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Simulated Annealing for Competitive p -Median Facility Location Problem

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Abstract. There is a number of models that take into account the market competition in the location theory. Aboularian R. et al. formulated the location and design problem (CFLDP), in which the share of the market being served elastically varies depending on the location. The models with elastic demand are considered to be rather complicated and advantageous. We developed this problem and proposed the new one which combines CFLDP and classical p -median problem. In the given formulation a number of new facilities is fixed and is equal to p , so competitive p -median facility location and design problem (CPFLDP) has been derived. Therefore, the task can be considered as a clustering problem. In this paper we develop local search algorithms for CPFLDP. A computational experiment is carried out, the results are discussed.

1. Introduction

Facility location problems take a special place in the location theory. During last few years the competitive models, which describe the modern economy processes most properly, have gained the ground. There are different types of the state above problems, depending on the rival parts and customers behavior, possible facility locations [1]. Aboularian R., Berman O., Krass D. have formulated the location and design problem, where the rival parts struggle for the demand share [2]. The share changes elastically depending on the customers choice of the facility in order to satisfy their demands. The authors [2] describe this type of demand through a nonlinear function, traditionally used in the special interaction models in marketing. The developed mathematical model with a nonlinear objective function complicates the optimum search. Most commonly the known commercial software is not able to find any possible solution [3, 4]. The approximate approach becomes actual in the given situation. It should be mentioned that the number of algorithms to solve the elastic demand problems is quite limited. Probabilistic weighted greedy heuristics was offered to solve the location and design problem in [2]. We developed and researched a number of local search algorithms for the given problem [3, 4]. In this work we continue investigations in the given direction for the new competitive p -median facility location problem.

2. Competitive p -Median Facility Location Problem

One of the most known facility location problems is the p -median problem, which has a wide employment range and is often formulated as a clustering problem. However, the competitive location models attract the main interest at this time. This work researches the offered



Competitive p -Median Facility Location and Design Problem (CPFLDP) which unites the statement of the elastic demand problem from [2] and the classical p -median problem. Let us introduce the problem formulation and its mathematical model.

A new company is to locate a limited number of its facilities according to the given volume of money. It can state both the locations and the types of the facilities, for example: a supermarket, a hypermarket, etc. It has to rival for the demand with the existing companies, taking into account the fact that the customers choose the facilities depending on the distance, attractiveness and other factors. That is why the share of the serviced demand changes elastically depending on the companies and customers decisions. The aim of the company is to define the facility locations and types in order to attract the biggest share of the total demand.

Let us introduce the following notation with the consideration of [2]: $N = \{1, 2, \dots, n\}$ is a discrete set of the demand points, so that each point is able to locate a facility; $C \subset N$ are the points of the demand occupied by competitor; $S = N \setminus C$ are the possible facility location points for the new company; R is a number of facility types; w_i is the demand weight in the point $i \in N$; c_{jr} is the facility opening cost if the variant $r \in R$ is used in the point $j \in S$; p is the facility number to be opened; β is the distance sensitivity parameter; λ is the demand elasticity coefficient.

Variables $x_{jr} = 1$, if the facility of the type $r \in R$ is located in the point $j \in S$, $x_{jr} = 0$ otherwise.

The usefulness u_{ij} of the company's facility $j \in S$ for a customer $i \in N$ is calculated the following way: $u_{ij} = \sum_{r \in R} k_{ijr} x_{jr}$, where $k_{ijr} = a_{jr}(d_{ij} + 1)^{-\beta}$ are the special coefficients which take into consideration the distance d_{ij} between the points i and j , the customers sensitivity to it β and the attractiveness a_{jr} of the facility type $r \in R$, $i, j \in N$. The total facility usefulness $U_i(S)$ of the point $i \in S$ is estimated through $U_i(C) = \sum_{j \in C} u_{ij} = \sum_{r \in R} k_{ijr} x_{jr}$. The notation for the competitors is similar.

In this model the demand function has an exponential form: $g(U_i) = 1 - \exp(-\lambda \cdot U_i)$, where U_i is the total utility for a customer at $i \in N$ from all open facilities:

$$U_i = \sum_{j \in S} \sum_{r=1}^R k_{ijr} x_{jr} + \sum_{j \in C} \sum_{r=1}^R k_{ijr} x_{jr}.$$

The share of the new facilities in the total volume of customer $i \in N$ servicing is equal to:

$$MS_i = \frac{\sum_{j \in S} \sum_{r=1}^R k_{ijr} x_{jr}}{\sum_{j \in S} \sum_{r=1}^R k_{ijr} x_{jr} + \sum_{j \in C} \sum_{r=1}^R k_{ijr} x_{jr}}.$$

Based on above notation, the mathematical model looks as follows:

$$\max \sum_{i \in N} w_i \cdot \left(1 - \exp \left(-\lambda \left(\sum_{j \in S} \sum_{r=1}^R k_{ijr} x_{jr} + \sum_{j \in C} \sum_{r=1}^R k_{ijr} x_{jr} \right) \right) \right). \quad (1)$$

$$\cdot \left(\frac{\sum_{j \in S} \sum_{r=1}^R k_{ijr} x_{jr}}{\sum_{j \in S} \sum_{r=1}^R k_{ijr} x_{jr} + \sum_{j \in C} \sum_{r=1}^R k_{ijr} x_{jr}} \right),$$

$$\sum_{j \in S} \sum_{r \in R} c_{jr} x_{jr} \leq B, \quad (2)$$

$$\sum_{r \in R} x_{jr} \leq 1, j \in S, \quad (3)$$

$$\sum_{j \in S} \sum_{r \in R} x_{jr} = p, \quad (4)$$

$$x_{jr} \in \{0, 1\}, \quad r \in R, j \in S. \quad (5)$$

The objective function (1) reflects the company's goal to maximize its demand share. Inequation (2) allows locate the facilities taking in account the budget amounts available. Conditions (3) show that there is a possibility to locate the facilities of only one type. Equation (4) sets the condition for the number of facilities to be opened.

Theoretically the given problem is NP-hard. For the implementation of experimental studies a set of 192 test examples have been used, which were constructed earlier on the basis of a real applied problem [3]. The set consists of two series, where the distances between the points are defined with the equally uniform distribution (Series 1) and satisfy the triangle inequality (Series 2). A set of 16 examples of the dimension $|N| = 60, 80, 100, 150, 200, 300$ with three possible projects and budget limits of 3, 5, 7 and 9 units have been formed.

3. Development of Simulated Annealing Algorithm

The research of the problem is frequently started from the attempt to use the existing software. The usage of the commercial software GAMS (solver CoinBonmin) [5] for the CPFLDP does not always allow find a permissible solution. Furthermore, the computational experiments [6] showed that CPFLDP is more complicated for the solver than the Competitive Facility Location and Design Problem (CFLDP), which is set by the conditions (1)-(3), (5). The data about average CPU time for Series 1 is given in the Table 1.

Table 1. Average CPU time of CoinBonmin, sec.

	60	80	100	150	200	300
CPFLDP	181	329	482	2351	2650	11831
CFLDP	106	161	295	1000	1325	4561

Taking into account the given above information the local search algorithms were chosen as the solution methods. The Variable Neighborhood Search (VNS) algorithm was offered in order to solve the CPFLDP in [6]. In this work we offer the realization of the simulated annealing (SA) algorithm. This algorithm is currently known for its successful usage for a wide range of optimization problems, for example [4, 7].

The simulated annealing algorithm starts from some initial solution and initial temperature parameter. At each value of temperature a certain number of iterations is carried out. On each iteration the algorithm selects from neighbourhood of the current solution a new solution randomly. This solution is accepted as a new current one according to some probabilistic law. Then the temperature decreases. The process continues until the system reaches the frozen state or until other stopping criteria are satisfied (e.g. maximal number of iterations, a maximal number of steps without improvements etc.).

The algorithm has following controlling parameters: τ^0 is the temperature of the frozen state; L is the length of a temperature interval; $\varphi(\tau)$ is the low of reduction of temperature.

Consider the scheme of the SA algorithm.

- (1) Set the initial solutions S and temperature τ .
- (2) While $l < L$ do
 - (a) choose solutions S' from the neighbourhood $N(S)$ of solutions S ;
 - (b) $\Delta := F(S') - F(S)$;
 - (c) if $\Delta \leq 0$ then $S := S'$;
 - (d) if $\Delta > 0$ then $S := S'$ with probability $p = \exp\{-\Delta/\tau\}$.

- (3) Reduce temperature $\tau := \varphi(\tau)$.
- (4) If time to terminate then return the best found solution S^* else go to step 2.

On the basis of the competitive p -median facility location and design problem specificity the special kinds of neighborhoods have been built. The Boolean matrix X can be associated an integer vector $z = (z_j)$, so that $z_j = r$, if $x_{jr} = 1$. Let us call the neighborhood N_p of a solution z a set of vectors received as a result of the following steps:

- (1) choose a facility opened in the point $j = s$ with the project variant z_s ;
- (2) choose a point $j = q$, where a facility is not opened yet;
- (3) open a facility in the point q with the project variant z_s and close the facility in the point s , i.e. execute the following transformation: $z_q := z_s$, $z_s := 0$.

It should be noted that there is a problem of selecting parameter values, so that the algorithm can give good results for most instances. After the series of preliminary experiments the following parameter values for the simulated annealing algorithm have been found: the threshold value equal to 5; for the simulated annealing algorithm the temperature interval length $L = 10$, the initial temperature $\tau = 150$, the cooling (minimal) temperature value $\tau^0 = 5$, the low of reduction of temperature is $\varphi(\tau) = 0.99 \cdot \tau$.

Table 2. Average deviations from upper bounds in case of uniform distribution distances.

	60	80	100	150	200	300
min	0.000	0.000	0.000	0.000	0.000	0.000
aver	0.483	0.101	0.450	0.263	0.190	0.384
max	3.326	0.825	2.421	2.212	3.011	3.162

Table 2 shows the minimum (min), average (aver) and maximum (max) values of deviations from the upper bounds (UB) for test instances with uniform distribution of distances in a single run of the algorithms. The average deviation for the uniform distribution of distances instances of 300 locations is 0.384%, and the maximal deviation not exceeds 3.162%.

The test instances with Euclidean distances proved to be difficult for all considered algorithms (see Table 3). The average deviations in this case for the same dimension is 15.775%, the maximal deviation is 18.912%. Previously it was shown that the upper bound used is rough for the Series 2 for the CFLDP problem [8]. The large deviations in table 3 may be due to the same reason.

Table 3. Average deviations from upper bounds in case of the Euclidean distances.

	60	80	100	150	200	300
min	19.274	18.802	10.027	18.349	10.178	9.330
aver	29.695	26.495	21.577	25.666	16.123	15.775
max	49.883	33.252	33.029	32.189	22.535	18.912

The described algorithm were tested on a computer with Intel Xeon X5675 @ 3.07 GHz processor, 32 GB RAM. In Table 4 the minimum, average and maximum running time for Series 1

are done. For example, the average CPU time of 300 locations is 166.465 seconds, compared to 11831 seconds of solver CoinBonmin of system GAMS (see Table 1). This advantage can be explained by the relatively simple type of neighborhood used. This fact once again confirms that the simulation annealing algorithm for should be applied to problems in which it is possible to construct a neighborhood that allows one to simply move from one solution to another. However, due to the presence of a large number of parameters in the algorithm, a wide computational experiment is required to configure them. For Series 2, similar results were obtained.

Table 4. Average CPU time for case of uniform distribution distances, sec.

	60	80	100	150	200	300
min	5.238	9.126	12.972	28.346	43.980	94.818
aver	12.174	18.855	25.287	51.234	75.961	166.465
max	24.052	33.777	48.124	106.724	146.520	312.111

4. Conclusions

In this paper we have suggested simulated annealing algorithm for new Competitive p -Median Facility Location Problem. This algorithm allow to find solutions close to the upper bounds on the special test instances in case of uniform distribution of distances. On another series in case of the Euclidean distances the deviations are greater. This confirms that the upper bounds used are rough for these examples. It was necessary to continue research using other rules for the upper bound construction. It should be noted that the proposed algorithm works on average faster than the GAMS up to 70 times. It is interesting to continue these studies for other threshold algorithms.

Acknowledgments

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