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Hydrodynamics singularities associated with a static or moving three-phase contact line

Valeriy Kiryushin

Moscow Technological University (MIREA), Institute of Cybernetics, Russian Federation E-mail: kiryushin.valera@yandex.ru

Abstract. Stationary flows of viscous fluids with a static or moving contact line are considered. Contact line separates three phases: vapor, liquid and solid. No-slip condition on the solidfluid surface and ordinary conditions with account constant surface tension for the fluid-vapor interface are supposed to be fulfilled. The flows presented in the report are induced by some physical mechanism concentrated in the very small region near the contact line. Such contact line is the origin of the flow and treated as a hydrodynamics singularity. As an example the flow in a two- dimensional viscous fluid drop which rests or steadily moves along a horizontal solid surface is considered. Motions of this type can be observed in experiments if the solid-fluid surface wettability is non-uniform. A sequence of solutions for the velocity field and the free surface shape with the successively increasing applicability region near the static or moving contact lines is obtained. At first stage the solution of the problem is found in the case when the distortion of the free surface of the drop during motion can be neglected. The problem is then reformulated using functions of a complex variable and expanded variables are introduced. In the new variables a more accurate solution of the same problem is found, with a much more narrow inapplicability region near the contact lines. Asymptotic behavior of the flow near the contact lines is discussed.

1. Introduction

There are many works devoted to describing the viscous fluid flows near a moving contact line [1]. Moving contact lines often appear in practice, for example, as a drop moves under gravity along an inclined plane, as a viscous liquid volume spreads over a horizontal surface, or as films of chemical species is deposited on a rigid substrate. The solution of these and similar problems meets serious mathematical difficulties in the formulation of boundary conditions on the unknown boundary, the free surface of the fluid. In practice, however, the consideration is often restricted to the case of small fluid velocities when the free surface shape can be assumed to be the same as at rest. The corresponding criterion is formulated as the smallness of the viscous forces that arise as the fluid flows as compared with the surface tension forces, that is, the capillary number Ca is small. The known boundary shape makes it possible to significantly simplify the problem formulation. However, the solutions obtained within the framework of this approximation have an important shortcoming: they are inapplicable near the contact line. No matter how slowly the contact line may move, near the moving line there is a region where the viscous forces are comparable with the surface tension forces and, hence, the shape of the free surface must considerably differ from that under static conditions [13]. The interest in the flow region adjacent to the contact line is also related with the fact that this region substantially

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determines some physical quantities. For example, within the framework of the models available, the friction force between the liquid and the rigid surface has the form of a divergent integral. It is now unknown how to overcome this deficiency in full measure. One of possible practical ways out is based on the hypothesis that the inapplicability region is so small that in this region fundamentally novel physical mechanisms begin to work, which leads to changes in the boundary conditions or in the system of equations. For example, some authors assume that on the rigid surface there is slipping [2 - 11] or that on the liquid a body force exerted by the rigid body acts [12, 13]. However, all these approaches assume that the size of the region within which the novel mechanisms work is greater that that of the inapplicability region and is comparable with the characteristic dimension of the problem. In this study we consider the problem of the spontaneous motion of a viscous fluid drop over a horizontal rigid surface. This motion may arise if the surface wettability is nonuniform. We call the wettability nonuniform if the contact angle at the line of separation between three phases, liquid, air, and solid, is nonuniform. From the hydrostatic equations it follows that on a horizontal plane rigid surface the stationary drop is symmetric, with the same contact angle at all its boundaries. Therefore, the drop with different contact angles at its boundaries must begin to move. Such spontaneous motion was observed in experiments [14]. The wettability inhomogeneity was created by the nonuniform deposition on a rigid surface of a certain species.

2. Formulation of the problem

Consider a two-dimensional volume of a viscous incompressible fluid on a horizontal rigid surface in the absence of gravity and other external body forces. The gravity force can be neglected if the drop diameter is small as compared with the capillary length. In this case, the cross-section of the free surface of the drop has the shape of a circular segment. The free surface of the drop forms with the rigid surface a static contact angle θ . Assume that in a very small neighborhood of the contact line there is a mechanism that creates the steady-state motion inside the drop. We consider two cases: the contact lines remain fixed and contact lines move along the rigid surface. It is natural to anticipate that the flow that arises within the entire drop volume is independent of the details of this mechanism if it is sufficiently small. We will use the reference system tied to the drop. The liquid flow can be described by the continuity equation and the stationary Stokes equation (the Reynolds number is assumed to be sufficiently small).

div
$$\mathbf{v} = 0; -\nabla p + \mu \triangle \mathbf{v} = \mathbf{0}$$
 (1)

Here, v is the liquid velocity, p the pressure, and μ the dynamic viscosity of the liquid. We assign on the rigid surface the no-slip and impermeability conditions. On the free surface the usual conditions for the pressure jump created by the constant surface tension are to take place. The air pressure is assumed to be constant and can, without loss of generality, be set equal to zero. We will also assume to be given the two-dimensional volume of the drop or its cross-sectional length 2L, the distance between between the two opposite contact lines. The system of equations and boundary conditions (1) is incomplete. In statics, the contact angle between the free surface of the liquid and the solid on the separation line of the three phases is specified. In dynamics, the free fluid surface may behave near the contact line in a special way and we cannot hope that this condition can be satisfied. The flow pattern and the drop shape are independent of the drop volume and are conserved as the scale varies. Introduce the Cartesian coordinate system so that the x and y axes be directed along the rigid surface and perpendicular to it and assume that the points x = 1 and x = 1 on the x axis correspond to the contact lines.

The continuity equation for plane flows makes it possible to introduce the stream function Ψ , which obeys the biharmonic equation $\Delta \Delta \Psi = 0$. On the free surface $\Psi = 0$ and for the

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tangential and normal stress tensor components the following conditions must be satisfied:

$$-2\frac{\partial^2 \Psi}{\partial x \partial y} n_x n_y + \frac{1}{2} \left(\frac{\partial^2 \Psi}{\partial y^2} - \frac{\partial^2 \Psi}{\partial x^2} \right) (n_x^2 - n_y^2) = 0$$
$$-p + \frac{\partial^2 \Psi}{\partial x \partial y} (n_x^2 - n_y^2) + \left(\frac{\partial^2 \Psi}{\partial y^2} - \frac{\partial^2 \Psi}{\partial x^2} \right) n_x n_y = -\sigma\kappa$$
(2)

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Here n_x and n_y are the components of the unit normal to the free surface $\Psi = 0$ with a curvature κ and surface tension σ . The pressure p is determined by the stream function.

3. Flows in droplets with a non-distorted circular shape

3.1. Quiescent droplet with internal flow

The problem set above is very complicated due to unknown shape of the free surface. We begin with a common assumption that the flow is weak and the free surface distorted slightly. This may be justified for the main volume of the droplet, but is not applicable to the vicinities of the contact lines where the sources are located. Keeping in mind this circumstance we suppose that the free surface of the droplet is an arc of the circle and can be described by the following equation $1 - x^2 - y^2 = 2y \cot \theta$, where θ denotes static contact angle. The second boundary condition (2) is discarded. Solution of the problem follows the method using complex variables and developed in the work [15]. The general solution of the biharmonic equation can be presented in the form:

$$\Psi = \operatorname{Re}\left[i(1-z^2)P(z)/2 + y\left[-iF(z) + \frac{d}{dz}(1-z^2)P(z)/2\right]\right]$$
(3)

Here F(z) and P(z) are analytical functions of the complex variable z = x + iy. The boundary conditions on the solid plane are fulfilled identically if these functions are real on the segment [-1,1] of the real axes. The solution sought in the form

$$F(z) = Ae^{\lambda\zeta} + \overline{A}e^{\overline{\lambda}\zeta}, P(z) = Be^{\lambda\zeta} + \overline{B}e^{\overline{\lambda}\zeta}, \zeta = \ln\frac{1+z}{1-z}$$
(4)

.

The bar over complex constants A, B, λ means complex conjugation. To satisfy the boundary conditions on the free surface the constants should be related by the equations

$$A = B(\cot \theta) - \lambda \cot(\lambda \theta)), \sin(2\lambda \theta) = \lambda \sin(2\theta)$$
(5)

The equation for λ has an infinite number of complex roots for any θ . As a result an infinite set of solutions is found. Moreover, due to linearity of the problem any algebraic combination of solutions is a solution too. An example of the flow is depicted schematically in Fig. 1. Streamlines of the flow are closed. An infinite vortex array can be seen.

3.2. Flow in the slowly moving droplet

We will use the reference system tied to the drop. The rigid surface then moves at a constant velocity V. To satisfy the no-slip condition on the solid plane the term Vy should be added in the equation (3) for the streamline function. One solution of the problem is found in the work [15]. Streamline function has a form

$$\Psi = Ca \ y + \frac{Cay\sin\theta}{\theta - \sin\theta\cos\theta} \left[\cos\theta - \left(\frac{1 - x^2 - y^2}{2y}\cos\theta + \sin\theta\right)\arctan\frac{2y}{1 - x^2 - y^2}\right]$$
(6)

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Figure 1. Streamlines of the flow in the quiescent droplet.



Figure 2. Streamlines of the flow in the moving droplet.

Distortion of the free surface droplet is neglected. It is valid in the main volume of the droplet if the capillary number $Ca = 2\mu V/\sigma$ is small. The streamlines of this solution are depicted in the Fig. 2. It may be noticed that near the advancing x = 1 and the receding x = 1 contact lines the solution obtained goes over asymptotically into the known solution [1] for the liquid flow inside the angle θ . But this solution is not unique: we may add an arbitrary solution for the quiescent droplet due to linearity of the problem. Resulting flow involves many vortexes and looks very complicated. Thus we found an infinite set of flows in the moving droplet.

4. Approaching the contact lines

Now consider the problem of accuracy of the solutions obtained. Discarded condition (2) for the normal stress jump allows to evaluate applicability of solutions found above. It is easy to see that the error (residual) is small in the main volume of the droplet for sufficiently small A, B, V but grows with approaching contact lines. Thus in the small vicinities of the contact lines distortion of the free surface becomes significant and the flow changes radically. To overcome this difficulty we introduce extended variable $Ca\zeta$. The region of the flow in the plane $Ca\zeta$ looks as an narrow infinite stripe. The values of the variables on both sides of the stripe can be related by Tailor expansions.

$$\Psi = \text{Re}W, W = \frac{i(1-z^2)\sin H}{2} + y\left[Ca - i\cos H - z\sin H + \frac{Ca\sin(2H)}{2H - \sin(2H)}\right]$$
(7)

The analytical function H is define by the formula

$$\int_{0}^{H} \frac{tdt}{\sin t} - \sin H = Ca\zeta + C \tag{8}$$

It is encountered in works [10, 18], C is an integration constant. The function H is multivalued. The branch real on the real axis is used. In addition, we obtain the equation for the shape of the drop free surface

$$H\left[\arctan\frac{2y}{1-x^2-y^2}\right] = \frac{Ca}{2}\ln\frac{(1+x)^2+y^2}{(1-x)^2+y^2} + C$$
(9)

Consider the obtained solution in the main flow region. It can be seen that the shape of the surface is close to that of the circle with the contact angle H() = C. In the main volume of the drop, solution (7) gives a flow pattern very close to that shown in Fig. 2. A difference

can only be observed in very small regions near the contact lines. Formulas (7,9) show that the angle of tangent inclination to the free surface (zero streamline) approaches 180^{0} when approaching the receding contact line and zero when approaching the advancing one. However, this change is so slow that it cannot practically be observed at any fixed scale. Thus the dynamic contact angle changes radically with a scale. More thorough analysis shows that the solutions found are still applicable as the dynamics angle does not exceeds $Ca^{1/3}$. Thus the region of applicability solutions enlarges significantly. Near the the receding contact line solution obtained is asymptotically applicable.

5. Conclusions and discussion

The obtained solution of the problem of liquid flow inside a moving plane drop allows to conclude that the assertion formulated in [1, 2, 412] about the incompatibility of the moving three-phase contact line with the liquid no-slip condition on the rigid surface is at least not quite correct. Well-known solutions are inapplicable near the contact lines, but we successfully reduce the region of inapplicability. The methods developed in the work allow to obtain next approximations which may reduce the region of inapplicability to zero. It seems that the flow near contact line becomes more and more complicated. Perhaps there is no simple asymptotic expression.

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