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Vectorial nature of nonparaxial ultrashort pulsed beam propagation in free space

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Abstract
We present a rigorous approach to the propagation of a fully vectorial nonparaxial ultrashort pulsed beam in free space. By using the Fourier transform and the vectorial angular-spectrum formalism, we derive an exact fully vectorial integral solution of Maxwell's equations for an ultrashort pulsed beam whose pulse duration is as short as a single optical oscillation period. From this general expression we develop a Taylor expansion of electric field, and obtain all-order corrections to the paraxial pulsed beam solution, which is assumed to be known. Furthermore, the influence of vectorial nature on nonparaxial pulsed beam propagation is analysed and the vectorial nonparaxial correction is given in this paper.

Keywords: Vectorial nonparaxial corrections, paraxial approximation, ultrashort pulsed beam

1. Introduction

With the experimental production of extremely short few-cycle, single-cycle, even half-cycle electromagnetic pulses [1, 2], it is nowadays well established that the propagation of ultrashort pulses can differ significantly from that of the quasi-monochromatic light. A great deal of research has been carried out on some new phenomena related to the spatiotemporal coupling during the pulsed beam propagation in free space or in a linear medium, such as time-dependent diffraction patterns, the time-derivative effect, decrease of the optical period along the beam axis, pulse time delay and redshift toward the beam periphery [3–12], etc. The above-mentioned studies are all based on the scalar paraxial theory, which is able to give an accurate description of the ultrashort pulsed beam propagation when the divergence angle is small and the beam width or diffraction length is much larger than the wavelength for each frequency. However, there do exist ultrashort pulsed beams with large divergence angles or with ultra-narrow waists, for which the scalar paraxial theory is invalid. Therefore the nonparaxial effect and the vectorial effect ought to be taken into account.

To deal with nonparaxial propagation of the monochromatic beam, Lax *et al* [13] developed a perturbation method to obtain corrections to the paraxial beam propagation by the use of a small dimensionless parameter $1/(kw_0)$, where k is the wavenumber in free space and w_0 is the waist width of the beam. Cao *et al* [14, 15] also proposed a related truncated operator to correct the paraxial beam solution. However, a theory of nonparaxial propagation should be vectorial and cannot be based on the scale Helmholtz equation [16, 17]. As for the ultrashort pulsed beam, Fu *et al* [18, 19] considered separately the nonparaxial and vectorial correction to the paraxial solution based on Cao's technique. In this paper, we employ the Fourier transform to deal with the full vectorial nonparaxial propagation of a ultrashort pulsed beam whose scalar paraxial solution is known. On the basis of a suitable angular spectrum and vectorial analysis, we develop a relatively simple transform from the scalar paraxial solution of the ultrashort pulsed beam to a correspondingly exact solution of the vectorial nonparaxial wave equation on condition that the evanescent wave is ignored. It is shown that the vectorial effect contributes much to the correction of the scalar paraxial solution.

This paper is organized as follows: in section 2, the equations governing the evolution of the transverse and longitudinal components are derived and the coupling relation between them is given in the frequency domain. In section 3,

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the rigorous integral solution of vectorial nonparaxial pulsed beams is presented. In section 4, the influence of vectorial effect on nonparaxial propagation is analysed. In section 5, we obtain a vectorial nonparaxial correction to an arbitrary scalar paraxial solution of an ultrashort pulsed beam. Finally, we come to a conclusion in section 6.

2. The vectorial nonparaxial propagation of the ultrashort pulsed beam

The propagation of an electromagnetic field $\vec{E}(\vec{r}, z, t)$ in free space is governed by the vectorial wave equation

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{E}(\vec{r}, z, t) = 0, \tag{1}$$

where $\vec{r} = x\hat{e}_x + y\hat{e}_y$ are the transverse coordinates and $\hat{e}_x$, $\hat{e}_y$ are the united vectors in the x and y directions, respectively. To derive equation (1), we have used

$$\nabla \cdot \vec{E}(\vec{r}, z, t) = 0. \tag{2}$$

Equation (2) should work together with (1) to obtain a self-consistent solution of the vector field. As usual, we introduce the local variables $t' = t - z/c$, $z' = z$ to extract from $\vec{E}$ its rapid variation along z that is due to the pulse transport as a whole at velocity c. In this wave, any dependence of $\vec{E}$ on the new variable z' represents diffraction changes on propagation that are to the finite transverse extent of the wave. Employing the Fourier transform on both equations (1) and (2) yields

$$\left[\nabla_\perp^2 + 2\mathrm{i}k(\omega)\frac{\partial}{\partial z'} + \frac{\partial^2}{\partial z'^2}\right]\vec{\Psi}(\vec{r}, z', \omega) = 0, \tag{3}$$

and

$$\mathrm{i}k(\omega)\Psi_{z'}(\vec{r}, z', \omega) + \frac{\partial}{\partial z'}\Psi_{z'}(\vec{r}, z', \omega) + \nabla_\perp \cdot \vec{\Psi}_\perp(\vec{r}, z', \omega) = 0, \tag{4}$$

where $\nabla_\perp^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the transverse Laplacian, $\nabla_\perp = \hat{e}_x\partial/\partial x + \hat{e}_y\partial/\partial y$. The Fourier transform of the electric field is represented as

$$\begin{aligned}\vec{E}(\vec{r}, z', t') &= \frac{1}{(2\pi)^{1/2}}\int \vec{\Psi}(\vec{r}, z', \omega)\exp[-\mathrm{i}\omega t']\,\mathrm{d}\omega,\\ &= \frac{1}{(2\pi)^{1/2}}\int [\vec{\Psi}_\perp(\vec{r}, z', \omega)\\ &\quad + \hat{e}_{z'}\Psi_{z'}(\vec{r}, z', \omega)]\exp[-\mathrm{i}\omega t']\,\mathrm{d}\omega,\end{aligned} \tag{5}$$

where $\vec{\Psi}_\perp(\vec{r}, z', \omega) = \hat{e}_x\Psi_x(\vec{r}, z', \omega) + \hat{e}_y\Psi_y(\vec{r}, z', \omega)$ is the transverse components of a vector field in the frequency domain and $\hat{e}_{z'}$ is the unit vector in the z' direction.

Equation (3) is a nonparaxial propagation equation for each vectorial component of the electric field at every frequency. In addition equation (4) manifests the coupling relation between the transverse and longitudinal components. Regarding a practical propagation problem, we will first solve equation (3) under a certain initial value condition, i.e. $\vec{E}(\vec{r}, 0, t')$, to get the transverse components and then evaluate the longitudinal component from equation (4).

If the paraxial approximation (PA)in frequency domain is valid, i.e.

$$\begin{aligned}&\left|\frac{\partial}{\partial z'}\Psi_j(\vec{r}, z', \omega)\right| \ll |k(\omega)\Psi_j(\vec{r}, z', \omega)|,\\ &\left|\frac{\partial^2}{\partial z'^2}\Psi_j(\vec{r}, z', \omega)\right| \ll \left|k(\omega)\frac{\partial}{\partial z'}\Psi_j(\vec{r}, z', \omega)\right|,\\ &\quad j = x, y, z,\end{aligned} \tag{6}$$

which indicates that the PA condition is satisfied for each component of the vectorial electric field at every frequency, the paraxial propagation equation for the transverse components in temporal-frequency domain yields

$$\left[\nabla_\perp^2 + 2\mathrm{i}k(\omega)\frac{\partial}{\partial z'}\right]\vec{\Psi}_\perp^{(p)}(\vec{r}, z', \omega) = 0. \tag{7}$$

Furthermore, the coupling relation between the transverse and longitudinal components is

$$\Psi_{z'}^{(p)}(\vec{r}, z', \omega) = \frac{\mathrm{i}}{k(\omega)}\nabla_\perp \cdot \vec{\Psi}_\perp^{(p)}(\vec{r}, z', \omega). \tag{8}$$

By the inverse Fourier transform, one gets

$$\left(\nabla_\perp^2 - \frac{2}{c}\frac{\partial^2}{\partial z'\partial t'}\right)\vec{E}_\perp^{(p)}(\vec{r}, z', t') = 0, \tag{9}$$

and

$$\frac{1}{c}\frac{\partial}{\partial t'}E_{z'}^{(p)}(\vec{r}, z', t') = \nabla_\perp \cdot \vec{E}_\perp^{(p)}(\vec{r}, z', t'). \tag{10}$$

Equation (9) has received extensive attention [7–12] since it allows one to extend the paraxial treatment of diffraction to arbitrary ultrashort pulses. Some solutions of this equation, such as the exact analytical solution called ultrashort pulse Gaussian beam [10] or integral solution [9], have been reported recently. In the following sections, we assume the scalar paraxial solutions $\vec{\Psi}_\perp^{(p)}(\vec{r}, z', \omega)$ and $\vec{E}_\perp^{(p)}(\vec{r}, z', t')$ are known and $\vec{\Psi}(\vec{r}, z', \omega)$ and $\vec{E}(\vec{r}, z', t')$ represent the exact solutions of vectorial nonparaxial propagation (governed by equations (3) and (4)) as the corrections to the paraxial solution.

3. Integral solution of vectorial nonparaxial pulsed beam

If only the positive propagation is considered, by using the angular-spectrum formalism, the integral solution of equation (3) can be written as [16]

$$\begin{aligned}&\vec{\Psi}(\vec{r}, z', \omega)\\ &= \frac{1}{2\pi}\int \mathrm{d}^2\vec{k}_\perp \exp\left[\mathrm{i}\vec{k}_\perp \cdot \vec{r} + \mathrm{i}kz\left(\sqrt{1 - k_\perp^2/k^2} - 1\right)\right]\\ &\quad \times\left[\vec{A}_\perp(\vec{k}_\perp, 0, \omega) - \hat{e}_z\frac{1}{k\sqrt{1 - k_\perp^2/k^2}}\vec{k}_\perp \cdot \vec{A}_\perp(\vec{k}_\perp, 0, \omega)\right],\end{aligned} \tag{11}$$

where $\vec{k}_\perp = k_x\hat{e}_x + k_y\hat{e}_y$, and $\vec{A}_\perp(\vec{k}_\perp, 0, \omega)$ is the spatial Fourier transform of the transverse electric field $\vec{\Psi}_\perp(\vec{r}, 0, \omega)$, that is

$$\vec{A}_\perp(\vec{k}_\perp, 0, \omega) = \frac{1}{2\pi}\int \vec{\Psi}_\perp(\vec{r}, 0, \omega)\mathrm{e}^{-\mathrm{i}\vec{k}_\perp \cdot \vec{r}}\,\mathrm{d}^2\vec{r}.$$

and

$$A_z(\vec{k}_\perp, 0, \omega) = \frac{1}{2\pi}\int \Psi_z(\vec{r}, 0, \omega)\mathrm{e}^{-\mathrm{i}\vec{k}_\perp\cdot\vec{r}}\,\mathrm{d}^2\vec{r}.$$

To derive equation (11), we have used the relation

$$A_z(\vec{k}_\perp, z', \omega) = -\frac{1}{k\sqrt{1-k_\perp^2/k^2}}\vec{k}_\perp\cdot\vec{A}_\perp(\vec{k}_\perp, z', \omega), \quad (12)$$

which is deduced from equations (3) and (4) through the bidimensional spatial Fourier transform method.

The solution equation (11) satisfies equations (3) and (4), simultaneously, for half-space $z > 0$. We can get the integral solution of electric field $\vec{E}(\vec{r}, z', t')$ in time domain by inverse Fourier transform, i.e. equation (5). Though it is difficult to work out the analytical solution directly from equation (11), it is possible for us to develop a Taylor expansion of electric field so as to obtain all-order corrections to the paraxial solution if the scalar paraxial solution $\vec{E}_\perp^{(p)}(\vec{r}, z', t')$ is known.

When the PA condition is met, from equation (7) comes the relation

$$|\nabla_\perp^2\Psi_j(\vec{r}, z', \omega)| \sim \left|k(\omega)\frac{\partial}{\partial z'}\Psi_j(\vec{r}, z', \omega)\right| \ll |k^2(\omega)\Psi_j(\vec{r}, z', \omega)|,$$

where $j = x, y, z$. In the angular-spectrum domain this relation can be presented as $\eta^2 \equiv |\vec{k}_\perp/k|^2 \ll 1$. From a mathematical point of view, the evaluation of vectorial and nonparaxial terms can be obtained by Taylor expanding the integral of equation (11) up to any order in the parameter $\eta = |\vec{k}_\perp/k|$.

Let us first recover the standard paraxial results from equation (11). Under the PA, i.e. $\eta^2 \ll 1$, and using $\sqrt{1-k_\perp^2/k^2} \cong 1 - \frac{1}{2}(k_\perp^2/k^2)$, we obtain the transverse components of the electric field from equation (11) as

$$\vec{\Psi}_\perp^{(p)}(\vec{r}, z', \omega) = \frac{1}{2\pi}\int \vec{A}_\perp(\vec{k}_\perp, 0, \omega)\mathrm{e}^{\mathrm{i}\vec{k}_\perp\cdot\vec{r}-\mathrm{i}zk_\perp^2/(2k)}\,\mathrm{d}^2\vec{k}_\perp, \quad (13)$$

$$= \frac{1}{2\pi}\frac{k(\omega)}{\mathrm{i}z'}\int \vec{\Psi}_\perp(\vec{r}', 0, \omega)\exp\left[\mathrm{i}\frac{k(\omega)}{2z'}(\vec{r}-\vec{r}')^2\right]\mathrm{d}^2\vec{r}'. \quad (14)$$

Equation (13) is the standard solution of the paraxial equation, i.e. equation (7), and equation (14) is the Fresnel diffraction integral for each frequency. After inverse time Fourier transforming, the integral solution for the paraxial pulsed beam in free space yields

$$\vec{E}_\perp^{(p)}(\vec{r}, t', z') = \frac{1}{\sqrt{2\pi}zc}\int \frac{\partial}{\partial\tau}\vec{E}_\perp(\vec{r}', 0, \tau)\,\mathrm{d}^2\vec{r}', \quad (15)$$

where $\tau = t' - (\vec{r}-\vec{r}')^2/(2cz')$ is the reduced time and $\vec{E}_\perp(\vec{r}', 0, t')$ is the electric field at the initial plane ($z = 0$). From this equation, the pulsed beam solution can be deduced for any initial value condition, whereas the analytical solution can only be derived for some specific case, such as an ultrashort pulsed Gaussian beam [10].

4. Influence of vectorial effect on nonparaxial propagation of pulsed beam

It is noticed that in the paraxial solution equation (15) we have completely neglected the longitudinal component of the field. However, the relation $\eta \ll 1$ is not determinately satisfied when the PA condition, i.e., $\eta^2 \ll 1$, is valid. In fact, under this condition, the longitudinal field can be obtained from equation (8) as

$$\vec{\Psi}_{z'}(\vec{r}, z', \omega) = \frac{1}{2\pi}\int \frac{1}{k}\vec{k}_\perp\cdot\vec{A}_\perp(\vec{k}_\perp, 0, \omega)\mathrm{e}^{\mathrm{i}\vec{k}_\perp\cdot\vec{r}-\mathrm{i}zk_\perp^2/(2k)}\,\mathrm{d}^2\vec{k}_\perp, \quad (16)$$

which is, due to the factor of $\vec{k}_\perp/k$, the first order in η (while the paraxial transverse one is zero order). This solution can also be derived from equation (12) by using the approximation

$$\frac{1}{k\sqrt{1-k_\perp^2/k^2}} \cong \frac{1}{k}.$$

It is obvious that, when the PA condition is only weakly satisfied, i.e. the relation $\eta \ll 1$ is not well satisfied, the longitudinal component cannot be neglected. In other words, when we weaken the PA condition, i.e. $\eta^2 \ll 1$, the beam is no longer completely paraxial and the first-order correction is the appearance of a longitudinal component stated in equation (16). Consequently, a theory of nonparaxial ultrashort pulsed beam propagation should be intrinsically vectorial and cannot be based on the scalar Helmholtz equation alone [16, 17].

By using equation (13),we can rewrite equation (16) as

$$\Psi_{z'}(\vec{r}, z', \omega) = \frac{\mathrm{i}}{2\pi k}\nabla_\perp\cdot\int \vec{A}_\perp(\vec{k}_\perp, 0, \omega)\mathrm{e}^{\mathrm{i}\vec{k}_\perp\cdot\vec{r}-\mathrm{i}zk_\perp^2/(2k)}\,\mathrm{d}^2\vec{k}_\perp, \quad (17)$$

$$= \frac{\mathrm{i}}{k}\nabla_\perp\cdot\vec{\Psi}_\perp^{(p)}(\vec{r}, z', \omega) \quad (18)$$

and the total vectorial electric field in frequency domain is

$$\vec{\Psi}(\vec{r}, z', \omega) = \left(1 + \hat{e}_{z'}\frac{\mathrm{i}}{k}\nabla_\perp\right)\cdot\vec{\Psi}_\perp^{(p)}(\vec{r}, z', \omega) \quad (19)$$

By means of the inverse Fourier transform of equation (5), we obtain the vector field in time domain as

$$\vec{E}(\vec{r}, z', t') = \vec{E}_\perp^{(p)}(\vec{r}, z', t') - c\sqrt{\frac{\pi}{2}}\mathrm{sign}(t') * [\nabla_\perp\cdot\vec{E}_\perp^{(p)}(\vec{r}, z', t')]\hat{e}_z$$

$$= \left[1 - \hat{e}_z c\sqrt{\frac{\pi}{2}}\mathrm{sign}(t') * \nabla_\perp\right]\cdot\vec{E}_\perp^{(p)}(\vec{r}, z', t'). \quad (20)$$

where

$$\mathrm{sign}(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$$

is the signal function and $*$ stand for the Fourier convolution. Equation (20) is valid under the condition that the PA $\eta^2 \ll 1$ is weakly satisfied whereas $\eta \ll 1$ is not satisfied.

5. The vectorial nonparaxial correction of ultrashort pulsed beams

When the PA condition $\eta^2 \ll 1$ is weakened, we ought to further refine the description of nonparaxial propagation, and to keep the next significant order in η^2 in the Taylor expansion of equation (11). Thus the transverse component of the electric field becomes

$$\vec{\Psi}_\perp(\vec{r}, z', \omega) = \frac{1}{2\pi}\int d^2\vec{k}_\perp e^{i\vec{k}_\perp\cdot\vec{r} - iz'k_\perp^2/(2k)} \times \exp[-iz'k_\perp^4/(8k^3)]\vec{A}_\perp(\vec{k}_\perp, 0, \omega). \quad (21)$$

By using the approximation $\exp[-iz'k_\perp^4/(8k^3)] \cong 1 - iz'k_\perp^4/(8k^3)$, equation (21) yields

$$\vec{\Psi}_\perp(\vec{r}, z', \omega) = \frac{1}{2\pi}\int d^2\vec{k}_\perp e^{i\vec{k}_\perp\cdot\vec{r} - iz'k_\perp^2/(2k)} \times [1 - iz'k_\perp^4/(8k^3)]\vec{A}_\perp(\vec{k}_\perp, 0, \omega), \quad (22)$$

which is equivalent to

$$\vec{\Psi}_\perp(\vec{r}, z', \omega) = \left[1 - ikz'\frac{1}{8}\left(\frac{\nabla_\perp^2}{k^2}\right)^2\right]\vec{\Psi}_\perp^{(p)}(\vec{r}, z', \omega). \quad (23)$$

By using equation (7), we can simplify equation (23) to

$$\vec{\Psi}_\perp(\vec{r}, z', \omega) = \left(1 + \frac{iz'}{2k}\frac{\partial^2}{\partial z'^2}\right)\vec{\Psi}_\perp^{(p)}(\vec{r}, z', \omega). \quad (24)$$

Through inverse time Fourier transforming, the nonparaxial correction to an ultrashort pulsed beam in the temporal domain is presented as

$$\vec{E}_\perp(\vec{r}, z', t') = \left[1 - \frac{cz'}{2}\sqrt{\frac{\pi}{2}}\mathrm{sign}(t') * \frac{\partial^2}{\partial z'^2}\right]\vec{E}_\perp^{(p)}(\vec{r}, z', t'). \quad (25)$$

As for the longitudinal component of the field, we make a Taylor expansion of the term $\frac{1}{\sqrt{1-k_\perp^2/k^2}} \cong 1 + k_\perp^2/(2k^2)$ in equation (12) and truncate it to the order of η^2, yielding

$$\Psi_{z'}(\vec{r}, z', \omega) = \left(1 - \frac{1}{2k^2}\nabla_\perp^2\right)\frac{i}{k}\nabla_\perp\cdot\vec{\Psi}_\perp(\vec{r}, z', \omega). \quad (26)$$

Substitute equation (24) into (26), and truncate the expansion to the third order of η since the next significant term is fifth order while equation (24) is fourth order. Hence

$$\Psi_{z'}(\vec{r}, z', \omega) = \left(1 + \frac{i}{k}\frac{\partial}{\partial z'}\right)\frac{i}{k}\nabla_\perp\cdot\vec{\Psi}_\perp^{(p)}(\vec{r}, z', \omega), \quad (27)$$

where we have taken the advantage of equation (7). With application of inverse Fourier transform equation (5), the longitudinal components in the temporal domain E_z are given as

$$E_z(\vec{r}, z', t') = c^2\sqrt{\frac{\pi}{2}}[t'\mathrm{sign}(t')] * \frac{\partial}{\partial z'}\nabla_\perp\cdot\vec{E}_\perp^{(p)}(\vec{r}, z', t') - c\sqrt{\frac{\pi}{2}}\mathrm{sign}(t') * \nabla_\perp\cdot\vec{E}_\perp^{(p)}(\vec{r}, z', t'). \quad (28)$$

Finally we can write the corrected nonparaxial vectorial electric field as

$$\vec{E}(\vec{r}, z', t') = \left[\vec{E}_\perp^{(p)} - \frac{cz'}{2}\sqrt{\frac{\pi}{2}}\mathrm{sign}(t') * \frac{\partial^2\vec{E}_\perp^{(p)}}{\partial z'^2}\right] + \hat{e}_{z'}c^2\sqrt{\frac{\pi}{2}} \times \left\{[t'\mathrm{sign}(t')] * \nabla_\perp\cdot\frac{\partial\vec{E}_\perp^{(p)}}{\partial z'} - \frac{\mathrm{sign}(t')}{c} * \nabla_\perp\cdot\vec{E}_\perp^{(p)}\right\}. \quad (29)$$

The expansion in equations (26) and (22) are truncated to the order of η^4. It is confirmed that equation (29) is qualified to describe the propagation of the ultra-short nonparaxial pulsed beams, even when the beam width of the pulsed beam is of the order of the wavelength. With our approach, it is fairly easy to deduce the higher-order correction to the pulsed beam to improve the precise of the description of the ultra-short pulsed beam propagation.

6. Conclusion

We present in this paper a rigorous approach to the propagation of a fully vectorial nonparaxial ultrashort pulsed beam in free space. By using the Fourier transform and the vectorial angular-spectrum formalism, we obtain an exact fully vectorial integral solution of Maxwell's equations for an ultrashort pulsed beam. From this general expression, we further develop a relatively simple transform from the scalar paraxial solution of the ultrashort pulsed beam, which is assumed to be known, to a corresponding correction solution of the vectorial nonparaxial wave equation. With this approach, one can easily deduce the higher-order correction to the pulsed beam to improve the precision of the description of the ultra-short pulsed beam propagation. The influence of vectorial effect on nonparaxial pulsed beam propagation are clearly analysed and the first-order vectorial nonparaxial corrections are obtained as well. It is proved that the vectorial effect contributes much to the correction of the scalar paraxial solution and vector effect should be taken in account for a nonparaxial propagation. In addition, our treatment allows us to deal with a fully vectorial situation in the presence of arbitrary boundary condition.

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