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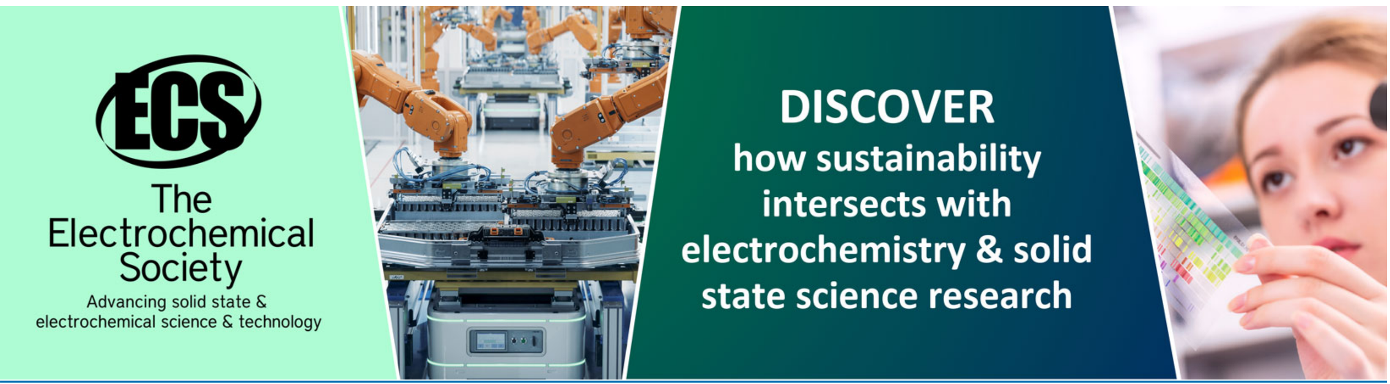

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Teachers' collective knowledge: the case of equivalent fractions

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Abstract. Research on teachers' mathematical knowledge has grown significantly over the last few decades. Many studies concern teachers' knowledge by using written tests, as for students. But in reality, teachers do not work in isolation but in institutions where professional knowledge is shared. How can this shared or collective knowledge be studied systematically and precisely? With this in mind, we designed so-called hypothetical teacher tasks (HTTs) which teachers solve in pairs. Each HTT can be used to investigate teachers' knowledge of some specific mathematical piece of knowledge (like how to add fractions) and knowledge about how to teach it. We present a case from Danish and Indonesian pre-service teachers' collaborative work on one HTT about equivalent fractions. We analyse how the shared mathematical and didactical knowledge of equivalent fractions differ between the two groups. Indeed, there are visible differences. The Danish pair proposed several didactical ideas to teach equivalent fractions, while the Indonesian pair focused on the direct instruction of standard algorithm.

1. Introduction

Research on teachers' knowledge has grown significantly over the last few decades especially after Shulman's seminal work introduced a new perspective on content knowledge in teaching [1]. This knowledge consists of subject matter content knowledge, pedagogical content knowledge, and curricular knowledge. These categories are closely related to learning and teaching practices in classrooms, and they become the primary focus of many studies including in mathematics education such as mathematical knowledge for teaching (MKT) [2].

Most studies on teachers' knowledge focus on teachers' knowledge by using written tests [e.g., 3], as for students, in which it is not in line with Shulman's view on how to study teachers' content knowledge and pedagogical content knowledge. For instance, he argued that content knowledge is not only about teachers' capability to define a correct answer in a specific subject, but "teachers must also be able to explain why a particular proposition is deemed warranted, why it is worth knowing, and how it relates to other propositions, both within discipline and without, both in theory and in practice" [1]. Therefore, to study teachers' knowledge one should let teachers share their knowledge that is related to what and how the teachers work in institutions where professional knowledge should not be isolated but shared. We try to address this issue in the present study.

Teachers' collective knowledge has been studied by Winsløw and colleagues [4,5] by designing so-called *hypothetical teacher tasks* (HTTs) which teachers solve in pairs. Each HTT aims to investigate

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teachers' mathematical knowledge, and how the teachers share this knowledge during their collaborative work. The mathematical task involved in the HTTs is both standard and elementary, that means in principle be addressed in many institutions [4]. But, it should give several possibilities, both practices, and theory, for teachers to solve the mathematical task and propose adequate didactical ideas. We adopt the idea of HTTs to study pre-service teachers' mathematical knowledge of rational numbers, specifically the case of equivalent fractions. We investigate how HTTs can be a useful tool to study and compare teachers' collective knowledge from two different institutions, Denmark and Indonesia.

In designing and studying teachers' collective knowledge, this study is developed based on the anthropological theory of the didactic (ATD) introduced by Chevallard [6], specifically the notion of *praxeology*. We apply the praxeology because it provides an epistemological model to study human practice and theory. The praxeology comes from a Greek word, *praxis,* and *logos*. The praxis or practical block is unified by *a type of tasks* (T) and *techniques* (τ). In the case of rational numbers, a mathematical type of task is to find a fraction that is equal to $\frac{a}{b}$. To solve it, one can employ several possible mathematical techniques such as multiplying both numerator and denominator by a positive integer or doubling each numerator and denominator. While, in the case of teachers' knowledge, the type of tasks is not only about mathematics but also didactics. An example of didactical tasks is to organise a teaching activity related to the equivalent fractions to support pupils learning, and the didactical technique could be to instruct pupils the standard mathematical technique of equivalent fraction $\left(\frac{a}{b} = \frac{na}{nb}, n \text{ is an integer}\right)$. While the logos or theoretical block is also unified by two components called a *technology* (θ) and a *theory* (Θ). The technology functions to justify or explain several techniques. An example of technological discourse related to the technique for the equivalent fraction task is that two fractions are equal if the value or size of the two fractions is the same. To show them, one can change both fractions into decimals. There is also a didactical technology associated with the didactical technique. For instance, a teacher who directly instructs pupils the standard mathematical technique may believe that the pupils need to receive the direct instruction on how to solve a mathematical task, and then use it to the similar task type. The second part of logos is the theory used to be a basis and support of the technology. Equivalent value of different representations of rational numbers can be seen as a theory to justify that mathematical technology, and the direct instruction theory is the example of didactic theory to justify the didactical technology. We use the mathematical and didactical praxeology as an analytical framework to the present study of teachers' mathematical and didactical knowledge of rational numbers, especially in the case of equivalent fractions, in the setting of collaborative work, because it provides us with a more detailed model to investigate teachers' knowledge, and the collaborative work provides opportunities for teachers to share and critically re-examine how they make claims from facts, and how they build on one another's ideas to construct more sophisticated ways of reasoning [7].

2. Method

The present study focuses on one among five HTTs designed for studying teachers' mathematical and didactical knowledge of rational numbers [8]. The design of HTT about equivalent fractions follows a similar way to the other HTTs. It is started by reviewing some previous studies on equivalent fractions that pupils have difficulties and misconceptions to find and justify the equivalent values of two fractions. One of the main misconceptions of finding an equivalent fraction is that pupils tend to add both numerator and denominator of a fraction by a positive integer $\left(\frac{a}{b} = \frac{a+n}{b+n}, \boldsymbol{n} \textbf{ is a positive integer}\right)$. By proposing this situation to PsTs, they may consider different mathematical and didactical praxeologies. The concrete HTT of equivalent fractions given to PsTs is presented in the following figure:

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You ask fourth grade pupils to find fractions which are equal to $\frac{3}{4}$.
A pupil claims that $\frac{3}{4} = \frac{8}{9}$ *because if you add 5 to both the top and the bottom of a fraction, the new fraction must be equal to the original.*

a. What do you think about this answer? Please explain! (*to be solved individually within 3 minutes*).
b. What would you do as a teacher to help the pupils from this case to understand the concept of equal fractions better? (*to be discussed and solved in pair within 5 minutes*).

Figure 1. HTT about equivalent fractions

The HTT about equivalent fractions along with other four HTTs of rational numbers has been tested to 31 Danish and 32 Indonesian PsTs. Almost all of them worked in pairs except for one Danish group consisting of three PsTs, so there were 15 Danish and 16 Indonesian pairs. However, in this particular study, we focus on presenting and analysing one pair from each group because the primary aim of this study is showing how HTTs can be used to study teachers' collective knowledge, and how this knowledge can be developed during their discussion.

From figure 1, we can describe two possible mathematical types of tasks and one didactical type of tasks. The first mathematical type of tasks (T_{m1}) is to find a fraction that is equal to $\frac{a}{b}$. The second mathematical type of tasks (T_{m2}) is to evaluate the two fractions $\frac{a}{b}$ and $\frac{a+n}{b+n}$ are equal, and it is based on the pupil's claim. Then, from question b the didactical task type (T_d) is to propose didactical praxeologies to help pupils to solve the task type T_{m1}, but some PsTs may relate it to T_{m2} when they discuss during their collaborative work. For each type of task, we can describe some possible techniques, and those can be found in Putra and Winsløw [8].

The data consists of PsTs' written answers from question a. and video recordings from question b. Especially for the video recordings, their utterances during the discussion were transcribed from time to time to produce detailed mathematical and didactical ideas. The transcripts were analysed in term of mathematical and didactical praxeologies (techniques, technologies, and a theory), both on mathematics and didactics. The first author did the principal analysis by reading the transcript several times and interpreting based on the praxeologies. Then some questionable points found in the data were discussed with the second author.

3. Findings

Now, we present our praxeological analysis of one Danish and one Indonesian pair working on the HTT about equivalent fractions. We select the pairs that can represent a commonality for their groups.

The Danish pair (D_1 and D_2). The written answer given by D_1 focused on mathematical technologies to clarify the pupil's incorrect mathematical technique of adding the positive integer 5 to the numerator and the denominator (τ_{m1}^{-}). He wrote "The size of a fraction depends solely on the relationship between the numerator and the denominator. If you add 5 to the numerator and the denominator, the size of the fraction increases because the numerator will be a larger percentage". D_1 had the mathematical technology to justify τ_{m1}^{-} based on the ratio, or a relationship between two quantitates, the numerator and the denominator. This is indicated that adding a number to both quantities does not give the same size of the two fractions. D_1 also wrote, "If the pupil had extended the fraction by multiplying the same factor in the numerator and the denominator, the relationship would still be the same, and the fraction size would be the same." This means that the appropriate mathematical technique to find an equivalent fraction is to use multiplication instead of addition (τ_{m1}) because it will give the same ratio between the two quantities. While D_2 also proposed a mathematical technology to justify τ_{m1}^{-} that was a little bit similar to what has been suggested by D_1. He said that pupils mix between addition and multiplication, but when one affects the numerator, the same goes to the denominator. Besides, D_2 also proposed a didactical technique that he would like to show to pupils that $\frac{8}{9}$ give a bigger number than $\frac{3}{4}$ by finding the quotients of the two fractions.

At the collaborative work, the Danish pair concerned on how to teach the pupils. They proposed three different didactical techniques to support pupils to understand the equivalent fractions. The first didactical technique was to present pupils with a common and straightforward fraction. It was stated by D_1 when they started the discussion. He said "Perhaps something about simplifying fractions. Perhaps [we] present them with the fractions of $\frac{10}{100}$, and then somehow to teach them, that is the same as a tenth". The didactical technology behind this technique was to have something easy for the pupils to grasp the meaning of equal fractions.

The second didactical technique was to support pupils to check the quotients of the two fractions $\frac{3}{4}$ and $\frac{8}{9}$. D2 explained it based on what he wrote in his written answer.

D_2: It's. They are on the track of the correct thing since they have the understanding that. They are just mixing up addition and multiplication, and they know that when you influence the numerator, that also works in the denominator, since [they] can prolong a fraction. In that sense, they are quite close, and I think that the only thing they are doing now is to test some hypotheses that they think, well earlier experiences in some way. Because, if you have two numbers then, and you add 5 to both then the difference should be the same. It just isn't when calculation [with] fractions. That's a shame, but what they have tested, which they could earlier.... and therefore, I think (..) almost right, to give the pupils an understanding... I think it's ok to make the pupils calculate these two numbers. Find out what it is as a decimal [number]. Perhaps do it together on the blackboard, have some pupils' approach [on the board], and use the calculator, so they will see the difference, and they see hmm, what it is then, that we perhaps did wrong.

D_1: Yes, perhaps start there, instead of giving them the answer to make them wonder.

D_2: Yes, because it is a little hypothesis they have [gotten] wrong. But it's something that is fine enough because if you have the number 5 and the number 10, and you have to prolong, you know make those numbers bigger, but it doesn't matter how there has to be the same value in between them. The proportion should be the same, well then it is okay to add the same in both. But it is just a little mistake when considering the rules of [operating] fractions.

D_1 : Yes. There it is the proportion between the numerator and the denominator which is important.

From the excerpt, D_2 supported the pupils to change both fractions into decimals because these could help them to see that the two fractions did not give the same value, the same decimal representation. The use of a calculator to check the value of the two fractions can be seen as a didactical tool to support pupils' understanding, but it also provides a challenge for teaching and learning activities, such as $\frac{8}{9}$ gives a result of repeated decimals. Besides, they also provided a didactical technology based on the mathematical idea of proportion. This means that the two fractions are equivalent when they have the same proportion.

The last didactical technique was to teach pupils using pizza representations, and the main purpose was to show that the two fractions $\frac{3}{4}$ and $\frac{8}{9}$ are not equal. During the discussion, D_1 drew the two diagrams to represent both fractions (figure 2). He suggested this didactical technique to let pupils find by themselves what went wrong. He believed that the visual representation could help the pupils learn a lot about the equivalent fractions especially for the pupils who were still in the fourth grade. So, it seems that the nature of teaching elementary school pupils based on contextual situations becomes a didactical technology to justify the didactical technique based on the pizza representations.

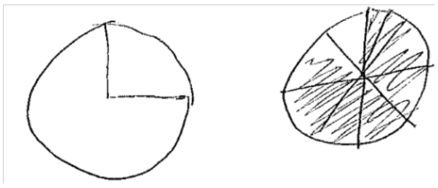

Figure 2. Pizza representations to support pupils' conception of equivalent fractions

The Indonesian pair (S_1 and S_2). The written answers given by both PsTs indicated that they recognised that the pupil's mathematical technique, adding both numerator and denominator by a positive integer ($\tau_{m1'}$), was incorrect. So, they proposed a correct mathematical technique, multiplying/dividing both numerator and denominator by a positive integer (τ_{m1}). S_1 also provided a technological discourse to justify the mathematical technique τ_{m1}. He argued that the two fractions are equivalent when the quotient of both fractions was the same (θ_{m1}). Besides, S_1 also stated that the pupil's explanation could be directed to find the least common multiple (LCM) because the pupil has understood that it has to be done an arithmetic operation to both numerator and denominator to get an equivalent fraction. This means that the pupil just chose an inappropriate operation, the addition instead of the multiplication. This explanation could be interpreted as a didactical technology to justify the pupil's mathematical thinking, and the mathematical idea of finding LCM seems less appropriate because the right term is just "multiple" or to multiply each numerator and each denominator.

At the collaborative work to address the task type T_d, S_1 dominated the discussion. He started it by asking S_2's opinion and continued by re-reading question b (T_d). Instead of giving an opportunity to S_2 to explain her thoughts, S_1 just proceeded to explain the technological discourse behind the pupil's answer based on what he has written on question a. He argued that the pupil has already understood how to find an equivalent fraction that it has to be done with the numerator and the denominator (adding the same number to both numerator and denominator), but it was incorrect. He continued the explanation by proposing a correct mathematical technique τ_{m1} that the pupil should apply to solve the task type T_{m1}, and the technique was correct if the quotient of the two fractions gave the same result (re-explain the technology θ_{m1}). While S_2 just seemed to agree with that explanation that it was indicated by her short answers "hmm" during the discussion. So, we can say that both PsTs agreed to instruct pupils with the correct and standard mathematical technique τ_{m1} to solve the task type T_{m1} as the main didactical technique for finding equivalent fractions.

The discussion was continued by proposing further possible justification behind the incorrect mathematical technique ($\tau_{m1'}$) given by the pupil. S_1 argued "[the arithmetic] operations needed [for the task] are multiplication and division," but when S_2 asked him a reason behind these two operations, S_1 claimed that it was based on "the rule." Although he did not elaborate on it, he tried to define the definition of equivalent fractions from the meaning of a fraction itself. He stated, "A fraction [$\frac{a}{b}$] is the division [a by b], and equivalent value to the division is the multiplication and the division itself." From this statement, we can interpret that S_1 tried to justify the mathematical technique τ_{m1} is correct from the concept of fraction as a division of two positive integers. They continued the discussion, but they still discussed the idea behind the reason to choose the multiplication or the division. For instance, we present a further discussion of the two Indonesian pair as follows:

S_2 : I think it is better to explain why it has to be multiplied not to be added. Do you understand it?

S_1 : [I] understand [it].Yes, it is like that, [but] why it should be multiplied. It is not only to be multiplied, but it is also possible to be divided.

S_2: Yes, it is divided when the fraction involves [two] bigger [digits]. Yes, it has to be divided by the same [positive] integer. If the case like this. The pupil asks to find a fraction that is equal to $\frac{3}{4}$. It involves [two] smaller [digits 3 and 4].

S_1: Maybe like this. A fraction is equal to the division. $\frac{3}{4}$ means that 3 is divided by 4. Which is equivalent to the division, … division, multiplication, subtraction, and addition…the same level to the division is multiplication, so only the multiplication can be used to find the equivalent fraction. Hmm, wait a minute, … How to explain it? Yes, it can be like that.

The excerpt shows that S_2 tried to propose an idea to teach pupils by explaining why the mathematical technique τ_{m1} should be used instead of $\tau_{m1'}$. This kind of didactical technique may support pupils to develop their technological discourse to accept the correct mathematical technique. Although S_1 tried to convince S_2 using the concept of fraction as a division of two positive integers, this technological discourse may be beyond the mathematical concept that could be accepted by S_2 and also by the pupils. It seems that S_2 was not satisfied with S_1, but she also could not provide some justification to the mathematical technique τ_{m1}. Then, at the end of their discussion S_1 argued "if we use addition to find [an equivalent fraction of $\frac{a}{b}$], the numerator and the denominator are added by the same positive integer and [got a new fraction $\frac{c}{d}$], and when we divide [*a* by *b* and *c* by *d*], we can show the quotients are different". So, this is one of didactical technology linked to mathematical technology to justify why such the technique proposed by the pupils on the HTT is incorrect.

4. Discussion and concluding remarks

The findings of the study show manifest differences between the Danish and Indonesian mathematical and didactical praxeologies for addressing the task of equivalent fractions. The main difference is that the Danish pair suggested three different possible didactical techniques, while the Indonesian pair only focused on one didactical technique based on the direct instruction of the mathematical technique τ_{m1}. That discrepancy is caused by their personal mathematical praxeologies. For instance, both Indonesian PsTs provided almost the same written answers to question a, and they just discussed the idea based on what they have written. The Danish pair, on the other hand, seems to have different personal mathematical praxeologies, so when they bring these praxeologies into the discussion, they come to different possible didactical praxeologies to teach the pupils. The differences between the two pairs can be caused by their mathematical technologies or theories. For instance, the Danish pair tends to justify their techniques based on the ratio or proportion. They tend to argue that the two fractions are equivalent when the ratio between the numerators and the denominators of the two fractions remains the same, and understanding a fraction as ratio between two numbers is considered a prerequisite for equivalent fractions [9]. While, the Indonesian pair focuses on the linguistic meaning of "fraction" as the division between the numerator and the denominator stated by S_1, and uses this technology to justify τ_{m1}. Their difficulty in explaining this makes them discuss the same thing for several times and could not achieve more advanced mathematical and didactical praxeologies.

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