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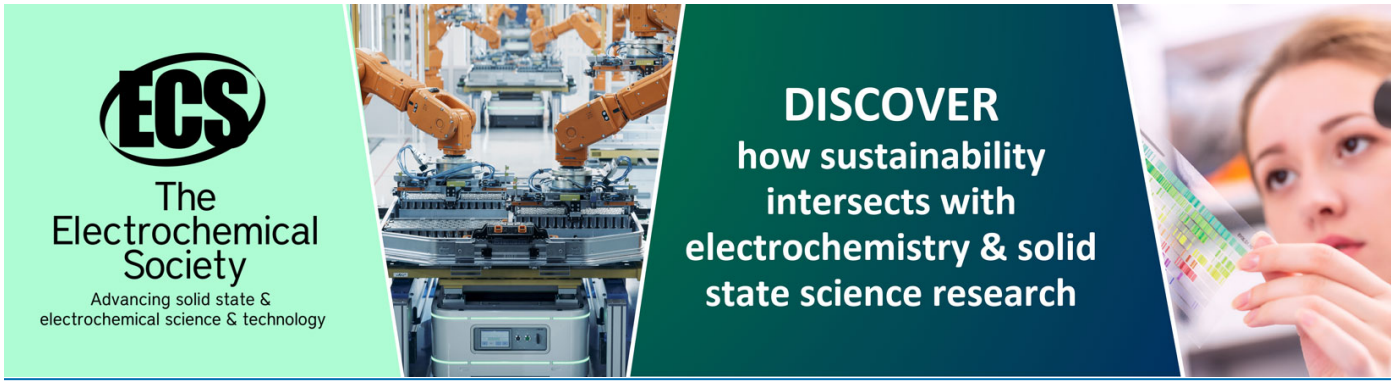

The Application of Differential Equation of Verhulst Population Model on Estimation of Bandar Lampung Population

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Abstract. This study is aimed to predict the population of Bandar Lampung up to the year 2025. This approach applies the Verhulst Population Model. The solutions to the model by using mathematical techniques of the differential equation. Results show that the Verhulst population model predicted the population of Bandar Lampung to be 1,147Million in 2025, with used to estimate the population in Bandar Lampung city is $(t) = \frac{1.547.263}{0,67602e^{-0,05092t}+1}$.

1. Introduction

Mathematics has an important role in life Mathematics has an important role in life [1]. Therefore mathematics is also an exact science that becomes the basis for other sciences so that it is interrelated with other sciences [2]. One of the benefits of differential equations in the field of demography is to find out the population in a particular area. So that, need to be tested to see the accuracy of the results[3, 4]. Benefits Logistics model (Verhulst) is usually to predict human population [5], with the projection (prediction) of the population useful in decision-making for socio-economic and demographic development [6]. Previous research describes that predator and prey logistic growth models can be used as a solution population model equilibrium [7], the population model can be modeled using the Verhulst model and its limits to determine unlimited population growth [8], as an alternative computationally reliable for problem population solving [9]. This research is different from previous studies, the researchers will look at population prediction in Bandar Lampung city only by using the Verhulst population model. Previous researchers used a combination of population models to predict population numbers.

The logistic equation or Verhulst equation is one of the growth population model, the form of the mathematical model is:

$$\frac{dN}{dt} = \alpha N(1 - \frac{N}{K}). \quad (1)$$

The continuous form is a differential equation and can be solved by integrating equation, this will give [10]:

$$N(t) = \frac{K}{Ae^{-at}+1}, \text{ where } A = \frac{K-N_0}{N_0}. \quad (2)$$

The logistic growth model is a population growth model which assumes "at some point the population will be close to the equilibrium point"[11].

Theorem 1. The equilibrium point x is unstable [12].

Proof. $\frac{d}{dx}\big(xM(x)\big) = M(x) + x\frac{dM(x)}{dx}$

$\frac{d}{dx}\big(xM(x)\big)|_{x=0} = M(0)$ (3)

Since $\tau = M(0)$ is positive, the point x= 0 is a stable point.

Theorem 2. *The positive equilibrium point $\bar{x}$ of (1) is globally asymptotically stable* [13].

Proof. Let $\{x_n\}$ be a positive solution of (1) and set

$$\limsup x_n = K, \qquad \liminf x_n = k \quad (4)$$

Then by the boundedness of $\{x_n\}$; $k, K$are positive real numbers. Thus, it is easy to see from

$$x_{n+1} = \frac{x_n x_{n-1}+p}{x_n+x_{n-1}}, n = 0, 1, \ldots$$

that

$$k \geq \frac{k^2+p}{2K}, K \leq \frac{K^2+P}{2k}, \quad (5)$$ Hence,

$$2kK - k^2 \geq p \geq 2kK - K^2$$
$$2kK - k^2 \geq 2kK - K^2$$

Thus, we have

$$k^2 \leq K^2$$

and so

$$k \leq K \quad (6)$$

From (6), If $k = K$, the proof is obvious. If $k < K$, this case contradicts the unique equilibrium solution of (1) whis is locally asymptotically stable. Consequently, it must be $k = K$. This completes the proof.

2. Verhulst (Logistic) Population Model

The assumption that the rate of growth of population decreases the limited population on the environment will be supported by approached logistic model. The logistic growth model is one of the population growth models which assumes that at certain times the population will approach equilibrium. The Verhulst population model (logistics) can be expressed by:

$$\frac{1}{N}\frac{dN}{dt} = \bar{a} - \frac{\bar{a}}{K}N$$

or

$$\frac{dN}{dt} = \bar{a}N\left(1 - \frac{N}{K}\right)$$

if $N(0) = N_0$ will get a special solution of a logistic equation that is:

$$N_t = \frac{K}{\left(\frac{K}{N_0}-1\right)e^{-\bar{a}t}+1} \quad (7)$$

Or

$$P_t = \frac{K}{\left(\frac{K}{N_0}-1\right)e^{-kt}+1} \quad (8)$$

With

$\bar{a} = k$ and $N_t = P_t$

for $\bar{a} > 0$ apply $\lim_{t\to\infty} N(t) = K$, so we will get a graph of a special solution that has a horizontal asymptot $N(t) = K$which is described in the picture below:

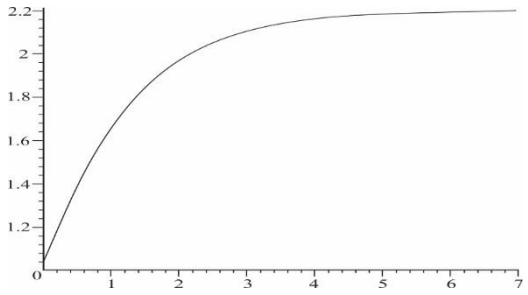

Figure 1 Logistics Growth Chart for $\bar{a} > 0$

Whereas if $\bar{a} < 0$apply$\lim_{t\to\infty} N(t) = 0$, so we will get a graph of a special solution, which is described in the picture below [7]:

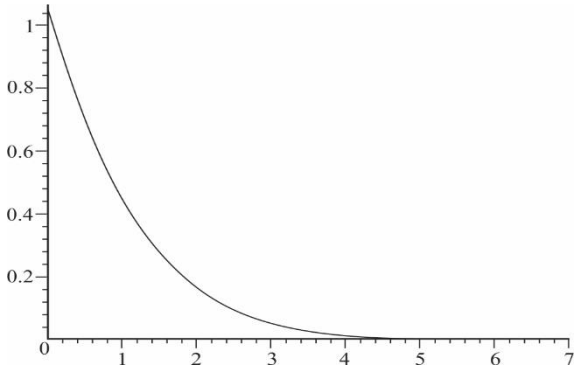

Figure 2 Logistics Growth Chart for$\bar{a} < 0$

3. Example of Population Growth of Bandar Lampung

The first step to determine the logistic population model from the data of the population of Bandar Lampung in the year of 2012 to 2016 is to determine the value of K (Capacity) and k value (population growth rate). First, find the K with the formula:

$$K = \frac{P_1(P_1P_0 - 2P_0P_2 + P_1P_2)}{{P_1}^2 - P_0P_2} \text{ and } k = \frac{P_0(P_2 - P_1)}{P_2(P_1 - P_0)}$$

Based on the population data from 2012 to 2016:

$P_0 = 942.039$
$P_1 = 923.175$
$P_2 = 960.695$

Thus we have,

$K = 1.547.263$ and $k = 0{,}05092$ or 5,09%.

The form of population logistic modeling used to predict the city of Bandar Lampung is

$$P(t) = \frac{1.547.263}{0{,}67602e^{-0{,}05092t} + 1}$$

Table 1 Result of Calculation of Bandar Lampung Population with Logistic Population Model

Year	Results of the Census	Results of The Logistics Model	error
2012	923.175	923.177	0,00022%
2013	942.039	942.041	0,00021%
2014	960.695	960.696	0,0001%

2015	979.287	979.124	0,012%
2016	997.728	997.304	0,042%

Based on table 1 it can be seen that the total population of Bandar Lampung city increases every year and the estimation result using logistic model has a relatively small error so that the estimation result of the population approaches the actual census result. By using the logistics population model, it can be predicted the number of residents of Bandar Lampung city in 2025 is :

$$P(13) = \frac{1.547.263}{0{,}67602e^{-0{,}05092(13)} + +1} = 1{,}147{,}210$$

So the prediction of the Bandar Lampung population with logistics model is 1,147,210 population. Based on the table, we can make the diagram of the different result between the census and logistic model.

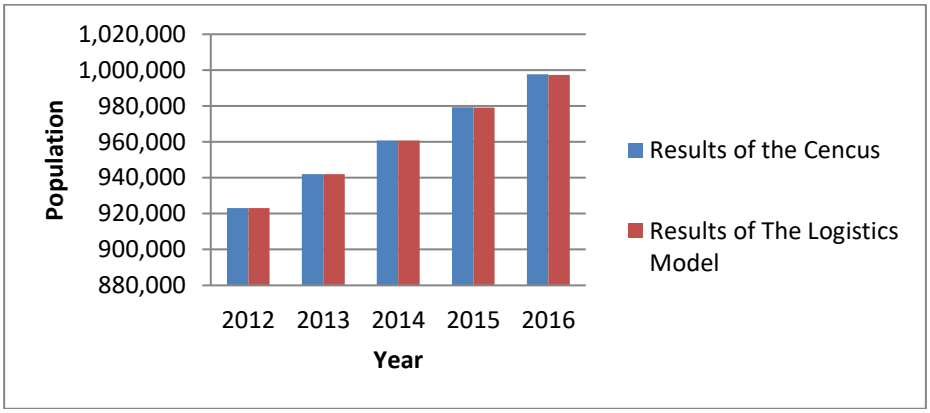

Figure 3 Graph The Number of Residents of Bandar Lampung Based on Census Result Data and Logistic Model

The chart indicates the number of people life in Bandar Lampung, which is the results of the census and the results of a logistic model in the 5 year period from 2012 to 2016. It can be seen that the number of residents grew steadily, showed the highest populations. In 2012 the number of people who live in Bandar Lampung to about 923,175 populations. There was a significant population twice to approximately 960,695 and 960,696 respectively in 2014. The number of people depicted in the graph above also shows that the city of Bandar Lampung always increased, therefore the need for estimation (prediction) of the city of Bandar Lampung in the next few years using the population logistics model. This is because there are many problems in an area like high unemployment due to high growth rate [12–14]. It is important to estimate (predict) a population in planning.

Conversely, the population of Bandar Lampung increases steadily. The several years averaged around % every year, rose in 2016. It is predicted that by 2025 in Bandar Lampung, residents who life will have increased. The Verhulst population model predicted the population of Bandar Lampung to be 1,147 Million in 2025, with used to estimate the population in Bandar Lampung city is $(t) = \frac{1.547.263}{0{,}67602e^{-0{,}05092t}+1}$.

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