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# Thermal model in electron beam welding with various dynamic positioning of the beam

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**Abstract**. The paper presents the differential equation which is a mathematical model of a whole class of phenomena of thermal conductivity and has an infinite number of solutions. Modelled thermal processes at electron beam welding and the resulting expression is the base for building thermal models that take into account different types of dynamic positioning electronic beam. At electron beam welding the beam oscillations on trajectories of various type are applied, in most cases, to eliminate the characteristic defects taking place when welding by a static beam. The following types of beam oscillations are most widely used: longitudinal, transverse and x-shaped. They are chosen for the construction of thermal models. The calculations showed that the proposed model of the heat source for electron beam welding with a static beam and the obtained on its basis solution of the heat problem, permit to describe the geometry of the weld. The method of Green functions as one of the universal ways to build thermal models in electron beam welding with various dynamic positioning of the beam is implemented.

## 1. Introduction

The importance of modeling the thermal processes is the ability to predict parameters such as the distribution of the temperature field in the welded product, the shape and size of the weld and the heat affected zone, etc.

Thermal processes during welding are most often described by the differential heat conduction equation in a moving coordinate system with a fixed heat source [1]:

$$\frac{\partial T}{\partial t} = a \cdot \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) + V\frac{\partial T}{\partial x} + \frac{1}{c\rho}F(x, y, z, t), \qquad (1)$$

where $F(x,y,z,t)$ is a function of heat source.

One of the methods for finding solutions of the problems of heat conduction is the Green functions method (sources). It allows to obtain analytical solutions, depending on various boundary value problems and the nature of the welding energy source, as well as to consider the patterns of change of the temperature field in the material at virtually any duration of exposure to heat source. It reduces the variety of ways of heating the material to some schemes, covering the main features of the heating processes [2].

## 2. Methods and result

The differential equation of heat conduction (1) is a mathematical model of a whole class of phenomena of heat conduction and has infinite number of solutions. To get from this set one particular solution describing a particular process, it is necessary to have additional data – the unique conditions including the body geometry, boundary and initial conditions. For modeling of the thermal processes at electron beam welding the following unique conditions are chosen: the geometry of the welded body is an infinite plate with thickness $L$:

$$-\infty < x < \infty; \quad -\infty < y < \infty; \quad 0 < z \leq L;$$

- the boundary conditions are of mixed type: the x and y axes boundary conditions of first kind equal to 0, in the z-axis the boundary conditions of the second kind equal to 0:

$$\frac{\partial T}{\partial z}(x, y, 0, t) = 0\ ; \quad \frac{\partial T}{\partial z}(x, y, L, t) = 0; \tag{2}$$

- the initial conditions: $T(x,y,z,0) = 0$.

The integral solution of the conduction problem (2) using heat sources is a set of standardized Green functions for a specific geometric and boundary conditions and the function of the heat source [3]:

$$T(x, y, z, t) = \frac{1}{c\rho}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty} G(x, y, z, x', y', z', t-\tau) \cdot exp\left(-\frac{V(x-x')}{4a} - \frac{V^2(t-\tau)}{2a}\right) \cdot$$
$$\cdot F(x', y', z', \tau)\partial x'\partial y'\partial z'\partial\tau \tag{3}$$

where $G(x, y, z, x', y', z', t-\tau)$ is the standardized Green function, $F(x', y', z', \tau)$ is the mathematical description of the function of the heat source using the Dirac delta function, *x', y', z'* are the coordinates of the heat source.

The solution of the specific heat problem is reduced to finding a standardized form of the Green function, which takes into account the boundary conditions and the definition of the mathematical description of the function for a given source of heat.

It is known that in the works of Rykalin [2, 4] for the distribution of temperatures at electron beam welding with deep penetration of the beam, it is offered to consider the action of two heat sources: a point heat source on the surface and a linear heat source, limited by the depth of the plate size. The solution of the thermal problem by an analytical method for electron beam welding with the application of the combined heat source can be found in the continuation of his work [3]. For the offered thermal model for an infinite plate constructed by the method of the Green functions, the heat source is point normally distributed on the surface and linear in depth (figure 1).

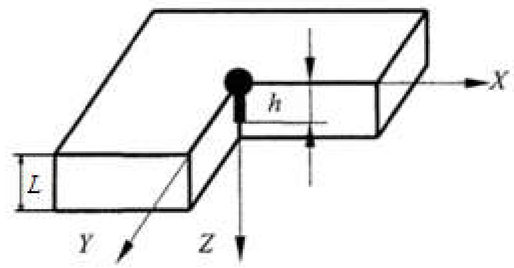


**Figure 1.** The scheme of the heat source at electron beam welding [3].

The mathematical description of function of the heat source in this case has the following appearance:

$$F(x, y, z, t) = \frac{q\eta}{c\rho}\begin{pmatrix} k_1\delta(x')\delta(y')\delta(z')E(\tau) + \\ +\frac{k_2}{h}\delta(x')\delta(y')E(z')E(\tau) \end{pmatrix} \tag{4}$$

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$$E(z') = \begin{cases} 1\, for\; 0 \le z' \le h \\ 0\, for\; h < z' < 0 \end{cases};$$

$$E(\tau) = \begin{cases} 1\, for\;\; t_0 \le \tau \le t \\ 0\, for\; \tau > t \end{cases};\quad t_0 = \frac{1}{4aK};\;\; K = \frac{12}{d^2},$$

where $q$ is the power of the electron beam, $\eta$ – the coefficient of efficiency, $c$ – the specific heat, $\rho$ - the density of the metal, $k_1$ и $k_2$ – the coefficients considering the distribution of the beam power between the superficial and linear sources, $h$ – the depth of the linear source, $E(z')$ и $E(\tau)$ – single functions.

For imitation of the influence of normal and circular source, the time of the fictitious source $t_0$ is calculated using the concentration coefficient $K$ for a given electron beam diameter $d$.

The integrated solution of the problem of the heat conductivity for electron beam welding by a static beam of an infinite plate with thickness $L$ is [3]:

$$T(x,y,z,t) = \frac{k_1 q\eta}{16r^2 c\rho\sqrt{\pi a}} \int_{t0}^{t} \frac{1}{\sqrt{\tau}} \cdot \left[erf\left(\frac{y+r}{2\sqrt{a\tau}}\right) - erf\left(\frac{y-r}{2\sqrt{a\tau}}\right)\right] \cdot \sum_{n=-\infty}^{\infty} exp\left(-\frac{(z+2nL)^2}{4a\tau}\right) \partial\tau +$$

$$+\frac{k_2 q\eta}{16h\pi\lambda} \cdot \sum_{n=-\infty}^{\infty} \begin{bmatrix} erf\left(\frac{z-S+h+2nL}{2\sqrt{a\tau}}\right) + erf\left(\frac{z+S+h+2nL}{2\sqrt{a\tau}}\right) - \\ -erf\left(\frac{z-S-h+2nL}{2\sqrt{a\tau}}\right) - erf\left(\frac{z+S-h+2nL}{2\sqrt{a\tau}}\right) \end{bmatrix} \partial\tau, \qquad (5)$$

It is established by the authors, that at electron beam powers from 5 to 10 kW for alloy steels, the average value of the coefficients $k_1$ and $k_2$ is 0.3 and 0.7, respectively, with the efficiency $\eta$ = 0.9. However, it should be noted that the numerous carried-out calculations for the alloy steels and non-ferrous metals have shown that the thermal model of the electron beam welding by a static beam (4) does not always give satisfactory results, when comparing with the geometry of the experimentally obtained welded seams, received with a settlement form.

For an increase in the accuracy of the solution of the thermal problem at electron beam welding the following form of the combined source is proposed (figure 2) consisting of:

- normally distributed on the surface circular source with radius r, enter in the origin;
- the linear depth of the source 'along the $Z$ axis, length $2{\cdot}h$', functioning under the surface at a distance $h1$, the input axis $(0,\ 0,\ S)$.

Both sources can be represented as a set of instantaneous point sources and, respectively, the function of these sources is described by means of delta function and single functions [5].

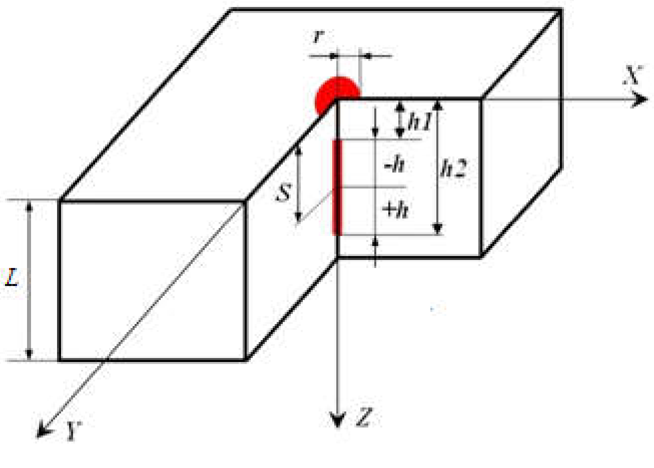



**Figure 2.** The scheme of the combined heat source in electron beam welding with a static beam: $r$ - the radius of the surface heat source, $h1$ – distance from the surface of the linear source, $h2$ – depth of action of the linear source, $S$ – the insertion point of the linear heat source with length $2{\cdot}h$, where $h = (h2 - h1) / 2$; $S = h + h1$.

The integral solution of the heat problem at electron beam welding of a static beam in the analytical form is as follows:

$$T(x,y,z,t)=\frac{k_1 q\eta}{16r^2 c\rho\sqrt{\pi a}}\int_{t0}^{t}\frac{1}{\sqrt{\tau}}\cdot\left[erf\left(\frac{y+r}{2\sqrt{a\tau}}\right)-erf\left(\frac{y-r}{2\sqrt{a\tau}}\right)\right]\cdot\sum_{n=-\infty}^{\infty}exp\left(-\frac{(z+2nL)^2}{4a\tau}\right)\partial\tau+$$

$$+\frac{k_2 q\eta}{16h\pi\lambda}\int_{t0}^{t}\frac{1}{\tau}exp\left(-\frac{(x+V\tau)^2+y^2}{4a\tau}\right)\sum_{n=-\infty}^{\infty}\begin{bmatrix}erf\left(\frac{z-S+h+2nL}{2\sqrt{a\tau}}\right)+erf\left(\frac{z+S+h+2nL}{2\sqrt{a\tau}}\right)-\\-erf\left(\frac{z-S-h+2nL}{2\sqrt{a\tau}}\right)-erf\left(\frac{z+S-h+2nL}{2\sqrt{a\tau}}\right)\end{bmatrix}\partial\tau \qquad (6)$$

where $q$ – beam power, $V$ – welding speed, $L$ – product thickness, $c$ – heat capacity, $\rho$ – density of metal, $a$ – thermal diffusivity, $\lambda$ – thermal conductivity coefficient, $k_1$ и $k_2$ – the coefficients considering the distribution of the beam power between the sources, $\eta$ – coefficient of efficiency.

From the schematic image of the heat source (figure 2) it is visible that for the definition of $h$ and $S$ it is necessary to know the distance from the surface $h1$, at which the source is operating on the axis $Z$, and the depth of the action of this source $h2$, which is the penetration depth.

The depth of the linear source $h2$ acting along the $Z$ axis can be described by the calculated penetration depth, which is related to the parameters of the electron beam welding by the criterion equation. At present, there is a large number of works devoted to determination of the penetration depth. The following equation, used to determine the depth of penetration during welding by a static beam, is presented in [6]:

$$H=\frac{UI}{r_b V_w}\cdot\left(c\rho T_{\text{cp}}+\rho L_h+\frac{5\lambda T_{\text{cp}}}{2a}+\frac{5\lambda T_{\text{cp}}}{2r_b V_w}\right)^{-1} \qquad (7)$$

$$H=IUlg\left(\frac{2.24\lambda}{c\rho V_w r_b}\right)\cdot\left(2\pi\lambda\left(T_{\text{cp}}-T_0\right)\right)^{-1} \qquad (8)$$

where $H$ is penetration depth, cm; $U$ – accelerating voltage, V; $I$ and $I_b$ – beam current; $V_{св}$ – welding velocity, cm/s; $r_b$ – radius of the electron beam, cm; $c$ – heat capacity, J/(g·°C); $\rho$ – density of the metal, g/cm$^3$; $\lambda$ – thermal conductivity coefficient, J/(cm·c·K); $a$ – thermal diffusivity cm$^2$/s; $T_t$ – fusion temperature, K; $T_{cp}$ = $1.5T_t$ – the average temperature of the welding pool, K; $T_0$ – initial temperature, K; $L_h$ – latent heat of melting, J/cm$^3$; $L_{кип}$ – latent heat of vaporization, J/cm$^3$.

To determine the coefficients $k1$, $r$, $h1$ in (5) and (6), regression equations were obtained as a function of $F(I_b, U, V_w, h2)$. The regression dependences were defined for medium-alloy steels with the same thermos-physical characteristics. The conclusions for the regression dependences were made on the basis of statistical data processing at selection of the coefficients, using the obtained experimental samples welded at various modes:

$$k_1 = 0.453+5.017\cdot I_b+7.742\cdot 10^{-3}\cdot U-0.08\cdot V_w-0.049\cdot h2 \qquad (9)$$

$$r = 2.129+41.62\cdot I_b+0.047\cdot U-0.311\cdot V_w-0.317\cdot h2 \qquad (10)$$

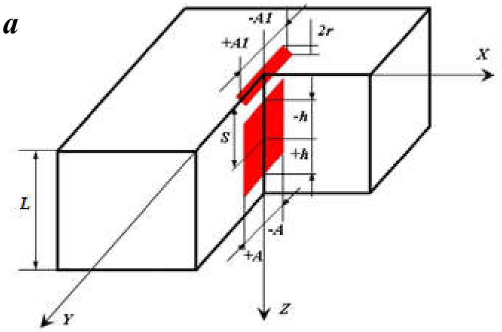
$$h1 = -0.057-1.35\cdot I_b+0.011\cdot U-0.101\cdot V_w+0.03\cdot h2 \qquad (11)$$

At electron beam welding the oscillations of the beam along trajectories with various forms are applied in most cases to eliminate the typical defects that occur during welding with a static beam. The following types of scan of the beam are most widely used: longitudinal, transverse and x-shaped with ellipse, arc and circle displacement of the beam. The amplitude of the oscillations of the beam often is in the range of 1-3 mm, frequency – from 50 Hz to 1 kHz and depends on the type of the welded material. For sufficiently small values of the frequency and the amplitude of the beam oscillations the size of the vapor channel remains virtually unchanged, while the expansion is with higher values.

The oscillations of the electron beam impacts the vapor volume and the hydrodynamic processes in the keyhole, improving its resistance and consequently, leads to a change in the configuration of the welding pool. However, in the cross-sectional shape of the weld the extension in the upper part remains, consequently, it can be assumed that at these modes of beam oscillations the processes associated with the shielding of the electron beam remain, although occurring to a lesser extent.

| ***The scheme of the heat source*** | ***Mathematical description of heat source*** |
|---|---|

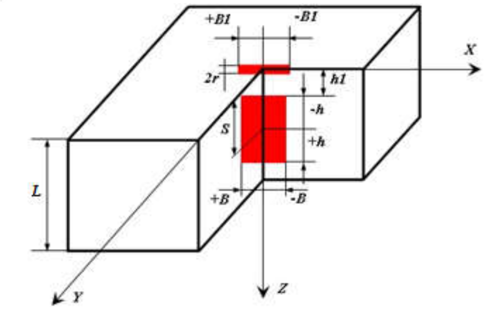
***a***




*Transverse oscillations of the beam*

$$F(x,y,z,t) == \frac{q\eta}{c\rho}\begin{pmatrix} \frac{k_1}{4rA1}E(x')E1(y')\delta(z')E(\tau) + \\ + \frac{k_2}{4Ah}\delta(x')E2(y')E(z')E(\tau) \end{pmatrix} \quad (12)$$

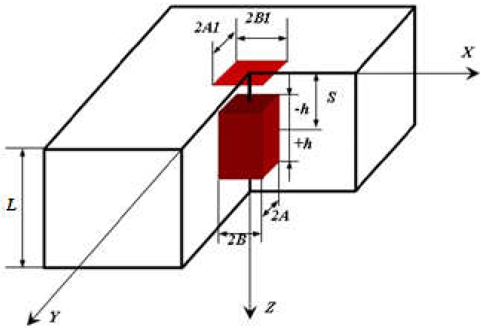
***b***




*Longitudinal oscillations of the beam*

$$F(x,y,z,t) = \frac{q\eta}{c\rho}\begin{pmatrix} \frac{k_1}{4rB1}E1(x')E(y')\delta(z')E(\tau) + \\ + \frac{k_2}{4Bh}E2(x')\delta(y')E(z')E(\tau) \end{pmatrix} \quad (13)$$

***c***




*x-shaped oscillations of the beam*

$$F(x,y,z,t) = \frac{q\eta}{c\rho}\begin{pmatrix} \frac{k_1}{4rB1}E1(x')E(y')\delta(z')E(\tau) + \\ + \frac{k_2}{4Bh}E2(x')\delta(y')E(z')E(\tau) \end{pmatrix} \quad (14)$$

$$where: E(x') = \begin{cases} 1\, for -r \le x' \le r \\ 0\, for\, r < x' < -r \end{cases}; \qquad E(y') = \begin{cases} 1\, for -r \le y' \le r \\ 0\, for\, r < y' < -r \end{cases};$$

$$E1(x') = \begin{cases} 1\, for - B1 \le x' \le B1 \\ 0\, for\, B1 < x' < -B1 \end{cases}; \qquad E1(y') = \begin{cases} 1\, for - A1 \le y' \le A1 \\ 0\, for\, A1 < y < -A1 \end{cases};$$

$$E2(x') = \begin{cases} 1\, for - B \le x' \le B \\ 0\, for\, B < x' < -B \end{cases}; \qquad E2(y') = \begin{cases} 1\, for - A \le y' \le A \\ 0\, for\, A < y < -A \end{cases};$$

$$E(z') = \begin{cases} 1\, for - h \le z' \le h \\ 0\, for\, h < z' < -h \end{cases}; \qquad E(\tau) = \begin{cases} 1\, for\, t_0 \le \tau \le t \\ 0\, for\, \tau > t \end{cases};$$

$r$ – width of a superficial source; $A$, $B$ – source oscillation amplitudes along Y and X axes, acting in depth, $A1$, $B1$ – oscillation amplitudes of the surface source ($B1 = B + r$; $A1 = A + r$)

**Figure 3.** Schemes and mathematical description of the forms of the heat sources for electron beam welding with oscillations of the electron beam.

A combined heat source was used for constructing models to solve the heat problems and selecting the shape of the heat source for welding with oscillations of the electron beam. The extension of the diameter of the heating surface and the effect of the source depth under the surface are taken into account.

The case, when the frequency of oscillations of the beam for different types of scanning is selected sufficiently high, within frequencies of self-oscillation process 400 - 800 Hz, is usually used in practice. Therefore, the thermal perturbations over time in the area of the beam can be neglected and the oscillatory process can be considered as quasi-stationary. Based on this the form of heat sources can be simplified in order to build thermal models of the selected longitudinal, transverse and x-shaped trajectories of the beam oscillations. In a weld, obtained by longitudinal and transverse oscillations of the electron beam, the heat source consists of a surface plane with a rectangular shape, the length along the corresponding X and Y axes and a plane acting in depth along the Z axis and width along the corresponding X and Y axes (figure 3 (a) and (b)).

Circular and elliptic oscillations of the beam can be represented as the variation of the fluctuations with *x*-shaped trajectory. This approach is possible, based on the following. To describe the heat source, which increases the diameter, in [3] it is proposed to use a rectangular shape of the source and include the action of the fictitious source with time $t_0$. Because of this, the shape of the heating spot becomes closer to circular, and the distribution of the power on the heating spot is with much less curvature compared to normal-circular heat source.

Substituting the functions of the heat sources (12), (13) and (14) into the equation representing the General form of the integral solution of the heat conduction problem in a moving coordinate system for an infinite plate (4), the following expression of the integral solutions of the problems of heat conduction for electron beam welding with beam oscillations is obtained:

- with longitudinal oscillations of an electron beam

$$T(x,y,z,t) == \frac{k_1 q\eta}{16rB1c\rho\sqrt{\pi a}}\int_{t0}^{t}\frac{1}{\sqrt{\tau}}\cdot\left[erf\left(\frac{x+B1+V\tau}{2\sqrt{a\tau}}\right)-erf\left(\frac{x-B1+V\tau}{2\sqrt{a\tau}}\right)\right]\cdot\left[erf\left(\frac{y+r}{2\sqrt{a\tau}}\right)-erf\left(\frac{y-r}{2\sqrt{a\tau}}\right)\right]\cdot \sum_{n=-\infty}^{\infty}exp\left(-\frac{(z+2nL)^2}{4a\tau}\right)\partial\tau+\frac{k_2 q\eta}{32Bhc\rho\sqrt{\pi a}}\int_{t0}^{t}\frac{1}{\sqrt{\tau}}\left[erf\left(\frac{x+B+V\tau}{2\sqrt{a\tau}}\right)-erf\left(\frac{x-B+V\tau}{2\sqrt{a\tau}}\right)\right]\cdot exp\left(-\frac{y^2}{4a\tau}\right)\cdot \cdot\sum_{n=-\infty}^{\infty}\begin{bmatrix}erf\left(\frac{z-S+h+2nL}{2\sqrt{a\tau}}\right)+erf\left(\frac{z+S+h+2nL}{2\sqrt{a\tau}}\right)-\\-erf\left(\frac{z-S-h+2nL}{2\sqrt{a\tau}}\right)-erf\left(\frac{z+S-h+2nL}{2\sqrt{a\tau}}\right)\end{bmatrix}\partial\tau. \tag{15}$$

- with transverse oscillations of the electron beam:

$$T(x,y,z,t) == \frac{k_1 q\eta}{16rA1c\rho\sqrt{\pi a}}\int_{t0}^{t}\frac{1}{\sqrt{\tau}}\cdot\left[erf\left(\frac{x+r+V\tau}{2\sqrt{a\tau}}\right)-erf\left(\frac{x-r+V\tau}{2\sqrt{a\tau}}\right)\right]\cdot\left[erf\left(\frac{y+A1}{2\sqrt{a\tau}}\right)-erf\left(\frac{y-A1}{2\sqrt{a\tau}}\right)\right]\cdot \sum_{n=-\infty}^{\infty}exp\left(-\frac{(z+2nL)^2}{4a\tau}\right)\partial\tau+\frac{k_2 q\eta}{32Ahc\rho\sqrt{\pi a}}\int_{t0}^{t}\frac{1}{\sqrt{\tau}}\cdot exp\left(-\frac{(x+V\tau)^2}{4a\tau}\right)\cdot\left[erf\left(\frac{y+A}{2\sqrt{a\tau}}\right)-erf\left(\frac{y-A}{2\sqrt{a\tau}}\right)\right]\cdot \cdot\sum_{n=-\infty}^{\infty}\begin{bmatrix}erf\left(\frac{z-S+h+2nL}{2\sqrt{a\tau}}\right)+erf\left(\frac{z+S+h+2nL}{2\sqrt{a\tau}}\right)-\\-erf\left(\frac{z-S-h+2nL}{2\sqrt{a\tau}}\right)-erf\left(\frac{z+S-h+2nL}{2\sqrt{a\tau}}\right)\end{bmatrix}\partial\tau. \tag{16}$$

- with x-shaped oscillations of the electron beam:

$$T(x,y,z,t) = \frac{k_1 q\eta}{16A1B1c\rho\sqrt{\pi a}}\int_{t0}^{t}\frac{1}{\sqrt{\tau}}\cdot\left[erf\left(\frac{x+B1+V\tau}{2\sqrt{a\tau}}\right)-erf\left(\frac{x-B1+V\tau}{2\sqrt{a\tau}}\right)\right]\cdot\cdot\left[erf\left(\frac{y+A1}{2\sqrt{a\tau}}\right)-erf\left(\frac{y-A1}{2\sqrt{a\tau}}\right)\right]\cdot \sum_{n=-\infty}^{\infty}exp\left(-\frac{(z+2nL)^2}{4a\tau}\right)\partial\tau+\frac{k_2 q\eta}{64ABhc\rho}\int_{t0}^{t}\left[erf\left(\frac{x+B+V\tau}{2\sqrt{a\tau}}\right)-erf\left(\frac{x-B+V\tau}{2\sqrt{a\tau}}\right)\right]\cdot\left[erf\left(\frac{y+A}{2\sqrt{a\tau}}\right)- erf\left(\frac{y-A}{2\sqrt{a\tau}}\right)\right]\cdot\cdot\sum_{n=-\infty}^{\infty}\begin{bmatrix}erf\left(\frac{z-S+h+2nL}{2\sqrt{a\tau}}\right)+erf\left(\frac{z+S+h+2nL}{2\sqrt{a\tau}}\right)-\\-erf\left(\frac{z-S-h+2nL}{2\sqrt{a\tau}}\right)-erf\left(\frac{z+S-h+2nL}{2\sqrt{a\tau}}\right)\end{bmatrix}\partial\tau. \tag{17}$$

The solution for welding with *x*-shaped oscillations of the electron beam can be used in the case of electron beam welding with a scan in circular and elliptical trajectories.

Figure 4 demonstrates the application of the formulas (6) to determine the shape of penetration 'the weld' and the construction of the thermal cycles at electron beam welding with a static beam. Welding

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was conducted on steel 41CrAlMo7 with thickness of 20 mm and electron beam power of 5600 W, the welding speed was 16 m/h and the current focus provided the maximum penetration depth.

For comparison, calculation of the geometry of the welded seam cross-section by the equation (5) was made, using combined superficial normally distributed and linear in depth heat source. The geometry of a welded seam in the cross-section was determined by a crystallization isotherm (figure 4); and the points show experimentally measured values of the width of the seam.

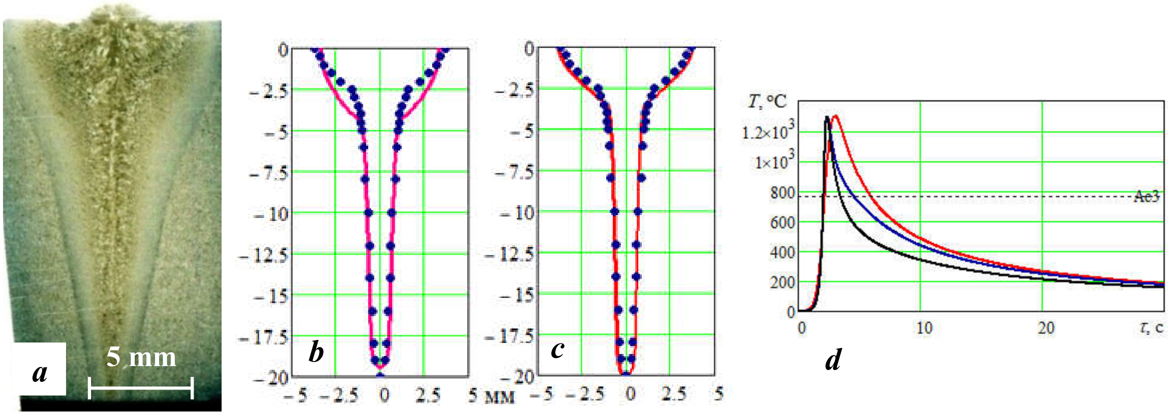


**Figure 4.** (a) Macrostructure of the weld, (b, c) the estimated form of the weld, (b) – taking into account a classical heat source: *k1* = 0.3, *k2* = 0.7, *h* = 20 mm; (c) – taking into account the offered heat source: *k1* = 0.29, *k2* = 0.71, *h1* = 0.2 mm, *h2* = 20 mm, *r* – 3.7 mm, (d) thermal cycles of welding for a zone of overheating of various sites in depth of the welded seam from steel 41CrAlMo7 at electron beam welding by a static beam.

Figure 5 shows the application of the formula (17) [5] for the definition of the form of pro-melting 'welded seam' and creation of thermal cycles at electron beam welding with x-shaped oscillation of the beam.

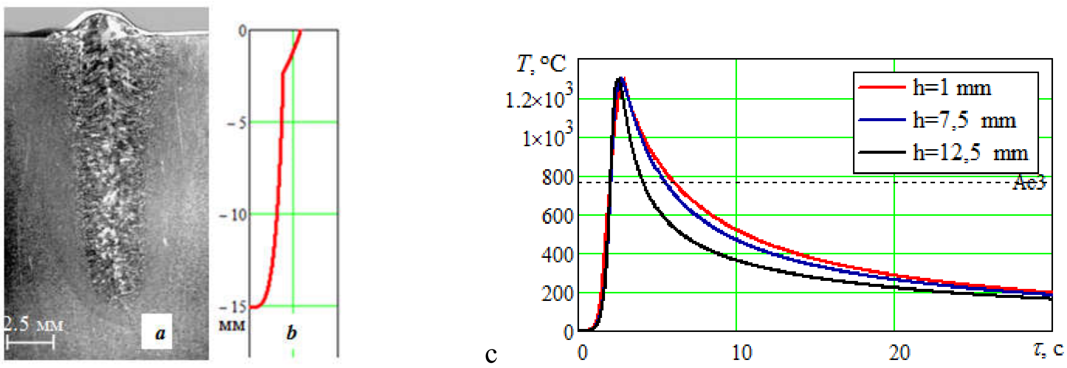


**Figure 5.** (a) Macrostructure of the weld, (b) the estimated form of the weld and (c) the thermal cycles of welding for the zone of overheating of various parts according the depth of the weld for steel 41CrAlMo7 at electron beam welding with x-shaped beam oscillations.

Welding was conducted on steel 41CrAlMo7 with thickness of 20 mm, with electron beam power 5600 W, the welding speed was 10 m/h, the focusing current provided the maximum penetration depth, the frequency of x-shaped oscillation was 600 Hz and the amplitude was 1.8 mm. The thermal cycles of welding are presented for the zone of overheating ‘*Tmax* = 1300 °C’ for different sites in depth of the weld. The calculations were performed by using the mathematical package Mathcad.

## 3. Conclusions

Thus, the possibility of applying the method of Green functions as one of the universal ways to build thermal models at electron beam welding with various dynamic positioning of the beam is presented. The calculations showed that the proposed model of the heat source for electron beam welding with a static beam and the obtained on its basis solution of the heat problem, permit to describe the geometry of the weld at electron beam welding.

## Acknowledgements

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## References

[1] Frolov V V 1985 Theory of welding processes (*Moscow: Higher school*) p 559
[2] Rykalin N N, Uglov A A, Zuev I V and Cocora A N 1985 Laser and electron beam processing of materials: reference book (*Moscow: Mashinostroenie*) p 496
[3] Yazovskih V M 2008 Mathematical modeling and engineering methods of calculation in welding *Thermal processes during welding and modeling in MathCad* (*Perm: Publishing house Perm State Technical University*) chapter 2 p 119
[4] Rykalin N N, Uglov A A and Zuev I V 1978 Bases of electron beam processing of materials (*Moscow: Mashinostroenie*) p 239
[5] Ol'shans'kaya T V, Fedoseeva E M and Koleva E G 2017 *Bulletin PNRPU. Mechanical engineering, materials science* **19**(3) 49-74
[6] Kaydalov A A 2004 *Electron beam welding and adjacent technologies* (Kiev: Ecotechnology) p 260