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Mathematical models for estimating production risks

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Abstract. When designing and operating machine-building facilities, their operation reliability is significant. The quantitative measure of reliability is production risks. The article proposes a mathematical model to determine probabilities of unfavorable events: the occurrence of a risk factor at a given time interval at some point in time; a worker's exposure at a given workplace in the adverse event area; negative changes in the worker's ability to fulfill his production duties. The technique to define production risks based on the knowledge of likelihood of adverse events occurrence is resulted. Examples of the application method for solving safety management problems of a typical technological operation of assembling an all-in-one connection have been presented.

1. Introduction

Risks are studied from different disciplines points, with discussion of various methods and tools used to optimize technological processes and to reduce adverse events [1]. The improvement of the technological process should be aimed at meeting requirements of international standards ISO 9001:2015 and ISO 14001:2015 and the draft standard ISO 45001, that consider a problem to minimize production risks [2].

The idea of using simulation to estimate the production risk in different engineering areas has been discussed by researchers from different countries. At the same time, it is important to realize the nature of the initial data included into the model, the field of its application and the result obtained [3]. However, the application of mathematical models to estimate occupational risks in the workplace for the management of occupational safety is currently lacking.

The quantification of production risks should be based on expert estimates obtained as a result of experiments or accumulated statistical data [4].

2. Problem statement

The technological process in time interval T is serviced with *n* working places $w_1,\ldots,w_n$ and to analyze its danger level there are m independently acting hazards $f_1,\ldots,f_m$.

This situation corresponds to a chart of technological process risk levels in time interval T

$$\begin{bmatrix} \mathrm{P}_{11} & \cdots & \mathrm{P}_{1n} \\ \cdots & \cdots & \cdots \\ \mathrm{P}_{m1} & \cdots & \mathrm{P}_{mn} \end{bmatrix}$$

where P_{ij} is probability of a risk factor f_i exposure to a working place w_j ($i = 1,\ldots, m$; $j = 1,\ldots, n$).

According to [5] $\mathrm{P}_{ij} = \mathrm{P}_i \cdot q_{ij}$, where P_i is a probability estimation of occurrence of the factor f_i, q_{ij} is assessment of the likelihood of worker's exposure in the working place w_j into the zone of its impact if it is. The overall level of risk (the hazards level of the technological process relatively $f_1,\dots,f_m$) is estimated with the formula

$$\overline{\mathrm{P}} = 1 - \prod_{i=1}^{m}\prod_{j=1}^{n}(1-\mathrm{P}_{ij}) \tag{1}$$

3. Theory

When solving problems of managing the production safety, the following questions are very important:

1. How will the overall level of risk $\overline{\mathrm{P}}$ decrease if some elements P_{ij} of risk levels charts are managed to be reduced?
2. What elements P_{ij} of risk levels chart and how many of them to be reduced to achieve reduction in the overall level of risk $\overline{\mathrm{P}}$ to the given value?

To obtain the dependence of changing the overall level of risk $\overline{\mathrm{P}}$ on changes in the elements of the risk level chart P_{ij} the following well-known fact from mathematical analysis is used: for the function $f(x_1,\dots,x_N)$ in the point $M_0(x_1^{(0)},\dots,x_N^{(0)})$ there is approximate equality

$$f(x_1^{(0)}+\Delta x_1,\dots,x_N^{(0)}+\Delta x_N) - f(x_1^{(0)},\dots,x_N^{(0)}) \approx \left.\frac{\partial f}{\partial x_1}\right|_{M_0}\cdot\Delta x_1 + \dots + \left.\frac{\partial f}{\partial x_N}\right|_{M_0}\cdot\Delta x_N$$

(the right-hand side of the equation is a total differential of the function $f(x_1,\dots,x_N)$ in the point M_0). The order of errors of this equation can be estimated as $\max_i\left((\Delta x_i)^2\right)$.

One considers $\overline{\mathrm{P}}$ as a function $N = m\cdot n$ of variables

$$f(x_{11},x_{12},\dots,x_{mn}) = 1 - \prod_{i=1}^{m}\prod_{j=1}^{n}(1-x_{ij}),$$

And for it

$$\frac{\partial f}{\partial x_{ij}} = \prod_{\substack{k=1\\k\neq i}}^{m}\ \prod_{\substack{s=1\\s\neq j}}^{n}(1-x_{ks}).$$

One calculates $\frac{\partial f}{\partial x_{ij}}$ in the point $M_0(\mathrm{P}_{11},\mathrm{P}_{12},\dots,\mathrm{P}_{mn})$ using equation (1).

$$\left.\frac{\partial f}{\partial x_{ij}}\right|_{M_0} = \prod_{\substack{k=1\\k\neq i}}^{m}\ \prod_{\substack{s=1\\s\neq j}}^{n}(1-\mathrm{P}_{ks}) = \frac{1-\overline{\mathrm{P}}}{1-\mathrm{P}_{ij}}$$

There is $\Delta\mathrm{P}_{ij} \geq 0$. Then

$$f(\mathrm{P}_{11}-\Delta\mathrm{P}_{11},\mathrm{P}_{12}-\Delta\mathrm{P}_{12},\dots,\mathrm{P}_{mn}-\Delta\mathrm{P}_{mn}) - f(\mathrm{P}_{11},\mathrm{P}_{12},\dots,\mathrm{P}_{mn}) \approx \sum_{i=1}^{m}\sum_{j=1}^{n}\frac{1-\overline{\mathrm{P}}}{1-\mathrm{P}_{ij}}\cdot(-\Delta\mathrm{P}_{ij})$$

Hence, denoting through $\overline{\mathrm{P}}_{\mathrm{H}}$ general level of risk when decreasing P_{ij} by $\Delta\mathrm{P}_{ij}$ (i = 1,…, m; j = = 1,…, n), the formula is obtained

$$\Delta\overline{\mathrm{P}} = \overline{\mathrm{P}} - \overline{\mathrm{P}}_{\mathrm{H}} \approx \sum_{i=1}^{m}\sum_{j=1}^{n}\frac{1-\overline{\mathrm{P}}}{1-\mathrm{P}_{ij}}\cdot(\Delta\mathrm{P}_{ij}). \tag{2}$$

The order of errors of the formula is $\max_{ij}\left((\Delta \mathrm{P}_{ij})^2\right)$.

This ratio analyzes quantitatively the following situations:

1. To estimate the magnitude of the reduction in the overall level of risk with a reduction in the likelihood of risk factors occurrence (change in technological production parameters) and (or) with a decrease in the likelihood of workers entering the zone of its exposure (organizational changes in the operation of the relevant technological processes).
2. To estimate the value $\Delta \mathrm{P}_{ij}$ of reducing the likelihood of exposure of any i-th risk factor to j-th working place (if possible) to reduce the overall level of risk by a given amount $\Delta \overline{\mathrm{P}}$.

There is formula (2) in the first situation, there is another formula obtained from formula (2) in the second situation:

$$\Delta \mathrm{P}_{ij} \approx \frac{1-\mathrm{P}_{ij}}{1-\overline{\mathrm{P}}} \cdot \Delta \overline{\mathrm{P}}. \tag{3}$$

4. Experimental results

These situations are presented on the example of a typical technological operation of assembling an fixed joint (tubular assembly) on a magnetic pulse installation.

Time T, the exposure of risk factors is estimated in 8 hours (working shift), there are 2 workers (an operator, a mechanical technician), number of manufactured products is 200 pcs.

The following dangerous risk factors are identified:

1) f_1 is a dangerous factor: fragments of the inductor formed during its destruction, an expert estimate of the number of possible fractures is 1 per 800 discharges;

2) f_2 is a dangerous factor: a situation when the electrical circuit is closed through the human body - 1 case for 2000 hours of operation.

As expert estimates of time for entering working places (an operator, a mechanical technician) into the zone of exposure to risk factors under these conditions, the following estimates are considered:

1) risk factor f_1: for the operator is 0.01 % working shift time; for the mechanical technician it is 0.03 % working shift time;

2) risk factor f_2: for the operator is 10 % working shift time; for the mechanical technician it is 5 % working shift time

A chart of technological operation risks levels relative to risk factors $f_1, f_2\,(m=2)$ at time interval T = 8 hours for working places w_1, w_2 (an operator, a mechanical technician, n = 2) is calculated.

Using the Poisson distribution, the probability of occurrence of a risk factor f_1 according to the formula $\mathrm{P}_1 = 1 - e^{-N\mathrm{P}_0}$ is calculated considering initial data, N = 200, $\mathrm{P}_0 = 1/800$:

$$\mathrm{P}_1 \approx 1 - e^{-0.25} \approx 0.221199.$$

Using an exponential distribution, the probability of occurrence of a risk factor f_2 according to the formula $\mathrm{P}_2 = 1 - e^{-\lambda \mathrm{T}}$ is calculated considering initial data, $\lambda = 1/2000$, T = 8:

$$\mathrm{P}_2 \approx 1 - e^{-0.004} \approx 0.003992.$$

Estimates of the probabilities of getting working places w_1, w_2 into the exposure area of risk factors f_1, f_2 if they are, according to the initial data are

$$q_{11} = 0.00001, \quad q_{12} = 0.0003, \quad q_{21} = 0.1, \quad q_{22} = 0.5.$$

A risk levels chart of the technological operation in question during the work shift T ($\mathrm{P}_{ij} = \mathrm{P}_i q_{ij}$) is:

$$\mathrm{P}=\begin{bmatrix}0.000022 & 0.000066\\ 0.000399 & 0.000199\end{bmatrix}.$$

The overall level of risk according to formula (1) is

$$\overline{\mathrm{P}}=1-\prod_{i=1}^{2}\prod_{j=1}^{2}(1-\mathrm{P}_{ij})=0.000686.$$

Coefficients $\frac{1-\overline{\mathrm{P}}}{1-\mathrm{P}_{ij}}$ are calculated according to formula (2):

$$\frac{1-\overline{\mathrm{P}}}{1-\mathrm{P}_{11}}\approx 0.959336,\quad \frac{1-\overline{\mathrm{P}}}{1-\mathrm{P}_{12}}\approx 0.959380,\quad \frac{1-\overline{\mathrm{P}}}{1-\mathrm{P}_{21}}\approx 0.959713,\quad \frac{1-\overline{\mathrm{P}}}{1-\mathrm{P}_{22}}\approx 0.959513.$$

Thus, formula (2) is in the case

$$\Delta\overline{\mathrm{P}}=0.959336\cdot\Delta\mathrm{P}_{11}+0.959380\cdot\Delta\mathrm{P}_{12}+0.59719\cdot\Delta\mathrm{P}_{21}+0.959513\cdot\Delta\mathrm{P}_{22}. \qquad (4)$$

Example 1. How will the overall level of risk (level of danger) of the said technological operation decrease if the inductor reliability is doubled (an estimate of the possibility of destruction is 1 per 1600 discharges)?

In this case

$$\mathrm{P}_1^{(\mathrm{H})}=1-e^{-200\cdot(1/1600)}\approx 1-e^{-0.25}\approx 0.117503.$$

The new risk level chart is

$$\mathrm{P}_{11}^{(\mathrm{H})}=\mathrm{P}_1^{(\mathrm{H})}\cdot q_{11}=0.000012,\quad \mathrm{P}_{12}^{(\mathrm{H})}=\mathrm{P}_1^{(\mathrm{H})}\cdot q_{12}=0.000035,$$

wherein,

$$\Delta\mathrm{P}_{11}=\mathrm{P}_{11}-\mathrm{P}_{11}^{(\mathrm{H})}=0.00001,\quad \Delta\mathrm{P}_{12}=\mathrm{P}_{12}-\mathrm{P}_{12}^{(\mathrm{H})}=0.000031,\quad \Delta\mathrm{P}_{21}=\Delta\mathrm{P}_{22}=0.$$

According to formula (4) one obtains

$$\Delta\overline{\mathrm{P}}=0.959336\cdot 0.00001+0.959380\cdot 0.000031=0.000039,$$

And it is 5.7 % from the total risk level, i.e. the level of danger will decrease by 5.7 %.

Example 2. How will the overall level of risk decrease if the probability of electric shock for an operator is reduced by 2?

In this case

$$\mathrm{P}_{21}^{(\mathrm{H})}=0.5\cdot\mathrm{P}_{21}=0.0001995,\ \Delta\mathrm{P}_{21}=\mathrm{P}_{21}-\mathrm{P}_{21}^{(\mathrm{H})}=0.0001995,\ \Delta\mathrm{P}_{11}=\Delta\mathrm{P}_{12}=\Delta\mathrm{P}_{22}=0.$$

According to formula (4) one obtains

$$\Delta\overline{\mathrm{P}}=0.959713\cdot 0.0001995=0.0001915,$$

And it is 27.9 % from the total risk level, i.e. the level of danger will decrease by 27.9 %.

Example 3. How will the overall level of risk decrease if the probability of electric shock for a mechanical-technician is reduced by 2?

In this case

$$\mathrm{P}_{22}^{(\mathrm{H})}=0.5\cdot\mathrm{P}_{22}=0.0001,\ \Delta\mathrm{P}_{22}=\mathrm{P}_{22}-\mathrm{P}_{22}^{(\mathrm{H})}=0.0001,\ \Delta\mathrm{P}_{11}=\Delta\mathrm{P}_{12}=\Delta\mathrm{P}_{21}=0.$$

According to formula (4) one obtains

$$\Delta\overline{\mathrm{P}}=0.959513\cdot 0.0001=0.000096,$$

And it is 14 % from the total risk level, i.e. the level of danger will decrease by 14 %

Example 4. Suppose that it is necessary to reduce the overall level of risk $\overline{\mathrm{P}}=0.000686$ by 10%, i.e. $\Delta\overline{\mathrm{P}}=0.000068$.

According to formula (3) one obtains

$$\Delta P_{11} = \frac{1-\overline{P}}{1-P_{11}} \cdot \Delta\overline{P} = 0.00071, \quad \Delta P_{12} = \frac{1-\overline{P}}{1-P_{12}} \cdot \Delta\overline{P} = 0.000071,$$

$$\Delta P_{21} = \frac{1-\overline{P}}{1-P_{21}} \cdot \Delta\overline{P} = 0.000070, \quad \Delta P_{22} = \frac{1-\overline{P}}{1-P_{22}} \cdot \Delta\overline{P} = 0.000070.$$

Comparing ΔP_{11}, ΔP_{12} with P_{11}, P_{12} relatively proves that a decrease in the likelihood of exposure to a risk factor f_1 to working places w_1 or w_2 it is impossible to achieve such a reduction in the overall level of risk.

Comparing ΔP_{21}, ΔP_{22} with P_{21}, P_{22} proves that to do this it is necessary to reduce the likelihood of exposure of a risk factor f_2 to working place w_1 by 17 % or to reduce the likelihood of exposure f_2 to working place w_2 by 35.2 %.

5. Discussion

The mathematical model for calculating operational risks was used by the authors for the quantitative estimation of operational risk in the working places in the engineering operations. The calculation results allow:

1. To implement an automated quantitative risk estimation using software based on the proposed use of mathematical models.
2. To apply mathematical simulating in the process of managing production risks, it is proposed by international standards.
3. To apply the proposed mathematical models to quantify the production risks in organizations with different activities for different processes.
4. To formulate recommendations for the development of effective events to manage the production risks in organizations.
5. To use the proposed methodological approach when training specialists in the field of occupational safety in the production management.

6. Conclusion

The proposed mathematical model for calculating production risks has both a theoretical and an applied nature. It gives a quantitative estimation of production risks in working places in the engineering operations according to the international standards requirements. The use of risk estimation techniques increases their management effectiveness.

The mathematical model offers an unambiguous algorithm for calculating production risks and can therefore be formalized using computational methods of mathematics.

7. References

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