_{k}^{\mathrm{T}}$$
(46)

Equation (43) can be further expressed as

$$\boldsymbol{x}_{k} = \hat{\boldsymbol{x}}_{k|k-1} + \overline{\boldsymbol{K}}_{k} \left( \boldsymbol{z}_{k} - \hat{\boldsymbol{z}}_{k} \right)$$
(47)

Where

$$\overline{\boldsymbol{K}}_{k} = \overline{\boldsymbol{P}}_{k|k-1} \boldsymbol{H}_{k}^{\mathrm{T}} \left( \boldsymbol{H}_{k} \overline{\boldsymbol{P}}_{k|k-1} \boldsymbol{H}_{k}^{\mathrm{T}} + \overline{\boldsymbol{R}}_{k} \right)^{-1}$$
(48)

$$\bar{P}_{k|k-1} = S_{l,k|k-1} C_{n,k}^{-1} S_{l,k|k-1}^{T}$$
(49)

$$\overline{\boldsymbol{R}}_{k} = \boldsymbol{S}_{u,k} \boldsymbol{C}_{m,k}^{-1} \boldsymbol{S}_{u,k}^{\mathrm{T}}$$
(50)

## 3.4 High-degree Kalman filtering algorithm based on MCC

Combining the above MCC method with high-degree Cubature Kalman filter, the measurement update process in HCKF is transformed into the problem of solving linear regression equations, and a new robust filtering algorithm is obtained. The specific flow of the algorithm is summarized as follows.

1 Initialization

Set the initial state and variance of the filter,  $\hat{\mathbf{x}}_0 = E[\mathbf{x}_0], \mathbf{P}_0 = E[(\mathbf{x}_0 - \hat{\mathbf{x}}_0)(\mathbf{x}_0 - \hat{\mathbf{x}}_0)^T]$ , select the kernel width  $\sigma$  and iteration threshold  $\varepsilon = 10^{-6}$ .

(2) Time update

The time updating process of MCC-HCKF is consistent with that of traditional HCKF in section 3.2. One-step state prediction  $x_{k|k-1}$  and covariance  $P_{k|k-1}$  of k moment are obtained by using equations (11)-(19) and cubature points and their weights (14) and (18), and then the Cholesky decomposition is adopted to obtain the matrix  $S_k$ .

③ Measurement update

1. The calculation of measurement prediction  $\hat{z}_{k|k-1}$  at time *k* according to equation (29), and the construction of the linear regression equation through equation (32).

2. The linear regression equation is rewritten into the form of equation (37), with the number of iterations t = 1, and the initial iteration value was set as

$$\hat{\boldsymbol{x}}_{k|k(0)} = \left(\boldsymbol{B}_{k}^{\mathrm{T}}\boldsymbol{B}_{k}\right)^{-1}\boldsymbol{B}_{k}^{\mathrm{T}}\boldsymbol{y}_{k}$$
(51)

3. When the cost function reaches the maximum, the t-th iteration state at time k is obtained according to equations (44), (45) and (48)-(50)

$$\hat{\boldsymbol{x}}_{k|k(t)} = \hat{\boldsymbol{x}}_{k|k-1} + \overline{\boldsymbol{K}}_k \left( \boldsymbol{z}_k - \hat{\boldsymbol{z}}_k \right)$$
(52)

4. When equation (53) is established,  $\hat{x}_{k|k} = \hat{x}_{k|k(t)}$ , otherwise, return to step 3.

$$\frac{\left\|\hat{\boldsymbol{x}}_{k|k(t)} - \hat{\boldsymbol{x}}_{k|k(t-1)}\right\|}{\left\|\hat{\boldsymbol{x}}_{k|k(t-1)}\right\|} \le \varepsilon$$
(53)

5. Finally, the state error covariance matrix  $P_{k|k}$  is obtained according to equation (46).

It is worth noting that the time update process of robust HCKF based on MCC continues the advantages of high accuracy of HCKF. Linear regression process was adopted in the measurement update process, and the estimated value of the current state was obtained through iterative solution, which has the advantages of MCC method and improved the robustness of the algorithm. When the

kernel width  $\sigma \rightarrow \infty$ , MCC-HCKF became the traditional HCKF, which was proved as follows. When  $\sigma \rightarrow \infty$ , kernel function

 $G_{\sigma}(e) = \exp\left(-\frac{e^2}{2\delta^2}\right) = 1$ , at this point, the

matrix  $C_k$ ,  $\overline{R}_k$  and  $\overline{P}_{k|k-1}$  satisfy

$$\begin{cases} \boldsymbol{C}_{k} = \boldsymbol{E}_{N \times N} \\ \boldsymbol{\overline{R}}_{k} = \boldsymbol{R}_{k} \\ \boldsymbol{\overline{P}}_{k|k-1} = \boldsymbol{P}_{k|k-1} \end{cases}$$
(54)

Substitute the above equation into equation (45)

$$\overline{\boldsymbol{K}}_{k} = \overline{\boldsymbol{P}}_{k|k-1} \boldsymbol{H}_{k}^{\mathrm{T}} \left( \boldsymbol{H}_{k} \overline{\boldsymbol{P}}_{k|k-1} \boldsymbol{H}_{k}^{\mathrm{T}} + \overline{\boldsymbol{R}}_{k} \right)^{-1}$$

$$= \boldsymbol{P}_{k|k-1} \boldsymbol{H}_{k}^{\mathrm{T}} \left( \boldsymbol{H}_{k} \boldsymbol{P}_{k|k-1} \boldsymbol{H}_{k}^{\mathrm{T}} + \boldsymbol{R}_{k} \right)^{-1}$$

$$= \boldsymbol{K}_{k}$$
(55)

Obviously, at this point, equation (49) is equal to equation (19).

In literature [6-8], the cost function J(e) and weight function  $\Psi(e)$  based on Huber robust filtering algorithm are given. According to equations (39), (44)-(45), the cost function and weight function based on MCC are known. The specific expressions are shown in Table 1 and Figure 2.

As can be seen from Figure 2, the weight function  $\Psi_{MSE}(e)$  of traditional filtering algorithm is a constant value, the weight function  $\Psi_{Huber}(e)$  based on Huber is composed of a two-stage weight function, and the weight function  $\Psi_{MCC}(e)$  based on MCC is composed of an exponential function. Since  $\Psi_{MSE}(e)=1$ , when abnormal measurement occurs, the traditional algorithm cannot reduce the abnormal interference, which will affect the filtering accuracy and robustness of the algorithm. When  $|e| > \alpha$ ,  $\Psi_{Huber}(e)$  would decrease with the increase of e in order to achieve the purpose of measuring abnormal interference,  $\Psi_{MCC}(e)$ , however, with the increase of e rendering index in the form of attenuation, compared to the  $\Psi_{Huber}(e)$  can be quickly reduced to near zero, has faster attenuation process, so can effectively inhibit the influence of measurement anomaly, so the algorithm has better robustness.

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Table 1. Cost function and weight function					
Estimation criterion	cost function $J(e)$	weight function $\psi(e)$			
MSE	$e^2$	1			
Huber	$\begin{cases} \frac{e^2}{2} ,  e  < \alpha \\ \alpha  e  - \frac{\alpha^2}{2} ,  e  > \alpha \end{cases}$	$\begin{cases} 1 ,  e  \le \alpha \\ \frac{\alpha}{ e },  e  > \alpha \end{cases}$			
MCC	$\frac{\left(1 - \exp\left(\frac{-e^2}{2\sigma^2}\right)\right)}{\sqrt{2\pi}\sigma}$	$\exp\!\left(\frac{-e^2}{2\sigma^2}\right)$			



Figure 2. Diagram of weight function

## 4.Simulation Results and Analysis

The initial position of the aircraft is set to 108° east longitude, 34° north latitude and 200m high. The initial velocity of the carrier is 10m/s, and the direction is north. The maneuvering consists of accelerated, uniform, roll, right-circular, left-circular, climb, descend, and decelerate. The specific maneuver is shown in Table 2, the flight path is shown in Figure 3, and the red arrow indicates the starting position of the aircraft.

Table 2. Flight path simulation				
time	motor	state		
$0s\sim 20s$	Uniform	$v = 5 \mathrm{m} / \mathrm{s}$		
$20s\sim 25s$	Right roll	$\gamma\!=\!0^\circ\!\sim\!1.02^\circ$		
$25s\sim70s$	right-circular	$v = 5 \mathrm{m} / \mathrm{s}$		
$70s\sim75s$	Left roll	$\gamma\!=\!1.02^{\circ}\sim0^{\circ}$		
$75s\sim95s$	Uniform	$v = 5 \mathrm{m} / \mathrm{s}$		
$95s{\sim}105s$	accelerate	$a = 1 \mathrm{m} / \mathrm{s}^2$		
$105s{\sim}115s$	Uniform	v = 15 m/s		
$115s{\sim}125s$	climb	$\theta{=}0^{\circ}{\sim}20^{\circ}$		
$125s{\sim}175s$	Uniform	v = 15 m/s		
$175s{\sim}185s$	descend	$\theta=20^{\circ}\sim0^{\circ}$		
$185s\sim215s$	Uniform	v = 15 m/s		
$215s\sim225s$	descend	$\theta = 0^\circ \sim -20^\circ$		
$225s\sim 275s$	Uniform	v = 15 m/s		
$275s\sim 285s$	climb	$\theta=-20^{\circ}\sim0^{\circ}$		
$285s\sim 385s$	Uniform	v = 15 m/s		
$385s\sim 390s$	Left roll	$\gamma = 0^{\circ} \sim 3.06^{\circ}$		
$390s\sim435s$	left-circular	v = 15 m/s		
$435s\sim440s$	Right roll	$\gamma\!=\!3.06^\circ\!\sim\!0^\circ$		
$440s\sim490s$	Uniform	v = 15 m/s		
$490s\sim500s$	slow down	$a = -1 \mathrm{m} / \mathrm{s}^2$		
$500s\sim 600s$	Uniform	$v = 5 \mathrm{m} / \mathrm{s}$		





Initial position error of SINS is 10 m, initial velocity error is 0.5 m/s, eastward, northward initial misalignment Angle is 1 °, azimuth misalignment angle is 1 °. The root-mean-square of the horizontal

position error of GPS is 10m, the root-mean-square of the height error is 3m, and the root-mean-square of the speed error is 0.1m/s. The calculation cycle of SINS is 0.01s, the sampling period of GPS signal is 0.1s, and the flight time is 600s. Initial variance matrix is P(0) and system noise matrix is  $Q_{t}$ , they are set as

$$P(0) = \text{diag}[(10^{\circ})^{2} (10^{\circ})^{2} (30^{\circ})^{2} (1\text{m/s})^{2} (1\text{m/s})^{2} (1\text{m/s})^{2} (1\text{m/s})^{2} (10\text{m})^{2} (100)^{2} (30\text{m})^{2} (0.1^{\circ})^{2} (0.1^{\circ})^{2} (0.1^{\circ})^{2} (0.1^{\circ})^{2} (1000ug)^{2} (1000ug)^{2} (1000ug)^{2}]$$

$$Q_{k} = \text{diag}[(0.01^{\circ} / h)^{2} (0.01^{\circ} / h)^{2} (0.01^{\circ} / h)^{2} (100\mu g)^{2} (100\mu g)^{2} 0_{9\times 1}]$$

In order to verify the effectiveness of the proposed robust filtering algorithm, two cases are set to experiment with the proposed algorithm. The results and analysis are as follows.

Define root-mean-square error (RMSE) of position at time k as

$$RMSE_{\delta p}(k) = \sqrt{\frac{1}{M} \sum_{i=1}^{M} \left( x_{k}^{i} - \hat{x}_{k}^{i} \right)^{2}}$$
(56)

Define minimum root-mean-square error (ARMSE) of position at time k as

$$\text{ARMSE}_{\delta p} = \sqrt{\frac{1}{MT} \sum_{k=1}^{T} \sum_{i=1}^{M} \left( x_k^i - \hat{x}_k^i \right)^2}$$
(57)

 $x_k^i$  and  $\hat{x}_k^i$  represent the real position error and estimated position error at the moment k when the *i* th Monte Carlo is running. M=100 represents the number of Monte Carlo, Similar to RMSE and ARMSE of position, RMSE and ARMSE of speed can be written.

Experiment 1 The measurement information is gaussian distribution.

In the flight process, it is assumed that the measurement information of GPS is normal. At this time, the measurement noise satisfies the following gaussian distribution

$$\begin{cases} \boldsymbol{v}_{\delta_{\mathcal{V}}} \sim N\left(0, (0.1m/s)^{2}\right) \\ \boldsymbol{v}_{\delta_{P_{E,N}}} \sim N\left(0, (10m)^{2}\right) \\ \boldsymbol{v}_{\delta_{P_{U}}} \sim N\left(0, (3m)^{2}\right) \end{cases}$$
(58)

The three algorithms of CKF, HCKF and MCC-HCKF ( $\sigma = 3$ ) proposed in this paper were respectively applied to the SINS/GPS integrated navigation system.  $\text{RMSE}_{\delta p}$  and  $\text{ARMSE}_{\delta p}$  of the position of the integrated navigation system were used as filter precision indexes. The curves of  $\text{RMSE}_{\delta p}$  of longitude, latitude and altitude were shown in Figure 4, and  $\text{ARMSE}_{\delta p}$  was shown in Table 3.





Figure 4. RMSE of position under gaussian noise conditions

		-	
Filtering	Longitude	Latitude	altitude
algorithm	ARMSE/m	ARMSE/m	ARMSE/m
CKF	0.3854	0.3680	0.1485
HCKF	0.3815	0.3644	0.1470
MCC-HCKF	0.3913	0.3820	0.1478

Table 3. ARMSE of position under gaussian noise condition

According to the results of experiment in Figure 4 and Table 3, after 100s, the filter of the integrated navigation system converges. At this point, the position error of HCKF is less than that of CKF. Therefore, it can be seen that the HCKF with high-order volumetric rules performs better than CKF with third-order volumetric rules. The performance of MCC-HCKF is roughly the same as that of HCKF, but the positioning precision is slightly lower than that of HCKF, this is because in the gaussian noise environment, HCKF is a filter based on the minimum mean variance, and its performance is optimal when the noise is gaussian distribution.

Experiment 2 The measurement information is the contaminated gaussian mixture distribution

In the process of flight, assuming that the GPS measurement information is contaminated gaussian white noise, the measurement information is gaussian mixture distribution, and variance is 10 times of the original gaussian distribution. Formula (59) is the probability distribution expression of the white gaussian noise, Where,  $\varepsilon$  is the mixture percentage, and its value range is  $0 \sim 1$ . When  $\varepsilon = 0$ , the noise is an ideal gaussian distribution, this paper take  $\varepsilon = 0.2$ , at the same time increase the H-HCKF in literature [13] as contrast algorithm, and  $\alpha = 1.345$ .

$$\begin{cases} \mathbf{v}_{\delta_{v}} \sim (1-\varepsilon) N \left( 0, (0.1m/s)^{2} \right) + \varepsilon N \left( 0, (1m/s)^{2} \right) \\ \mathbf{v}_{\delta_{P_{E,N}}} \sim (1-\varepsilon) N \left( 0, (10m)^{2} \right) + \varepsilon N \left( 0, (100m)^{2} \right) \\ \mathbf{v}_{\delta_{P_{U}}} \sim (1-\varepsilon) N \left( 0, (3m)^{2} \right) + \varepsilon N \left( 0, (30m)^{2} \right) \end{cases}$$
(59)





Figure 5. RMSE of Position under non-gaussian noise conditions

From the Figure 5, the measurement noise does not satisfy the gaussian distribution, a decrease in the filtering precision of traditional HCKF, show that algorithm can't effectively restrain the influence of gaussian noise on the system, the MCC-HCKF filter showed good filtering performance and filtering precision higher than HCKF, this is because the MCC-HCKF filtering algorithm is updated at the same time as HCKF, a linear regression estimation based on MCC is carried out in the measurement update process through numerical iterative solution method. This process can reduce the influence of the deviation between the observation noise and the assumed distribution on the filter, but it also increases the amount of computation. With the decrease of  $\sigma$ , the robustness and accuracy of the filter are improved. As the  $\sigma$  increases, the robustness decreases and the performance of the filter becomes closer to HCKF, which is consistent with the above theoretical proof. By comparing the estimated curves of H-HCKF and MCC-HCKF ( $\sigma$ =4) and lower than MCC-HCKF ( $\sigma$ =3), indicating that MCC-HCKF( $\sigma$ =3) is slightly better than H-HCKF at this time.



Figure 6. ARMSE of position under different  $\sigma$ 

To further analyze the effect of  $\sigma$  on the positioning accuracy of integrated navigation, Figure 6 shows the ARMSE of position under different  $\sigma$ . It can be seen from the figure that the ARMSE of longitude, latitude and altitude reached the minimum near  $\sigma = 2.5$ , and the value greater than or less than this value will increase the positioning error. It needs to be pointed out that when  $\sigma \rightarrow 0$ , the filter will have numerical stability problems and the filtering precision will drop sharply, indicating that the selection of  $\sigma$  has a great impact on the positioning precision of integrated navigation, and the system will be less affected by non-gaussian noise when  $\sigma = 2.5$ .

#### **5.**Conclusion

Traditional High-degree CKF algorithm assumes that process noise and measurement noise are gaussian white noise with known statistical characteristics. In practice this assumption is difficult to satisfy. In this paper, MCC and High-degree Cubature criterion are combined, the HCKF is improved, the advantages of both are fully utilized, a new robust high-degree Cubature Kalman filter is proposed, and Monte Carlo simulation is carried out with the loose combination navigation as the research background. The simulation results show that the algorithm can effectively deal with the problem of non-linearity and non-gaussian measurement noise of the integrated navigation system, and has achieved high filtering precision.

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