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Scalar–tensor–vector gravity theory

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Abstract. A covariant scalar–tensor–vector gravity theory is developed which allows the gravitational constant G, a vector field coupling ω and the vector field mass μ to vary with space and time. The equations of motion for a test particle lead to a modified gravitational acceleration law that can fit galaxy rotation curves and cluster data without non-baryonic dark matter. The theory is consistent with solar system observational tests. The linear evolutions of the metric, vector field and scalar field perturbations and their consequences for the observations of the cosmic microwave background are investigated.

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Contents

1. Introduction

Two theories of gravity called the non-symmetric gravity theory (NGT) [1] and the metric–skew–tensor gravity (MSTG) theory [2] have been proposed to explain the rotational velocity curves of galaxies, clusters of galaxies and cosmology without dark matter. A fitting routine for galaxy rotation curves has been used to fit a large number of galaxy rotational velocity curve data, including low surface brightness (LSB), high surface brightness (HSB) and dwarf galaxies with both photometric data and a two-parameter core model without non-baryonic dark matter [2, 3]. The fits to the data are remarkably good and for the photometric data only the one parameter, the mass-to-light ratio $\langle M/L \rangle$, is used for the fitting, once two parameters M_0 and r_0 are universally fixed for galaxies and dwarf galaxies. A large sample of x-ray mass profile cluster data has also been fitted [4].

    The gravity theories require that Newton's constant G, the coupling constant γ_c that measures the strength of the coupling of the skew field to matter and the mass μ of the skew field, vary with distance and time, so that agreement with the solar system and the binary pulsar PSR 1913+16 data can be achieved, as well as fits to galaxy rotation curve data and galaxy cluster data. In [2], the variation of these constants was based on a renormalization group (RG) flow description of quantum gravity theory formulated in terms of an effective classical action [5]. Large infrared renormalization effects can cause the effective G, γ_c, μ and the cosmological constant Λ to run with momentum k and a cut-off procedure leads to a space and time varying G, γ_c and μ, where $\mu = 1/r_0$ and r_0 is the effective range of the skew symmetric field.

    In the following, we shall pursue an alternative relativistic gravity theory based on scalar–tensor–vector gravity (STVG), in which G, a vector field coupling constant ω and the mass μ of the vector field are dynamical scalar fields that allow for an *effective*

description of the variation of these 'constants' with space and time. We shall not at present consider the variation of the cosmological constant Λ with space and time.

The gravity theory leads to the same modified acceleration law obtained from NGT and MSTG for weak gravitational fields and the same fits to galaxy rotation curve and galaxy cluster data, as well as to agreement with the solar system and pulsar PSR 1913+16 observations. An important constraint on gravity theories is the bounds obtained from weak equivalence principle tests and the existence of a 'fifth' force, due to the exchange of a massive vector boson [6]. These bounds are only useful for distances $\leq$100 A.U. and they cannot rule out gravity theories that violate the weak equivalence principle or contain a fifth force at galactic and cosmological distance scales. Since the variation of G in our modified gravity theory leads to consistency with solar system data, then we can explore the consequences of our STVG theory without violating any known local observational constraints.

An important feature of the NGT, MSTG and STVG theories is that the modified acceleration law for weak gravitational fields has a *repulsive* Yukawa force added to the Newtonian acceleration law. This corresponds to the exchange of a massive spin 1 boson, whose effective mass and coupling to matter can vary with distance scale. A scalar component added to the Newtonian force law would correspond to an *attractive* Yukawa force and the exchange of a spin 0 particle. The latter acceleration law cannot lead to a satisfactory fit to galaxy rotation curves and galaxy cluster data.

In section 8, we investigate a cosmological solution based on a homogeneous and isotropic Friedmann–Lemaître–Robertson–Walker (FLRW) spacetime. We present a solution that can possibly fit the acoustic peaks in the CMB power spectrum by avoiding significant suppression of the baryon perturbations [7], and which can possibly be made to fit the recent combined satellite data for the power spectrum without non-baryonic dark matter.

All the current applications of the three gravity theories that can be directly confronted with experiment are based on weak gravitational fields. To distinguish the theories, it will be necessary to obtain experimental data for strong gravitational fields e.g. black holes. Moreover, confronting the theories with cosmological data may also allow a falsification of the gravity theories. Recently, the NGT and MSTG were studied to derive quantum fluctuations in the early universe from an inflationary-type scenario [8].

2. Action and field equations

Our action takes the form

$$S = S_{\text{Grav}} + S_\phi + S_{\text{S}} + S_M, \tag{1}$$

where

$$S_{\text{Grav}} = \frac{1}{16\pi} \int \mathrm{d}^4x \, \sqrt{-g} \left[\frac{1}{G}(R + 2\Lambda) \right], \tag{2}$$

$$S_\phi = - \int \mathrm{d}^4x \, \sqrt{-g} [\omega(\tfrac{1}{4} B^{\mu\nu} B_{\mu\nu} + V(\phi))], \tag{3}$$

and

$$S_{\mathrm{S}} = \int \mathrm{d}^4 x \, \sqrt{-g} \left[\frac{1}{G^3} \left(\frac{1}{2} g^{\mu\nu} \nabla_\mu G \nabla_\nu G - V(G) \right) \right.$$
$$\left. + \frac{1}{G} \left(\frac{1}{2} g^{\mu\nu} \nabla_\mu \omega \nabla_\nu \omega - V(\omega) \right) + \frac{1}{\mu^2 G} \left(\frac{1}{2} g^{\mu\nu} \nabla_\mu \mu \nabla_\nu \mu - V(\mu) \right) \right]. \tag{4}$$

Here, we have chosen units with $c = 1$, ∇_μ denotes the covariant derivative with respect to the metric $g_{\mu\nu}$. We adopt the metric signature $\eta_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$ where $\eta_{\mu\nu}$ is the Minkowski spacetime metric. We have

$$R_{\mu\nu} = \partial_\lambda \Gamma^\lambda{}_{\mu\nu} - \partial_\nu \Gamma^\lambda{}_{\mu\lambda} + \Gamma^\lambda{}_{\mu\nu} \Gamma^\sigma{}_{\lambda\sigma} - \Gamma^\sigma{}_{\mu\lambda} \Gamma^\lambda{}_{\nu\sigma}, \tag{5}$$

where $\Gamma^\lambda{}_{\mu\nu}$ denotes the Christoffel connection:

$$\Gamma^\lambda{}_{\mu\nu} = \tfrac{1}{2} g^{\lambda\sigma} (\partial_\nu g_{\mu\sigma} + \partial_\mu g_{\nu\sigma} - \partial_\sigma g_{\mu\nu}), \tag{6}$$

and $R = g^{\mu\nu} R_{\mu\nu}$. Moreover, $V(\phi)$ denotes a potential for the vector field ϕ^μ, while $V(G), V(\omega)$ and $V(\mu)$ denote the three potentials associated with the three scalar fields $G(x), \omega(x)$ and $\mu(x)$, respectively. The field $\omega(x)$ is dimensionless and Λ denotes the cosmological constant. Moreover,

$$B_{\mu\nu} = \partial_\mu \phi_\nu - \partial_\nu \phi_\mu. \tag{7}$$

The total energy–momentum tensor is given by

$$T_{\mu\nu} = T_{M\mu\nu} + T_{\phi\mu\nu} + T_{S\mu\nu}, \tag{8}$$

where $T_{M\mu\nu}$ and $T_{\phi\mu\nu}$ denote the ordinary matter energy–momentum tensor and the energy–momentum tensor contribution of the ϕ_μ field, respectively, while $T_{S\mu\nu}$ denotes the scalar G, ω and μ contributions to the energy–momentum tensor. We have

$$\frac{2}{\sqrt{-g}} \frac{\delta S_M}{\delta g^{\mu\nu}} = -T_{M\mu\nu}, \qquad \frac{2}{\sqrt{-g}} \frac{\delta S_\phi}{\delta g^{\mu\nu}} = -T_{\phi\mu\nu}, \qquad \frac{2}{\sqrt{-g}} \frac{\delta S_{\mathrm{S}}}{\delta g^{\mu\nu}} = -T_{S\mu\nu}. \tag{9}$$

The matter current density J^μ is defined in terms of the matter action S_M:

$$\frac{1}{\sqrt{-g}} \frac{\delta S_M}{\delta \phi_\mu} = -J^\mu. \tag{10}$$

We obtain from the variation of $g^{\mu\nu}$, the field equations

$$G_{\mu\nu} - g_{\mu\nu} \Lambda + Q_{\mu\nu} = 8\pi G T_{\mu\nu}, \tag{11}$$

where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$. We have

$$Q_{\mu\nu} = G(\nabla^\alpha \nabla_\alpha \Theta g_{\mu\nu} - \nabla_\mu \nabla_\nu \Theta), \tag{12}$$

where $\Theta(x) = 1/G(x)$. The quantity $Q_{\mu\nu}$ results from a boundary contribution arising from the presence of second derivatives of the metric tensor in R in S_{Grav}. These boundary contributions are equivalent to those that occur in Brans–Dicke gravity theory [9]. We also have

$$T_{\phi\mu\nu} = \omega \left[B_\mu{}^\alpha B_{\nu\alpha} - g_{\mu\nu} \left(\frac{1}{4} B^{\rho\sigma} B_{\rho\sigma} + V(\phi) \right) + 2 \frac{\partial V(\phi)}{\partial g^{\mu\nu}} \right]. \tag{13}$$

The $G(x)$ field yields the energy–momentum tensor:

$$T_{G\mu\nu} = -\frac{1}{G^3}\left[\nabla_\mu G\nabla_\nu G - 2\frac{\partial V(G)}{\partial g^{\mu\nu}} - g_{\mu\nu}\left(\frac{1}{2}\nabla_\alpha G\nabla^\alpha G - V(G)\right)\right]. \quad (14)$$

Similar expressions can be obtained for $T_{\omega\,\mu\nu}$ and $T_{\mu\,\mu\nu}$.

From the Bianchi identities

$$\nabla_\nu G^{\mu\nu} = 0, \quad (15)$$

and from the field equations (11), we obtain

$$\nabla_\nu T^{\mu\nu} + \frac{1}{G}\nabla_\nu G T^{\mu\nu} - \frac{1}{8\pi G}\nabla_\nu Q^{\mu\nu} = 0. \quad (16)$$

A variation with respect to ϕ_μ yields the equations

$$\nabla_\nu B^{\mu\nu} + \frac{\partial V(\phi)}{\partial\phi_\mu} + \frac{1}{\omega}\nabla_\nu\omega B^{\mu\nu} = -\frac{1}{\omega}J^\mu. \quad (17)$$

Taking the divergence of both sides with respect to ∇_μ, we get

$$\nabla_\mu\left(\frac{\partial V(\phi)}{\partial\phi_\mu}\right) + \nabla_\mu\left(\frac{1}{\omega}\nabla_\nu\omega B^{\mu\nu}\right) = -\nabla_\mu\left(\frac{1}{\omega}J^\mu\right). \quad (18)$$

If we assume that the current density J^μ is conserved, we obtain the equation

$$\nabla_\mu\left(\frac{\partial V(\phi)}{\partial\phi_\mu}\right) + \nabla_\mu\left(\frac{1}{\omega}\nabla_\nu\omega B^{\mu\nu}\right) = -\nabla_\mu\left(\frac{1}{\omega}\right)J^\mu. \quad (19)$$

In standard Maxwell–Proca theory, the conservation of the current J^μ is a separate physical assumption.

We shall choose for the potential $V(\phi)$:

$$V(\phi) = -\tfrac{1}{2}\mu^2\phi^\mu\phi_\mu + W(\phi), \quad (20)$$

where $W(\phi)$ denotes a vector field ϕ^μ self-interaction contribution. We can choose as a model for the self-interaction:

$$W(\phi) = \tfrac{1}{4}g(\phi^\mu\phi_\mu)^2, \quad (21)$$

where g is a coupling constant.

The effective gravitational 'constant' $G(x)$ satisfies the field equations

$$\nabla_\alpha\nabla^\alpha G + V'(G) + N = \frac{1}{2}G^2\left(T + \frac{\Lambda}{4\pi G}\right), \quad (22)$$

where

$$N = -3\Theta\left(\tfrac{1}{2}\nabla_\alpha G\nabla^\alpha G + V(G)\right) + G\left(\tfrac{1}{2}\nabla_\alpha\omega\nabla^\alpha\omega - V(\omega)\right)$$
$$+ \frac{G}{\mu^2}\left(\frac{1}{2}\nabla_\alpha\mu\nabla^\alpha\mu - V(\mu)\right) + \frac{3G^2}{16\pi}\nabla_\alpha\nabla^\alpha\Theta, \quad (23)$$

and $T = g^{\mu\nu}T_{\mu\nu}$. The scalar field $\omega(x)$ obeys the field equations

$$\nabla_\alpha\nabla^\alpha\omega + V'(\omega) + F = 0, \quad (24)$$

where

$$F = -\Theta\nabla_\alpha G\nabla^\alpha\omega + G(\tfrac{1}{4}B^{\mu\nu}B_{\mu\nu} + V(\phi)). \tag{25}$$

The field $\mu(x)$ satisfies the equations

$$\nabla_\alpha\nabla^\alpha\mu + V'(\mu) + P = 0, \tag{26}$$

where

$$P = -\left[\Theta\nabla^\alpha G\nabla_\alpha\mu + \frac{2}{\mu}\nabla^\alpha\mu\nabla_\alpha\mu + \omega\mu^2 G\frac{\partial V(\phi)}{\partial\mu}\right], \tag{27}$$

and the last term arises from the μ dependence of $V(\phi)$ in (20).

If we adopt the condition

$$\nabla^\nu\phi_\nu = 0, \tag{28}$$

then (17) takes the form

$$\nabla^\nu\nabla_\nu\phi_\mu - R_\mu{}^\nu\phi_\nu + \mu^2\phi_\mu - \frac{\partial W(\phi)}{\partial\phi^\mu} - \frac{1}{\omega}\nabla^\nu\omega B_{\mu\nu} = \frac{1}{\omega}J_\mu. \tag{29}$$

The test particle action is given by

$$S_{\text{TP}} = -m\int \mathrm{d}\tau - \lambda\int \mathrm{d}\tau\,\omega\phi_\mu\frac{\mathrm{d}x^\mu}{\mathrm{d}\tau}, \tag{30}$$

where τ is the proper time along the world line of the test particle and m and λ denote the test particle mass and coupling constant, respectively. The stationarity condition $\delta S_{\text{TP}}/\delta x^\mu = 0$ yields the equations of motion for the test particle

$$m\left(\frac{\mathrm{d}^2x^\mu}{\mathrm{d}\tau^2} + \Gamma^\mu_{\alpha\beta}\frac{\mathrm{d}x^\alpha}{\mathrm{d}\tau}\frac{\mathrm{d}x^\beta}{\mathrm{d}\tau}\right) = f^\mu, \tag{31}$$

where

$$f^\mu = \lambda\omega B^\mu{}_\nu\frac{\mathrm{d}x^\nu}{\mathrm{d}\tau} + \lambda\nabla^\mu\omega\left(\phi_\alpha\frac{\mathrm{d}x^\alpha}{\mathrm{d}\tau}\right) - \lambda\nabla_\alpha\omega\left(\phi^\mu\frac{\mathrm{d}x^\alpha}{\mathrm{d}\tau}\right). \tag{32}$$

The action for the field $B_{\mu\nu}$ is of the Maxwell–Proca form for a massive vector field ϕ_μ. It can be proved that this theory possesses a stable vacuum and the Hamiltonian is bounded from below. Even though the action is not gauge invariant, it can be shown that the longitudinal mode ϕ_0 (where $\phi_\mu = (\phi_0, \phi_i)$ ($i = 1, 2, 3$)) does not propagate and the theory is free of ghosts. Similar arguments apply to the MSTG theory [2].

The Hamilton–Dirac (HD) method is a tool for investigating the constraints and the degrees of freedom of a field theory [10]. The HD procedure checks the theory for consistency by producing the explicit constraints, and counting the number of degrees of freedom. It is a canonical initial value analysis. When the field theory is coupled dynamically to gravity, many vector field theories are ruled out, because the constraints produced by the canonical, Cauchy initial value formalism yield 'derivative coupled' theories (the Christoffel connections do not cancel out). The Maxwell and Maxwell–Proca theories are prime examples [11] of consistent vector field theories. With or without the gravitational field coupling, Maxwell's theory has two degrees of freedom and Maxwell–Proca has three, and they are stable and satisfy a consistent Cauchy evolution analysis.

There are no pathological singularities in the Maxwell–Proca theory coupled to gravity, when one solves for the second time derivative in the canonical initial value formulation. In other vector theories, singularities occur that spoil the stability of the theory and rule them out as physically unviable theories. In the present theory, the scalar fields G, ω and μ enter the action as dynamically determined degrees of freedom. The coupling of the scalar field μ to ϕ^μ in the action for $\mu \neq 0$ leads to a Hamiltonian bounded from below with no pathological singularities. It is possible to attribute the mass μ to a spontaneous symmetry breaking mechanism, but we shall not pursue this possibility at present.

3. Equations of motion, weak fields and the modified gravitational acceleration

Let us assume that we are in a distance scale regime for which the fields G, ω and μ take their approximate renormalized constant values:

$$G \sim G_0(1 + Z), \qquad \omega \sim \omega_0 A, \qquad \mu \sim \mu_0 B, \tag{33}$$

where G_0, ω_0 and μ_0 denote the 'bare' values of G, ω and μ, respectively, and Z, A and B are the associated renormalization constants.

For a static spherically symmetric field the line element is given by

$$ds^2 = \gamma(r)\,dt^2 - \alpha(r)\,dr^2 - r^2(d\theta^2 + \sin^2\theta\,d\phi^2). \tag{34}$$

The equations of motion for a test particle obtained from (31) to (33) are given by

$$\frac{d^2r}{d\tau^2} + \frac{\alpha'}{2\alpha}\left(\frac{dr}{d\tau}\right)^2 - \frac{r}{\alpha}\left(\frac{d\theta}{d\tau}\right)^2 - r\left(\frac{\sin^2\theta}{\alpha}\right)\left(\frac{\sigma d\phi}{d\tau}\right)^2 + \frac{\gamma'}{2\alpha}\left(\frac{dt}{d\tau}\right)^2$$
$$+ \sigma\frac{1}{\alpha}\left(\frac{d\phi_0}{dr}\right)\left(\frac{dt}{d\tau}\right) = 0, \tag{35}$$

$$\frac{d^2t}{d\tau^2} + \frac{\gamma'}{\gamma}\left(\frac{dt}{d\tau}\right)\left(\frac{dr}{d\tau}\right) + \sigma\frac{1}{\gamma}\left(\frac{d\phi_0}{dr}\right)\left(\frac{dr}{d\tau}\right) = 0, \tag{36}$$

$$\frac{d^2\theta}{d\tau^2} + \frac{2}{r}\left(\frac{d\theta}{d\tau}\right)\left(\frac{dr}{d\tau}\right) - \sin\theta\cos\theta\left(\frac{d\phi}{d\tau}\right)^2 = 0, \tag{37}$$

$$\frac{d^2\phi}{d\tau^2} + \frac{2}{r}\left(\frac{d\phi}{d\tau}\right)\left(\frac{dr}{d\tau}\right) + 2\cot\theta\left(\frac{d\phi}{d\tau}\right)\left(\frac{d\theta}{d\tau}\right) = 0, \tag{38}$$

where $\sigma = \lambda\omega/m$.

The orbit of the test particle can be shown to lie in a plane and by an appropriate choice of axes, we can make $\theta = \pi/2$. Integrating equation (38) gives

$$r^2\frac{d\phi}{d\tau} = J, \tag{39}$$

where J is the conserved orbital angular momentum. Integration of equation (36) gives

$$\frac{dt}{d\tau} = -\frac{1}{\gamma}(\sigma\phi_0 + E), \tag{40}$$

where E is a constant.

By substituting (40) into (35) and using (39), we obtain

$$\frac{\mathrm{d}^2 r}{\mathrm{d}\tau^2} + \frac{\alpha'}{2\alpha}\left(\frac{\mathrm{d}r}{\mathrm{d}\tau}\right)^2 - \frac{J^2}{\alpha r^3} + \frac{\gamma'}{2\alpha\gamma^2}(\sigma\phi_0 + E)^2 = \sigma\frac{1}{\alpha\gamma}\left(\frac{\mathrm{d}\phi_0}{\mathrm{d}r}\right)(\sigma\phi_0 + E). \tag{41}$$

We do not have an exact, spherically symmetric static solution to our field equations for a non-zero $V(\phi)$. However, if we consider $V(\phi)$ as a small but non-vanishing contribution that can be neglected in equation (17) and set $\Lambda = 0$ in equation (11), then the static, spherically symmetric (Reissner–Nordström) solution in empty space yields the line element

$$\mathrm{d}s^2 = \left(1 - \frac{2GM}{r} + \frac{Q^2}{r^2}\right)\mathrm{d}t^2 - \left(1 - \frac{2GM}{r} + \frac{Q^2}{r^2}\right)^{-1}\mathrm{d}r^2 - r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2), \tag{42}$$

where M is a constant of integration and

$$Q^2 = 4\pi G\omega\epsilon^2. \tag{43}$$

Here, ϵ denotes the 'charge' of the spin-1 vector particle given by

$$\epsilon = \int \mathrm{d}^3 x q, \tag{44}$$

where the matter current density J^μ is identified as $J^\mu = (q, J^i)$.

For large enough values of r, the solution (42) approximates the Schwarzschild metric components α and γ:

$$\alpha(r) \sim \frac{1}{1 - 2GM/r}, \qquad \gamma(r) \sim 1 - \frac{2GM}{r}. \tag{45}$$

It is not unreasonable to expect that a static, spherically symmetric solution of the field equations including the mass term μ in the field equations (17) will approximate for large values of r the Schwarzschild metric components (45).

We assume that $2GM/r \ll 1$ and the slow motion approximation $\mathrm{d}r/\mathrm{d}s \sim \mathrm{d}r/\mathrm{d}t \ll 1$. Then for material test particles, we obtain from (31)–(33), (41) and (45):

$$\frac{\mathrm{d}^2 r}{\mathrm{d}t^2} - \frac{J_{\mathrm{N}}^2}{r^3} + \frac{GM}{r^2} = \sigma\frac{\mathrm{d}\phi_0}{\mathrm{d}r}, \tag{46}$$

where J_{N} is the Newtonian orbital angular momentum.

For weak gravitational fields to first order, the static equations for ϕ_0 obtained from (29) are given for the source-free case by

$$\vec{\nabla}^2\phi_0 - \mu^2\phi_0 = 0, \tag{47}$$

where $\vec{\nabla}^2$ is the Laplacian operator, and we have neglected any contribution from the self-interaction potential $W(\phi)$. For a spherically symmetric static field ϕ_0, we obtain

$$\phi_0'' + \frac{2}{r}\phi_0' - \mu^2\phi_0 = 0. \tag{48}$$

This has the Yukawa solution

$$\phi_0(r) = -\beta\frac{\exp(-\mu r)}{r}, \tag{49}$$

where β is a constant. We obtain from (46):

$$\frac{\mathrm{d}^2 r}{\mathrm{d}t^2} - \frac{J_{\mathrm{N}}^2}{r^3} + \frac{GM}{r^2} = K\frac{\exp(-\mu r)}{r^2}(1 + \mu r),$$
(50)

where $K = \sigma\beta$.

We observe that the additional Yukawa force term in equation (50) is *repulsive* in accordance with the exchange of a spin 1 massive boson. We shall find that this repulsive component of the gravitational field is necessary to obtain a fit to galaxy rotation curves.

We shall write for the radial acceleration derived from (50):

$$a(r) = -\frac{G_\infty M}{r^2} + K\frac{\exp(-r/r_0)}{r^2}\left(1 + \frac{r}{r_0}\right),$$
(51)

and G_∞ is defined to be the effective gravitational constant at infinity

$$G_\infty = G_0\left(1 + \sqrt{\frac{M_0}{M}}\right).$$
(52)

Here, M_0 denotes a parameter that vanishes when $\omega = 0$ and G_0 is Newton's gravitational 'bare' constant. The constant K is chosen to be

$$K = G_0\sqrt{MM_0}.$$
(53)

The choice of K, which determines the strength of the coupling of $B_{\mu\nu}$ to matter and the magnitude of the Yukawa force modification of weak Newtonian gravity, is based on phenomenology and is not at present derivable from the STVG action formalism.

By using (52), we can rewrite the acceleration in the form

$$a(r) = -\frac{G_0 M}{r^2}\left\{1 + \sqrt{\frac{M_0}{M}}\left[1 - \exp(-r/r_0)\left(1 + \frac{r}{r_0}\right)\right]\right\}.$$
(54)

We can generalize this to the case of a mass distribution by replacing the factor $G_0 M/r^2$ in (54) by $G_0\mathcal{M}(r)/r^2$. The rotational velocity of a star v_{c} is obtained from $v_{\mathrm{c}}^2(r)/r = a(r)$ and is given by

$$v_{\mathrm{c}} = \sqrt{\frac{G_0\mathcal{M}(r)}{r}}\left\{1 + \sqrt{\frac{M_0}{M}}\left[1 - \exp(-r/r_0)\left(1 + \frac{r}{r_0}\right)\right]\right\}^{1/2}.$$
(55)

The gravitational potential for a point source obtained from the modified acceleration law (54) is given by

$$\Phi(r) = \frac{G_0 M}{r}\left[1 + \sqrt{\frac{M_0}{M}}(1 - \exp(-r/r_0))\right].$$
(56)

The acceleration law (54) can be written as

$$a(r) = -\frac{G(r)M}{r^2},$$
(57)

where

$$G(r) = G_0 \left[1 + \sqrt{\frac{M_0}{M}} \left(1 - \exp(-r/r_0) \left(1 + \frac{r}{r_0} \right) \right) \right] \tag{58}$$

is an *effective* expression for the variation of G with respect to r. A good fit to a large number of galaxies has been achieved with the parameters [2, 3]:

$$M_0 = 9.60 \times 10^{11} \, M_\odot, \qquad r_0 = 13.92 \text{ kpc} = 4.30 \times 10^{22} \text{ cm}. \tag{59}$$

In the fitting of the galaxy rotation curves for both LSB and HSB galaxies, using photometric data to determine the mass distribution $\mathcal{M}(r)$ [3], only the mass-to-light ratio $\langle M/L \rangle$ is employed, once the values of M_0 and r_0 are fixed universally for all LSB and HSB galaxies. Dwarf galaxies are also fitted with the parameters [3]:

$$M_0 = 2.40 \times 10^{11} \, M_\odot, \qquad r_0 = 6.96 \text{ kpc} = 2.15 \times 10^{22} \text{ cm}. \tag{60}$$

By choosing values for the parameters G_∞, $(M_0)_{\text{clust}}$ and $(r_0)_{\text{clust}}$, we are able to obtain satisfactory fits to a large sample of x-ray cluster data [4].

4. Orbital equations of motion

We set $\theta = \pi/2$ in (34), divide the resulting expression by $\mathrm{d}\tau^2$ and use equations (39) and (40) to obtain

$$\left(\frac{\mathrm{d}r}{\mathrm{d}\tau} \right)^2 + \frac{J^2}{\alpha r^2} - \frac{1}{\alpha\gamma}(\sigma\phi_0 + E)^2 = -\frac{E}{\alpha}. \tag{61}$$

We have $\mathrm{d}s^2 = E\mathrm{d}\tau^2$, so that $\mathrm{d}s/\mathrm{d}\tau$ is a constant. For material particles $E > 0$ and for massless photons $E = 0$.

Let us set $u = 1/r$ and by using (39), we have $\mathrm{d}r/\mathrm{d}\tau = -J \, \mathrm{d}u/\mathrm{d}\phi$. Substituting this into (61), we obtain

$$\left(\frac{\mathrm{d}u}{\mathrm{d}\phi} \right)^2 = \frac{1}{\alpha\gamma J^2}(E + \sigma\phi_0)^2 - \frac{1}{\alpha r^2} - \frac{E}{\alpha J^2}. \tag{62}$$

On substituting (45) and $\mathrm{d}r/\mathrm{d}\phi = -(1/u^2) \, \mathrm{d}u/\mathrm{d}\phi$ into (62), we get after some manipulation:

$$\frac{\mathrm{d}^2 u}{\mathrm{d}\phi^2} + u = \frac{EGM}{J^2} - \frac{EK}{J^2} \exp\left(-\frac{1}{r_0 u} \right) \left(1 + \frac{1}{r_0 u} \right) + 3GMu^2, \tag{63}$$

where $r_0 = 1/\mu$.

For material test particles $E = 1$ and we obtain

$$\frac{\mathrm{d}^2 u}{\mathrm{d}\phi^2} + u = \frac{GM}{J^2} + 3GMu^2 - \frac{K}{J^2} \exp\left(-\frac{1}{r_0 u} \right) \left(1 + \frac{1}{r_0 u} \right). \tag{64}$$

On the other hand, for massless photons $\mathrm{d}s^2 = 0$ and $E = 0$ and (63) gives

$$\frac{\mathrm{d}^2 u}{\mathrm{d}\phi^2} + u = 3GMu^2. \tag{65}$$

5. Solar system and binary pulsar observations

We obtain from equation (64) the orbit equation (we reinsert the speed of light c):

$$\frac{\mathrm{d}^2 u}{\mathrm{d}\phi^2} + u = \frac{GM}{c^2 J^2} - \frac{K}{c^2 J^2} \exp(-r/r_0) \left[1 + \left(\frac{r}{r_0}\right)\right] + \frac{3GM}{c^2} u^2 \tag{66}$$

where K is given by (53). Using the large r weak field approximation, and the expansion

$$\exp(-r/r_0) = 1 - \frac{r}{r_0} + \frac{1}{2}\left(\frac{r}{r_0}\right)^2 + \cdots \tag{67}$$

we obtain the orbit equation for $r \ll r_0$:

$$\frac{\mathrm{d}^2 u}{\mathrm{d}\phi^2} + u = N + 3\frac{GM}{c^2} u^2, \tag{68}$$

where

$$N = \frac{GM}{c^2 J_{\mathrm{N}}^2} - \frac{K}{c^2 J_{\mathrm{N}}^2}. \tag{69}$$

We can solve equation (68) by perturbation theory and find for the perihelion advance of a planetary orbit

$$\Delta\omega = \frac{6\pi}{c^2 L}(GM_\odot - K_\odot), \tag{70}$$

where $J_{\mathrm{N}} = (GM_\odot L/c^2)^{1/2}$, $L = a(1-e^2)$ and a and e denote the semimajor axis and the eccentricity of the planetary orbit, respectively.

We now use the running of the effective gravitational coupling constant $G = G(r)$, determined by (58) and find that for the solar system $r \ll r_0$, we have $G \sim G_0$ within the experimental errors for the measurement of Newton's constant G_0. We choose for the solar system

$$\frac{K_\odot}{c^2} \ll 1.5 \text{ km} \tag{71}$$

and use $G = G_0$ to obtain from (70) a perihelion advance of Mercury in agreement with GR. The bound (71) requires that the coupling constant ω varies with distance in such a way that it is sufficiently small in the solar system regime and determines a value for M_0, in equation (53), that is in accord with the bound (71).

For terrestrial experiments and orbits of satellites, we have also that $G \sim G_0$ and for $K_\oplus$ sufficiently small, we then achieve agreement with all gravitational terrestrial experiments including Eötvös free-fall experiments and 'fifth force' experiments.

For the binary pulsar PSR 1913+16 the formula (70) can be adapted to the periastron shift of a binary system. Combining this with the STVG gravitational wave radiation formula, which will approximate closely the GR formula, we can obtain agreement with the observations for the binary pulsar. The mean orbital radius for the binary pulsar is equal to the projected semimajor axis of the binary, $\langle r \rangle_{\mathrm{N}} = 7 \times 10^{10}$ cm, and we choose $\langle r \rangle_{\mathrm{N}} \ll r_0$. Thus, for $G = G_0$ within the experimental errors, we obtain agreement with the binary pulsar data for the periastron shift when

$$\frac{K_{\mathrm{N}}}{c^2} \ll 4.2 \text{ km}. \tag{72}$$

For a massless photon $E = 0$ and we have

$$\frac{\mathrm{d}^2 u}{\mathrm{d}\phi^2} + u = 3\frac{GM}{c^2}u^2. \tag{73}$$

For the solar system using $G = G_0$ within the experimental errors gives the light deflection:

$$\Delta_\odot = \frac{4G_0 M_\odot}{c^2 R_\odot} \tag{74}$$

in agreement with GR.

6. Galaxy clusters and lensing

The bending angle of a light ray as it passes near a massive system along an approximately straight path is given to lowest order in v^2/c^2 by

$$\theta = \frac{2}{c^2} \int |a^\perp| \, \mathrm{d}z, \tag{75}$$

where $\perp$ denotes the perpendicular component to the ray's direction, and $\mathrm{d}z$ is the element of length along the ray and a denotes the acceleration.

From (73), we obtain the light deflection

$$\Delta = \frac{4GM}{c^2 R} = \frac{4G_0 \overline{M}}{c^2 R}, \tag{76}$$

where

$$\overline{M} = M \left(1 + \sqrt{\frac{M_0}{M}} \right). \tag{77}$$

The value of $\overline{M}$ follows from (58) for clusters as $r \gg r_0$ and

$$G(r) \to G_\infty = G_0 \left(1 + \sqrt{\frac{M_0}{M}} \right). \tag{78}$$

We choose for a cluster $M_0 = 3.6 \times 10^{15} \, M_\odot$ and a cluster mass $M_{\mathrm{clust}} \sim 10^{14} \, M_\odot$, and obtain

$$\left(\sqrt{\frac{M_0}{M}} \right)_{\mathrm{clust}} \sim 6. \tag{79}$$

We see that $\overline{M} \sim 7M$ and we can explain the increase in the light bending without exotic dark matter.

From the formula equation (54) for $r \gg r_0$ we get

$$a(r) = -\frac{G_0 \overline{M}}{r^2}. \tag{80}$$

We expect to obtain from this result a satisfactory description of lensing phenomena using equation (75).

An analysis of a large number of clusters shows that the MSTG and STVG theories fit well the cluster data in terms of the cluster mass, M_{clust}, and a value for the parameter M_0 scaled by the total cluster gas mass [4].

7. Running of the effective constants G, ω and μ

The scaling with distance of the effective gravitational constant G, the effective coupling constant ω and the effective mass μ is seen to play an important role in describing consistently the solar system and the galaxy and cluster dynamics, without the postulate of exotic dark matter. We have to solve the field equations (22), (24) and (26) with given potentials $V(G), V(\omega)$ and $V(\mu)$ to determine the variation of the effective constants with space and time. These equations are complicated, so we shall make simplifying approximations. In equation (22), we shall neglect the contributions from N and obtain for $T = \Lambda = 0$:

$$\nabla_\nu \nabla^\nu G + V'(G) = 0, \tag{81}$$

where $f'(y) = \mathrm{d}f/\mathrm{d}y$. The effective variation of G with r is determined by equation (58). We obtain from (81) for the static spherically symmetric equations

$$\vec{\nabla}^2 G(r) - V'(G) \equiv G''(r) + \frac{2}{r}G'(r) - V'(G) = 0. \tag{82}$$

By choosing the potential

$$V(G) = -\frac{1}{2}\left(\frac{G_0}{r_0^2}\right)\left(\frac{M_0}{M}\right)\exp(-2r/r_0)\left(1 + \frac{2r}{r_0} - \frac{r^2}{r_0^2}\right), \tag{83}$$

we obtain a solution to (82) for $G(r)$ given by (58). The neglect of the contributions N can only be justified by solving the complete set of coupled equations by a perturbation calculation. We will not attempt to do this in the present work, but we plan to investigate this issue in a future publication.

We see from (58) that for $r \ll r_0$ we obtain $G(r) \sim G_0$. As the distance scale approaches the regime of the solar system $r < 100$ A.U. where 1 A.U. $= 1.496 \times 10^{13}$ cm $= 4.85 \times 10^{-9}$ kpc, then (54) becomes the Newtonian acceleration law:

$$a(r) = -\frac{G_0 M}{r^2}, \tag{84}$$

in agreement with solar physics observations for the inner planets.

Let us make the approximation of neglecting $F(\phi)$ in equation (24). In the static spherically symmetric case this gives

$$\omega''(r) + \frac{2}{r}\omega'(r) - V'(\omega) = 0. \tag{85}$$

We choose as a solution for $\omega(r)$:

$$\omega(r) = \omega_0\{1 + \overline{\omega}[1 - \exp(-\overline{\mu}r)(1 + \overline{\mu}r)]\}, \tag{86}$$

where $\overline{\omega}$ and $\overline{\mu}$ are positive constants. The potential $V(\omega)$ has the form

$$V(\omega) = -\tfrac{1}{2}\omega_0^2\overline{\mu}^2\overline{\omega}^2\exp(-2\overline{\mu}r)(1 + 2\overline{\mu}r - \overline{\mu}^2 r^2). \tag{87}$$

For the variation of the renormalized mass $\mu = \mu(r)$, we find that a satisfactory solution to equation (26) should correspond to a $\mu(r) = 1/r_0(r)$ that decreases from a value for the inner planets of the solar system, consistent with solar system observations,

to a small value corresponding to r_0 for the galaxy fits, $r_0 = 14$ kpc, and to an even smaller value for the cluster data fits.

The spatial variations of $G(r), \omega(r)$ and $\mu(r) = 1/r_0(r)$ can be determined numerically from the equations (22), (24) and (26) with given potentials $V(G), V(\omega)$ and $V(\mu)$, such that for the solar system and the binary pulsar PSR 1913+16 the bounds (71) and (72) are satisfied by the solutions for $G(r), \omega(r)$ and $\mu(r)$. The spatial variations of G, the coupling constant ω and the range r_0 are required to guarantee consistency with solar system observations. On the other hand, their increase at galactic and cosmological distance and time scales can account for galaxy rotation curves, cluster lensing and cosmology without non-baryonic dark matter.

We have constructed a classical action for gravity that can be considered as an *effective* field theory description of an RG flow quantum gravity scenario as described in [5] and [2].

The fitting of the solar system, galaxy and the clusters of galaxies data depends on the running of the 'constants' G, ω and r_0. They should increase from one distance scale to the next according to the renormalization group flow diagrams, or the solutions of the classical field equations in the present article. In a future article, the author plans to provide a more complete determination of the running of the constants. However, the present article describes the basic scenario and the ideas underlying the theory.

8. Cosmology

Let us now consider a cosmological solution to our STVG theory. We adopt a homogeneous and isotropic FLRW background geometry with the line element

$$ds^2 = dt^2 - a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right),\tag{88}$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta\, d\phi^2$ and $k = 0, -1, +1$ for a spatially flat, open and closed universe, respectively. Due to the symmetry of the FLRW background spacetime, we have $\phi_0 \equiv \psi \neq 0$, $\phi_i = 0$ and $B_{\mu\nu} = 0$.

We define the energy–momentum tensor for a perfect fluid by

$$T^{\mu\nu} = (\rho + p)u^\mu u^\nu - pg^{\mu\nu},\tag{89}$$

where $u^\mu = dx^\mu/ds$ is the 4-velocity of a fluid element and $g_{\mu\nu}u^\mu u^\nu = 1$. Moreover, we have

$$\rho = \rho_M + \rho_\phi + \rho_S, \qquad p = p_M + p_\phi + p_S,\tag{90}$$

where ρ_i and p_i denote the components of density and pressure associated with the matter, the ϕ^μ field and the scalar fields G, ω and μ, respectively.

The Friedmann equations take the form

$$H^2(t) + \frac{k}{a^2(t)} = \frac{8\pi G(t)\rho(t)}{3} + H(t)\frac{\dot{G}}{G} + \frac{\Lambda}{3},\tag{91}$$

$$\frac{\ddot{a}(t)}{a(t)} = -\frac{4\pi G(t)}{3}(\rho(t) + 3p(t)) + \frac{1}{2}\left(\frac{\ddot{G}}{G} - \frac{\dot{G}^2}{G^2} + 2H(t)\frac{\dot{G}}{G}\right) + \frac{\Lambda}{3},\tag{92}$$

where $H(t) = \dot{a}(t)/a(t)$.

Let us make the simplifying approximation for equations (22):

$$\ddot{\mathcal{G}} + 3H\dot{\mathcal{G}} + V'(\mathcal{G}) = \frac{1}{2}G_0\mathcal{G}^2 \left[\rho - 3p + \frac{\Lambda}{4\pi G_0\mathcal{G}} \right], \tag{93}$$

where $\mathcal{G}(t) = G(t)/G_0$. A solution for $\mathcal{G}$ in terms of a given potential $V(\mathcal{G})$ and for given values of ρ and p can be obtained from (93).

The solution for $\mathcal{G}$ must satisfy a constraint at the time of big bang nucleosynthesis [12]. The number of relativistic degrees of freedom is very sensitive to the cosmic expansion rate at 1 MeV. This can be used to constrain the time dependence of G. Recent measurements of the ^{4}He mass fraction and the deuterium abundance at 1 MeV leads to the constraint $G(t) \sim G_0$. We impose the conditions $\mathcal{G}(t) \to 1$ as $t \to t_{\mathrm{BBN}}$ and $\mathcal{G}(t) \to 1 + \bar{\omega}$ as $t \to t_{\mathrm{SLS}}$ where t_{BBN} and t_{SLS} denote the times of the big bang nucleosynthesis and the surface of last scattering, respectively. A possible solution for $\mathcal{G}$ can take the form

$$\mathcal{G}(t) = 1 + \bar{\omega} \left[1 - \exp(-t/T) \left(1 + \frac{t}{T} \right) \right], \tag{94}$$

where $\bar{\omega}$ and T are constants and $\bar{\omega}$ is a measure of the magnitude of the scalar field ψ. We have for $t \gg T$ that $\mathcal{G} \to 1 + \bar{\omega}$ and for $t \ll T$ that $\mathcal{G} \to 1$. We get

$$\dot{\mathcal{G}} = \frac{\bar{\omega}t}{T^2}\exp(-t/T), \qquad \ddot{\mathcal{G}} = \frac{\bar{\omega}}{T^2}\exp(-t/T)\left(1 - \frac{t}{T}\right). \tag{95}$$

It follows that $\dot{\mathcal{G}}(t) \to 0$ for $t \gg T$, which allows us for a suitable choice of T to satisfy the experimental bound from the Cassini spacecraft measurements [13]:

$$|\dot{G}/G| \simeq 10^{-13}\ \mathrm{yr}^{-1}. \tag{96}$$

A linear perturbation on the FLRW background will link the theory with observations of anisotropies in the CMB as well as galaxy clustering on large scales. The basic fields are perturbed around the background spacetime (denoted for a quantity Y by $\tilde{Y}$). In the conformal metric with the time transformation $\mathrm{d}\eta = \mathrm{d}t/a(t)$:

$$\mathrm{d}s^2 = a^2(\eta)(\mathrm{d}\eta^2 - \mathrm{d}\vec{x}^2), \tag{97}$$

the metric perturbations are in the conformal Newtonian gauge

$$g_{00}(\vec{x}, t) = a^2(t)(1 + 2\Phi(\vec{x}, t)), \qquad g_{ij}(\vec{x}, t) = a^2(t)(1 - 2\Phi(\vec{x}, t))\delta_{ij}, \tag{98}$$

where Φ is the gravitational potential. The vector field perturbations are defined by

$$\phi_\mu(\vec{x}, t) = a(t)(\tilde{\phi}_\mu(t) + \delta\phi_\mu(\vec{x}, t)), \tag{99}$$

where $\tilde{\phi}_i(t) = 0$. Denoting by χ_i the scalar fields $\chi_1 = G$, $\chi_2 = \omega$ and $\chi_3 = \mu$, the scalar field perturbations are

$$\chi_i(\vec{x}, t) = \tilde{\chi}_i(t) + \delta\chi_i(\vec{x}, t). \tag{100}$$

The problem that gravity theories such as MSTG and STVG face in describing cosmology with no cold dark matter (CDM) (non-baryonic dark matter) is the damping of perturbations during the recombination era. In a pure baryonic universe evolving according to Einstein's gravitational field equations, the coupling of baryons to photons

during the recombination era will suffer Silk damping, causing the collisional propagation of radiation from overdense to underdense regions [7, 14]. In the CDM model, the perturbations δ_{CDM} are undamped during recombination, because the CDM particles interact with gravity and only weakly with matter (photons). The Newtonian potential in the CDM model is approximately given by

$$k^2\Phi \sim 4\pi G_0(\rho_{\mathrm{b}}\delta_{\mathrm{b}} + \rho_{\mathrm{CDM}}\delta_{\mathrm{CDM}}), \tag{101}$$

where k^2 denotes the square of the wavenumber, ρ_{b} and ρ_{CDM} denote the densities of baryons and cold dark matter, respectively, and δ_i denotes the perturbation density contrast for each component i of matter. If ρ_{CDM} is sufficiently large, then δ_{CDM} will not be erased, whereas δ_{b} decreases during recombination [15]. In STVG, the imperfect fluid plasma before recombination has two components: the dominant neutral vector field component that does not couple to photons and the baryon–photon component. The vector field component has zero pressure and zero shear viscosity, so the vector field perturbations are not Silk damped like the baryon radiation perturbations, for the latter have non-vanishing pressure and shear viscosity.

In STVG, we have

$$\Omega_{\mathrm{b}} = \frac{8\pi G\rho_{\mathrm{b}}}{3H^2}, \qquad \Omega_{\psi} = \frac{8\pi G\rho_{\psi}}{3H^2}, \qquad \Omega_{\mathrm{S}} = \frac{8\pi G\rho_{\mathrm{S}}}{3H^2}. \tag{102}$$

We also have a possible contribution from massive neutrinos

$$\Omega_{\nu} = \frac{8\pi G\rho_{\nu}}{3H^2}, \tag{103}$$

where ρ_{ν} denotes the density of neutrinos. We assume that the vector field density dominates.

The Newtonian potential in our modified gravitational model becomes

$$k^2\Phi \sim 4\pi G_{\mathrm{ren}}[\rho_{\mathrm{b}}\delta_{\mathrm{b}} + \rho_{\nu}\delta_{\nu} + \rho_{\psi}\delta_{\psi} + \rho_{\mathrm{S}}\delta_{\mathrm{S}}], \tag{104}$$

where G_{ren} is the renormalized value of the gravitational constant. Assuming that the density ρ_{ψ} is significant before and during recombination, we can consider fitting the acoustic peaks in the power spectrum in a spatially flat universe with the parameters

$$\Omega = \Omega_{\mathrm{b}} + \Omega_{\nu} + \Omega_{\psi} + \Omega_{\Lambda} = 1. \tag{105}$$

The fitting of the acoustic peaks in the CMB power spectrum does not permit a too large value of Ω_{Λ}. Moreover, the neutrino contribution is constrained by the three neutrinos having a mass <2 eV. We could choose, for example, $\Omega_{\mathrm{b}} = 0.04, \Omega_{\psi} = 0.25, \Omega_{\nu} = 0.01, \Omega_{\Lambda} = 0.7$ as a possible choice of parameters to fit the data. There are now new data from the balloon borne Boomerang CMB observations [16] that together with other ground based observations and WMAP data determine more accurately the height of the third acoustic peak in the angular CMB power spectrum [17]. The ratio of the height of the first peak to the second peak determines the baryon content $\Omega_{\mathrm{b}} \sim 0.04$. The height of the third peak is determined by the amount of cold dark matter, and in the modified gravity theory by the possible amounts of scalar ψ, G, ω and μ contributions. In particular, the dominant neutral ψ vector component perturbations will not be washed out before recombination.

Can the effects of gravitational constant renormalization together with the possible effects of the densities ρ_{ψ} and ρ_{S} describe a universe which can reproduce the current galaxy and CMB observations? The answer to this problem will be addressed in a future publication.

9. Conclusions

A modified gravity theory based an a $D = 4$ pseudo-Riemannian metric, a spin 1 vector field and a corresponding second-rank skew field $B_{\mu\nu}$ and dynamical scalar fields G, ω and μ, yields a static spherically symmetric gravitational field with an added Yukawa potential and with an effective coupling strength and distance range. This modified acceleration law leads to remarkably good fits to a large number of galaxies [3] and galaxy clusters [4]. The previously published gravitational theories NGT [1] and MSTG [2] yielded the same modified weak gravitational field acceleration law and, therefore, the same successful fits to galaxy and cluster data. The MSTG and STVG gravity theories can both be identified generically as metric–skew–tensor gravity theories, for they both describe gravity as a metric theory with an additional degree of freedom associated with a skew field coupling to matter. For MSTG, this is a third-rank skew field $F_{\mu\nu\lambda}$, while for STVG the skew field is a second-rank tensor $B_{\mu\nu}$. However, MSTG is distinguished from STVG as being the weak field approximation to the non-symmetric gravitational theory (NGT).

An action $S_{\rm S}$ for the scalar fields $G(x)$, $\omega(x)$ and $\mu(x) = 1/r_0(x)$ and the field equations resulting from a variation of the action, $\delta S_{\rm S} = 0$, can be incorporated into the NGT and MSTG theories. The dynamical solutions for the scalar fields give an *effective* description of the running of the constants in an RG flow quantum gravity scenario, in which strong infrared renormalization effects and increasing large scale spatial values of G and ω lead, together with the modified acceleration law, to a satisfactory description of galaxy rotation curves and cluster dynamics without non-baryonic dark matter.

We have demonstrated that a cosmological model with the renormalized gravitational constant $G_{\rm ren}$ and contributions from the fields ϕ, G, ω and μ can possibly lead to a satisfactory description of the distribution of galaxies and the CMB power spectrum.

The neutral vector particle ϕ does not couple to radiation and it has zero pressure p and zero shear viscosity. Since it dominates the period of recombination, its perturbations associated with the plasma fluid will not be washed out by Silk damping. In contrast to standard dark matter models, we should not search for *new stable particles* such as weakly interacting massive particles (WIMPS) or neutralinos, because the fifth force charge in STVG that is the source of the neutral vector field (skew field) is carried by the known stable baryons (and electrons and neutrinos). This new charge is the source of a fifth force skew field that modifies the gravitational field in the universe.

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