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Traveling wave solutions to nonlinear directional couplers by modified Kudryashov method

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Abstract

This work finds several new traveling wave solutions for nonlinear directional couplers with optical metamaterials by means of the modified Kudryashov method. This model can be used to distribute light from a main fiber into one or more branch fibers. Two forms of optical couplers are considered, namely the twin- and multiple- core couplers. These couplers, which have applications as intensity-dependent switches and as limiters, are studied with four nonlinear items namely the Kerr, power, parabolic, and dual-power laws. The restrictions on the parameters for the existence of solutions are also examined. The 3D- and 2D figures are introduced to discuss the physical meaning for some of the gained solutions. The performance of the method shows the adequacy , power, and ability for applying to many other nonlinear evolution models.

Keywords: traveling wave solution, the modified kudryashov method, twin-core couplers, multiple-core couplers, optical metamaterials

(Some figures may appear in colour only in the online journal)

1. Introduction

The wave propagation in optical couplers is a well-known topic of research in nonlinear optics. Several results were published addressing this problem during the past couple of decades [1–24]. More recently, the study of wave propagation in couplers with optical metamaterials gained particular attention and important achievements were reported [1, 2, 19].

It was observed that by varying the intensity of the input light pulses in a nonlinear coupler one can achieve pulse switching between the cores [10] and, therefore, optical couplers can be used as an optical switch. In the past, it was shown that soliton switching in dual-core optical fibers yielded excellent switching characteristics with high efficiency for a wide range of input energies. After comparing the switching behavior amongst fundamental, second-order and quasi-solitons, it was concluded that a fundamental soliton carry the most ideal features for conducting optical switching. In fact, it was also established that pulse breakup can be avoided provided the input signal is a soliton [20]. Therefore, the extraction of soliton solutions of optical couplers is an important topic that carries relevant benefits in the area of telecommunications. We find several integration algorithms to fulfill this task such as the trial function, undetermined coefficients, sine-cosine function, Bernoulli equation, -expansion, and several others techniques [2, 12–14].

The main motivation of this paper is to employ a powerful mathematical algorithm, namely the modified Kudryashov method [25–27], to unravel the traveling wave solutions of couplers in optical metamaterials.

The paper is organized in the following order. Section 2, gives a brief description of the modified Kudryashov method. Sections 3 investigates the twin-, multiple- (coupling with nearest neighbors), and multiple- (coupling with all neighbors) core couplers. Section 4, investigates the physical interpretation for some of the obtained solutions. Section 5, provides the conclusions of the paper.

2. Description of the modified Kudryashov method

Here, we give a briefdescription of the modified Kudryashov method. Let us consider a nonlinear partial differential equation (PDE),

$$\Xi(u, u_t, u_x, u_{tt}, u_{xx}, \cdots) = 0, \tag{1}$$

where Ξ is a polynomial in its arguments.

The essence of modified Kudryashov method can be presented in the following steps

Step 1. By means of the variable transformation

$$\zeta = kx \pm \lambda t, \tag{2}$$

where k and λ are constants.

Equation (1) can be converted to

$$\Upsilon(\Lambda, \Lambda', \Lambda'', \Lambda''', \cdots) = 0, \tag{3}$$

where Υ is in general a polynomial function of its arguments and $\Lambda' = \dfrac{d\Lambda}{d\zeta}$.

Step 2. Suppose that the solution of (3) can be expressed by a polynomial in $\varphi(\zeta)$ as follows

$$\Lambda(\zeta) = \sum_{i=0}^{N} B_i \varphi^i(\zeta), \tag{4}$$

where B_i, $i = 0, 1, 2, 3, \cdots, N$ are real constants with $B_N \neq 0$ to be determined, and N is a positive integer to be determined. The function $\varphi(\zeta)$ is the solution of the auxiliary linear ordinary differential equation

$$\frac{d\varphi}{d\zeta} = (\varphi^2(\zeta) - \varphi(\zeta)) \ln(A), \tag{5}$$

where A is a non-zero constant with the conditions $A > 0$ and $A \neq 1$.

Equation (5) gives the following solution:

$$\varphi(\zeta) = \frac{1}{1 + dA^{\zeta}}, \quad d \in \mathbb{R}. \tag{6}$$

Step 3. After substituting (4) into (3) and (5) and collecting all terms with the same order of $\varphi(\zeta)$, the left-hand side of equation (3) is converted into another polynomial in $\varphi(\zeta)$. Equating each coefficient of this polynomial to zero, yields a set of algebraic equations for B_i, d, k, and λ by using a software package such as Maple.

Step 4. Substituting the values of the constants together with the solutions of equation (5), we will obtain the exact traveling wave solutions of the nonlinear PDE (1).

3. Applications and discussion

3.1. Twin-core couplers

Consider the following twin-core couplers in optical metamaterials [2, 12–14]

$$\begin{aligned} &i\Gamma_t + a_1\Gamma_{xx} + F_1(|\Gamma|^2)\Gamma \\ &\quad = \xi_1(|\Gamma|^2\Gamma)_{xx} + \eta_1 |\Gamma|^2\Gamma_{xx} + \zeta_1\Gamma^2\Gamma^*_{xx} + \kappa_1\Omega, \end{aligned} \tag{7}$$

$$\begin{aligned} &i\Omega_t + a_2\Omega_{xx} + F_2(|\Omega|^2)\Omega \\ &\quad = \xi_2(|\Omega|^2\Omega)_{xx} + \eta_2 |\Omega|^2\Omega_{xx} + \zeta_2\Omega^2\Omega^*_{xx} + \kappa_2\Gamma. \end{aligned} \tag{8}$$

The complex-valued functions $\Gamma = \Gamma(x, t)$ and $\Omega = \Omega(x, t)$ represent the optical fields in two respective cores and $i = \sqrt{-1}$. The symbols a_1 and a_2 denote the coefficients of group velocity dispersion. Moreover, κ_j, $j = 1, 2,$ are the coupling coefficients, and the terms including ξ_j, μ_j, and ζ_j characterise the properties of optical metamaterials [14].

In order to handle the governing model, we consider the following starting hypotheses

$$\Gamma(x, t) = U_1(\xi)e^{i\Phi(x,t)}, \tag{9}$$

$$\Omega(x, t) = U_2(\xi)e^{i\Phi(x,t)}, \tag{10}$$

where U_1 (ξ) and U_2 (ξ) represent the shape of the pulse with the phase $\Phi(x, t)$ so that

$$\Phi(x, t) = -px + \omega t + \theta_0.$$

The traveling coordinate ξ is given by

$$\xi = k(x - vt),$$

where v, p, ω, and θ_0 represent the soliton velocity, frequency, wave number, and phase constant, respectively.

Substituting (9) and (10) into (7) and (8), respectively, the imaginary component yields

$$\begin{aligned} &-(vk + 2a_s pk)U_s' + 2pk^2(3\xi_s + \eta_s - \zeta_s)U_s^2 U_s'' = 0, \\ &s = 1, 2. \end{aligned} \tag{11}$$

After setting the coefficients of the linearly independent functions in (11) to zero, we verify that it is possible to determine the speed of the soliton $v = -2a_s p$ and the constraint conditions

$$3\xi_s + \eta_s - \zeta_s = 0. \tag{12}$$

Equating the two values of the soliton velocity $v = -2a_s p$ leads to

$$a_1 = a_2 = a. \tag{13}$$

Consequently, it reduces to

$$v = -2ap. \tag{14}$$

The real parts imply

$$\begin{aligned} &ak^2U_s'' - (\omega + ap^2)U_s + F_s(U_s^2)U_s \\ &+ (\xi_s + \eta_s + \zeta_s)p^2U_s^3 - 6\xi_s k^2 U_s(U_s') \\ &- k^2(3\xi_s + \eta_s + \zeta_s)U_s^2U_s'' - \kappa_s U_{s^*} = 0, \end{aligned} \tag{15}$$

where $a_1 = a_2 = a$ and $s^* = 3 - s$, $s = 1, 2$.

The balancing principle leads to $U_s = U_{s^*}$. Consequently, equation (15) can be written as

$$\begin{aligned} &ak^2U_s'' - (\omega + ap^2 + \kappa_s)U_s + F_s(U_s^2)U_s + 2(\zeta_s - \xi_s)p^2U_s^3 \\ &- 6\xi_s k^2U_s(U_s') - 2k^2\zeta_s U_s^2U_s'' = 0. \end{aligned} \tag{16}$$

In the following subsections, this equation will be studied when considering four different types of nonlinearity.

3.1.1. Kerr law nonlinearity. In this case we have $F_s(\Delta) = b_s\Delta$ $(s = 1, 2)$ and inserting it into equations (7) and (8) yields

$$\begin{aligned} &i\Gamma_t + a\Gamma_{xx} + b_1|\Gamma|^2\Gamma \\ &= \xi_1(|\Gamma|^2\Gamma)_{xx} + \eta_1|\Gamma|^2\Gamma_{xx} + \zeta_1\Gamma^2\Gamma_{xx}^* + \kappa_1\Omega, \end{aligned} \tag{17}$$

$$\begin{aligned} &i\Omega_t + a\Omega_{xx} + b_2|\Omega|^2\Omega \\ &= \xi_2(|\Omega|^2\Omega)_{xx} + \eta_2|\Omega|^2\Omega_{xx} + \zeta_2\Omega^2\Omega_{xx}^* + \kappa_2\Gamma. \end{aligned} \tag{18}$$

Equation (16) takes the following form

$$\begin{aligned} &ak^2U_s'' - (\omega + ap^2 + \kappa_s)U_s + (b_s + 2(\zeta_s - \xi_s)p^2)U_s^3 \\ &- 6\xi_s k^2U_s(U_s') - 2k^2\zeta_s U_s^2U_s'' = 0. \end{aligned} \tag{19}$$

To obtain an analytic solution, we apply the transformations $\zeta_s = \xi_s = 0$ in equation (19) and we obtain

$$ak^2U_s'' - (\omega + ap^2 + \kappa_s)U_s + b_sU_s^3 = 0. \tag{20}$$

Balancing U_s'' with U_s^3 in equation (20) give

$$N + 2 = 3N \Leftrightarrow N = 1.$$

We seek solutions of the form

$$U_1(\zeta) = \sigma_0 + \sigma_1\varphi(\zeta), \tag{21}$$

$$U_2(\zeta) = \rho_0 + \rho_1\varphi(\zeta), \tag{22}$$

where σ_0, σ_1, ρ_0, and ρ_1 are constants to be determined.

Substituting the solutions (21) and (22) in (20) gives the polynomial equation in $\varphi(\zeta)$. Thus, we find algebraic equations system by setting all the coefficients to zero. The solution of this system for σ_0, σ_1, ρ_0, ρ_1, and ω with σ_1, $\rho_1 \neq 0$ gives

$$\begin{aligned} &\sigma_0 = \pm\frac{k}{2}\sqrt{\frac{2a}{b_1}}\ln(A), \quad \sigma_1 = \mp k\sqrt{\frac{2a}{b_1}}\ln(A), \\ &\rho_0 = \pm\frac{k}{2}\sqrt{\frac{2a}{b_2}}\ln(A), \quad \rho_1 = \mp k\sqrt{\frac{2a}{b_2}}\ln(A), \\ &\omega = -ap^2 + \kappa_s - \frac{1}{2}ak^2\ln^2(A). \end{aligned} \tag{23}$$

After substituting equations (23), (9), and (10) into equations (21) and (22), we obtain the exact solutions of equations (17) and (18)

$$\begin{aligned} \Gamma_1(x, t) &= \pm k\sqrt{\frac{2a}{b_1}}\ln(A)\left[\frac{1}{2} - \frac{1}{1 + dA^{k(x+2apt)}}\right] \\ &\times \exp\left(i\left(-px + \left(-ap^2 + \kappa_1 - \frac{1}{2}ak^2\ln^2(A)\right)t + \theta_0\right)\right), \\ \Omega_1(x, t) &= \pm k\sqrt{\frac{2a}{b_2}}\ln(A)\left[\frac{1}{2} - \frac{1}{1 + dA^{k(x+2apt)}}\right] \\ &\times \exp\left(i\left(-px + \left(-ap^2 + \kappa_2 - \frac{1}{2}ak^2\ln^2(A)\right)t + \theta_0\right)\right). \end{aligned}$$

These solutions are valid for $a\, b_s > 0$, $s = 1, 2$.

3.1.2. Power law nonlinearity. In this case we have $F_s(\Delta) = b_s\,\Delta^n$ $(s = 1, 2)$, (n indicates the power law nonlinearity factor) and inserting it into equations (7) and (8) yields

$$\begin{aligned} &i\Gamma_t + a\Gamma_{xx} + b_1|\Gamma|^{2n}\Gamma \\ &= \xi_1(|\Gamma|^2\Gamma)_{xx} + \eta_1|\Gamma|^2\Gamma_{xx} + \zeta_1\Gamma^2\Gamma_{xx}^* + \kappa_1\Omega, \end{aligned} \tag{24}$$

$$\begin{aligned} &i\Omega_t + a\Omega_{xx} + b_2|\Omega|^{2n}\Omega \\ &= \xi_2(|\Omega|^2\Omega)_{xx} + \eta_2|\Omega|^2\Omega_{xx} + \zeta_2\Omega^2\Omega_{xx}^* + \kappa_2\Gamma. \end{aligned} \tag{25}$$

Equation (16) takes the form

$$\begin{aligned} &ak^2U_s'' - (\omega + ap^2 + \kappa_s)U_s + b_sU_s^{2n+1} + 2(\zeta_s - \xi_s)p^2U_s^3 \\ &- 6\xi_s k^2U_s(U_s')^2 - 2k^2\zeta_s U_s^2U_s'' = 0. \end{aligned} \tag{26}$$

To obtain an analytic solution, we apply the transformations $\zeta_s = \xi_s = 0$ in equation (26) to find

$$ak^2U_s'' - (\omega + ap^2 + \kappa_s)U_s + b_sU_s^{2n+1} = 0. \tag{27}$$

In order to obtain the closed form solutions, the following transformation

$$U_s = V_s^{\frac{1}{2n}}, \tag{28}$$

is applied. Therefore, equation (27) becomes

$$\begin{aligned} &ak^2(2nV_sV_s'' + (1 - 2n)(V_s')^2) \\ &- 4n^2(\omega + ap^2 + \kappa_s)V_s^2 + 4n^2b_sV_s^3 = 0. \end{aligned} \tag{29}$$

Balancing V_s^3 with $(V_s')^2$ gives $N = 2$. We seek solutions of the form

$$V_1(\zeta) = \sigma_0 + \sigma_1\varphi(\zeta) + \sigma_2\varphi^2(\zeta), \tag{30}$$

$$V_2(\zeta) = \rho_0 + \rho_1\varphi(\zeta) + \rho_2\varphi^2(\zeta), \tag{31}$$

where σ_0, σ_1, σ_2, ρ_0, ρ_1, and ρ_2 are constants to be determined.

Substituting the solution (30) and (31) into (27) gives the polynomial equation in $\varphi(\zeta)$. Thus, we find algebraic equations system by setting all the coefficients to zero. The solution of this system for σ_0, σ_1, σ_2, ρ_0, ρ_1, ρ_2, and ω with $\sigma_2, \rho_2 \neq 0$ gives

$$\begin{aligned}
&\sigma_0 = 0, \quad \sigma_1 = \frac{ak^2(n+1)\ln^2(A)}{n^2 b_1},\\
&\sigma_2 = -\frac{ak^2(n+1)\ln^2(A)}{n^2 b_1},\\
&\rho_0 = 0, \quad \rho_1 = \frac{ak^2(n+1)\ln^2(A)}{n^2 b_2},\\
&\rho_2 = -\frac{ak^2(n+1)\ln^2(A)}{n^2 b_2},\\
&\omega = \frac{ak^2\ln^2(A) - 4n^2(ap^2+\kappa_s)}{4n^2}.
\end{aligned} \tag{32}$$

After substituting equations (32), (9), and (10) into equations (30) and (31), we obtain the following exact solutions of equations (24) and (25)

$$\Gamma_2(x, t) = \left[\frac{ak^2(n+1)\ln^2(A)dA^{k(x+2apt)}}{n^2 b_1(1+dA^{k(x+2apt)})^2}\right]^{\frac{1}{2n}} \times \exp\left(i\left(-px + \left(\frac{ak^2\ln^2(A) - 4n^2(ap^2+\kappa_1)}{4n^2}\right)t + \theta_0\right)\right),$$

$$\Omega_2(x, t) = \left[\frac{ak^2(n+1)\ln^2(A)dA^{k(x+2apt)}}{n^2 b_2(1+dA^{k(x+2apt)})^2}\right]^{\frac{1}{2n}} \times \exp\left(i\left(-px + \left(\frac{ak^2\ln^2(A) - 4n^2(ap^2+\kappa_2)}{4n^2}\right)t + \theta_0\right)\right).$$

3.1.3. Parabolic law nonlinearity. In this case we have $F_s(\Delta) = b_s\Delta + c_s\Delta^2 \quad (s = 1, 2)$ and inserting it into equations (7) and (8) yields

$$\begin{aligned}
&i\Gamma_t + a\Gamma_{xx} + (b_1|\Gamma|^2 + c_1|\Gamma|^4)\Gamma\\
&\quad = \xi_1(|\Gamma|^2\Gamma)_{xx} + \eta_1|\Gamma|^2\Gamma_{xx} + \zeta_1\Gamma^2\Gamma^*_{xx} + \kappa_1\Omega,
\end{aligned} \tag{33}$$

$$\begin{aligned}
&i\Omega_t + a\Omega_{xx} + (b_2|\Omega|^2 + c_2|\Omega|^4)\Omega\\
&\quad = \xi_2(|\Omega|^2\Omega)_{xx} + \eta_2|\Omega|^2\Omega_{xx} + \zeta_2\Omega^2\Omega^*_{xx} + \kappa_2\Gamma.
\end{aligned} \tag{34}$$

Equation (16) becomes:

$$\begin{aligned}
&ak^2U_s'' - (\omega + ap^2 + \kappa_s)U_s + (b_s + 2(\zeta_s - \xi_s)p^2)U_s^3\\
&\quad + c_sU_s^5 - 6\xi_s k^2 U_s(U_s')^2 - 2k^2\zeta_s U_s^2U_s'' = 0.
\end{aligned} \tag{35}$$

To obtain an analytic solution, we apply the transformations $\zeta_s = \xi_s = 0$ in equation (35) to find

$$ak^2U_s'' - (\omega + ap^2 + \kappa_s)U_s + b_sU_s^3 + c_sU_s^5 = 0. \tag{36}$$

In order to obtain the closed form solutions, the following transformation

$$U_s = V_s^{\frac{1}{2}}, \tag{37}$$

is applied. The above equation gives

$$\begin{aligned}
&ak^2(2V_sV_s'' - (V_s')^2) - 4(\omega + ap^2 + \kappa_s)V_s^2\\
&\quad + 4b_sV_s^3 + 4c_sV_s^4 = 0.
\end{aligned} \tag{38}$$

Balancing V_s^4 with V_sV_s'' gives $N = 1$. We seek solutions of the form

$$V_1(\zeta) = \sigma_0 + \sigma_1\varphi(\zeta), \tag{39}$$

$$V_2(\zeta) = \rho_0 + \rho_1\varphi(\zeta), \tag{40}$$

where σ_0, σ_1, ρ_0, and ρ_1 are constants to be determined.

Substituting the solution (39) and (40) into (36) gives the polynomial equation in $\varphi(\zeta)$. Thus, we find algebraic equations system by setting all the coefficients to zero. The solution of this system for σ_0, σ_1, ρ_0, ρ_1, ω, and k with $\sigma_1, \rho_1 \neq 0$ gives

Case 1.

$$\begin{aligned}
&\sigma_0 = -\frac{3}{4}\frac{b_1}{c_1}, \quad \sigma_1 = \frac{3}{4}\frac{b_1}{c_1},\\
&\rho_0 = -\frac{3}{4}\frac{b_2}{c_2}, \quad \rho_1 = \frac{3}{4}\frac{b_2}{c_2},\\
&\omega = -\frac{1}{16}\frac{16c_c(ap^2+\kappa_s) + 3b_s^2}{c_s},\\
&k = \pm\frac{b_s}{\ln(A)}\sqrt{-\frac{3}{4c_sa}}.
\end{aligned} \tag{41}$$

Case 2.

$$\begin{aligned}
&\sigma_0 = -\frac{3}{4}\frac{b_1}{c_1}, \quad \sigma_1 = \frac{3}{4}\frac{b_1}{c_1},\\
&\rho_0 = 0, \quad \rho_1 = -\frac{3}{4}\frac{b_2}{c_2},\\
&\omega = -\frac{1}{16}\frac{16c_c(ap^2+\kappa_s) + 3b_s^2}{c_s},\\
&k = \pm\frac{b_s}{\ln(A)}\sqrt{-\frac{3}{4c_sa}}.
\end{aligned} \tag{42}$$

Substituting equations (41), (9), and (10) into equations (39) and (40), leads to the following exact solutions of equations (33) and (34)

$$\Gamma_3(x, t) = \left(\frac{3}{4}\frac{b_1}{c_1}\left[-1 + \frac{1}{1 + dA^{\pm\frac{b_1}{\ln(A)}\sqrt{-\frac{3}{4c_1a}}(x+2apt)}}\right]\right)^{\frac{1}{2}} \times \exp\left(i\left(-px - \frac{1}{16}\frac{16c_1(ap^2+\kappa_1) + 3b_1^2}{c_1}t + \theta_0\right)\right),$$

$$\Omega_3(x, t) = \left(\frac{3}{4}\frac{b_2}{c_2}\left[-1 + \frac{1}{1 + dA^{\pm\frac{b_2}{\ln(A)}\sqrt{-\frac{3}{4c_2a}}(x+2apt)}}\right]\right)^{\frac{1}{2}} \times \exp\left(i\left(-px - \frac{1}{16}\frac{16c_2(ap^2+\kappa_2) + 3b_2^2}{c_2}t + \theta_0\right)\right).$$

Substituting equations (42), (9), and (10) into equations (39) and (40), results in the following exact solutions of equations (33) and (34)

$$\Gamma_4(x,t) = \left(-\frac{3b_1}{4c_1\left(1 + dA^{\pm\frac{b_1}{\ln(A)}\sqrt{-\frac{3}{4c_1a}}(x+2apt)}\right)}\right)^{\frac{1}{2}} \times \exp\left(i\left(-px - \frac{1}{16}\frac{16c_1(ap^2+\kappa_1)+3b_1^2}{c_1}t + \theta_0\right)\right),$$

$$\Omega_4(x,t) = \left(-\frac{3b_2}{4c_2\left(1 + dA^{\pm\frac{b_2}{\ln(A)}\sqrt{-\frac{3}{4c_2a}}(x+2apt)}\right)}\right)^{\frac{1}{2}} \times \exp\left(i\left(-px - \frac{1}{16}\frac{16c_2(ap^2+\kappa_2)+3b_2^2}{c_2}t + \theta_0\right)\right).$$

These solutions are valid for $a\ c_s < 0,\ s = 1, 2$.

3.1.4. Dual-power law nonlinearity. In this case we have $F_s(\Delta) = b_s\Delta^n + c_s\Delta^{2n}\ \ (s = 1,\ 2)$ and inserting it into equations (7) and (8) yields

$$i\Gamma_t + a\Gamma_{xx} + (b_1|\Gamma|^{2n} + c_1|\Gamma|^{4n})\Gamma = \xi_1(|\Gamma|^2\Gamma)_{xx} + \eta_1|\Gamma|^2\Gamma_{xx} + \zeta_1\Gamma^2\Gamma^*_{xx} + \kappa_1\Omega, \tag{43}$$

$$i\Omega_t + a\Omega_{xx} + (b_2|\Omega|^{2n} + c_2|\Omega|^{4n})\Omega = \xi_2(|\Omega|^2\Omega)_{xx} + \eta_2|\Omega|^2\Omega_{xx} + \zeta_2\Omega^2\Omega^*_{xx} + \kappa_2\Gamma. \tag{44}$$

Equation (16) takes the form

$$ak^2U_s'' - (\omega + ap^2 + \kappa_s)U_s + b_sU_s^{2n+1} + 2(\zeta_s - \xi_s)p^2U_s^3 + c_sU_s^{4n+1} - 6\xi_sk^2U_s(U_s')^2 - 2k^2\zeta_sU_s^2U_s'' = 0. \tag{45}$$

To obtain an analytic solution, we apply the transformations $\zeta_s = \xi_s = 0$ in equation (45) obtaining

$$ak^2U_s'' - (\omega + ap^2 + \kappa_s)U_s + b_sU_s^{2n+1} + c_sU_s^{4n+1} = 0. \tag{46}$$

In order to obtain the closed form solutions, the following transformation

$$U_s = V_s^{\frac{1}{2n}}, \tag{47}$$

is applied. Therefore the above equation can be written as

$$ak^2(2nV_sV_s'' + (1-2n)(V_s')^2) - 4n^2(\omega + ap^2 + \kappa_s)V_s^2 + 4n^2b_sV_s^3 + 4n^2c_sV_s^4 = 0. \tag{48}$$

Balancing V_s^4 with V_sV_s'' gives $N = 1$. We seek solutions of the form

$$V_1(\zeta) = \sigma_0 + \sigma_1\varphi(\zeta), \tag{49}$$

$$V_2(\zeta) = \rho_0 + \rho_1\varphi(\zeta), \tag{50}$$

where σ_0, σ_1, ρ_0, and ρ_1 are constants to be determined.

Substituting the solution (49) and (50) into (46) gives the polynomial equation in $\varphi(\zeta)$. Thus, we find algebraic equations system by setting all the coefficients to zero. The solution of this system for σ_0, σ_1, ρ_0, ρ_1, and k with $\sigma_1,\ \rho_1 \neq 0$ gives ***Case 1.***

$$\sigma_0 = -\frac{(2n+1)b_1}{2(n+1)c_1},\quad \sigma_1 = \frac{(2n+1)b_1}{2(n+1)c_1},$$
$$\rho_0 = -\frac{(2n+1)b_2}{2(n+1)c_2},\quad \rho_1 = \frac{(2n+1)b_2}{2(n+1)c_2},$$
$$\omega = -\frac{1}{4}\frac{4c_s(n+1)^2(ap^2+\kappa_s) + (2n+1)b_s^2}{(n+1)^2c_s},$$
$$k = \pm\frac{nb_s}{(n+1)\ln(A)}\sqrt{-\frac{2n+1}{4c_sa}}. \tag{51}$$

Case 2.

$$\sigma_0 = 0,\quad \sigma_1 = \frac{(2n+1)b_1}{2(n+1)c_1},$$
$$\rho_0 = 0,\quad \rho_1 = \frac{(2n+1)b_2}{2(n+1)c_2},$$
$$\omega = -\frac{1}{4}\frac{4c_s(n+1)^2(ap^2+\kappa_s) + (2n+1)b_s^2}{(n+1)^2c_s},$$
$$k = \pm\frac{nb_s}{(n+1)\ln(A)}\sqrt{-\frac{2n+1}{4c_sa}}. \tag{52}$$

Substituting equations (51), (9), and (10) into equations (49) and (50), we obtain the following exact solutions to equations (43) and (44)

$$\Gamma_5(x,t) = \left(\frac{(2n+1)b_1}{2(n+1)c_1}\left[-1 + \frac{1}{1 + dA^{\pm\frac{nb_1}{(n+1)\ln(A)}\sqrt{-\frac{2n+1}{4c_1a}}(x+2apt)}}\right]\right)^{\frac{1}{2n}} \times \exp\left(i\left(-px - \frac{1}{4}\frac{4c_1(n+1)^2(ap^2+\kappa_1)+(2n+1)b_1^2}{(n+1)^2c_1}t + \theta_0\right)\right),$$

$$\Omega_5(x,t) = \left(\frac{(2n+1)b_2}{2(n+1)c_2}\left[-1 + \frac{1}{1 + dA^{\pm\frac{nb_2}{(n+1)\ln(A)}\sqrt{-\frac{2n+1}{4c_2a}}(x+2apt)}}\right]\right)^{\frac{1}{2n}} \times \exp\left(i\left(-px - \frac{1}{4}\frac{4c_2(n+1)^2(ap^2+\kappa_2)+(2n+1)b_2^2}{(n+1)^2c_2}t + \theta_0\right)\right).$$

Substituting equations (52), (9), and (10) into equations (49) and (50), we obtain the following exact solutions to equations (43) and (44)

$$\Gamma_6(x,t) = \left(-\frac{3b_1}{4c_1\left(1 + dA^{\pm\frac{nb_1}{(n+1)\ln(A)}\sqrt{-\frac{2n+1}{4c_1a}}(x+2apt)}\right)}\right)^{\frac{1}{2n}} \times \exp\left(i\left(-px - \frac{1}{4}\frac{4c_1(n+1)^2(ap^2+\kappa_1)+(2n+1)b_1^2}{(n+1)^2c_1}t + \theta_0\right)\right),$$

$$\Omega_6(x,t)=\left(-\frac{3b_2}{4c_2\left(1+dA^{\pm\frac{nb_2}{(n+1)\ln(A)}\sqrt{-\frac{2n+1}{4c_2a}}(x+2apt)}\right)}\right)^{\frac{1}{2n}}$$
$$\times\exp\left(i\left(-px-\frac{1}{4}\frac{4c_2(n+1)^2(ap^2+\kappa_2)+(2n+1)b_2^2}{(n+1)^2c_2}t+\theta_0\right)\right).$$

These solutions are valid for $a\,c_s<0,\ s=1,2$.

3.2. Multiple-core couplers (coupling with nearest neighbors)

Consider the following Multiple-core couplers in optical metamaterials [2, 12]

$$\begin{aligned}&i\Gamma_\tau^{(s)}+a_s\Gamma_{xx}^{(s)}+F_s(|\Gamma^{(s)}|^2)\Gamma^{(s)}\\&\quad=\xi_s(|\Gamma^{(s)}|^2\Gamma^{(s)})_{xx}+\eta_s|\Gamma^{(s)}|^2\Gamma_{xx}^{(s)}+\zeta_s\Gamma^{(s)2}\Gamma_{xx}^{(s)*}\\&\qquad+K[\Gamma^{(s-1)}-2\Gamma^{(s)}+\Gamma^{(s+1)}],\end{aligned}\tag{53}$$

where $1\leqslant s\leqslant M$ and K is the coupling coefficient. In order to handle the governing model, the following are our starting hypothesis

$$\Gamma^{(s)}(x,\tau)=U_s(\xi)e^{i\Phi(x,\tau)},\tag{54}$$

where $U_s(\xi)$ represents the shape of the pulse. The phase $\Phi(x,\tau)$ is expressed as

$$\Phi(x,\tau)=-px+\omega\tau+\theta_0.$$

The traveling coordinate ξ is given by

$$\xi=k(x-v\tau),$$

where v, p, ω, and θ_0 represent velocity, frequency, wave number, and phase constant of the soliton, respectively.

Substituting (54) into (53), the imaginary component offers

$$\begin{aligned}&-(vk+2a_spk)U_s'+2pk^2(3\xi_s+\eta_s-\zeta_s)U_s^2U_s''=0,\\&1\leqslant s\leqslant M.\end{aligned}\tag{55}$$

Setting the coefficients of the linearly independent functions, in (55) to zero, we obtain the speed of the soliton

$$v=-2a_sp,\tag{56}$$

and the constraint conditions

$$3\xi_s+\eta_s-\zeta_s=0.\tag{57}$$

The real parts imply

$$\begin{aligned}&ak^2U_s''-(\omega+a_sp^2)U_s+F_s(U_s^2)U_s\\&\quad+2(\zeta_s-\xi_s)p^2U_s^3-6\xi_sk^2U_s(U_s')^2\\&\quad-2k^2\zeta_sU_s^2U_s''-K(U_{s-1}-2U_s+U_{s+1})=0.\end{aligned}\tag{58}$$

Next, balancing principle leads to

$$U_{s-1}=U_s=U_{s+1}.\tag{59}$$

Consequently, equation (58) is

$$\begin{aligned}&ak^2U_s''-(\omega+a_sp^2)U_s+F_s(U_s^2)U_s+2(\zeta_s-\xi_s)p^2U_s^3\\&\quad-6\xi_sk^2U_s(U_s')^2-2k^2\zeta_sU_s^2U_s''=0.\end{aligned}\tag{60}$$

In the following subsections, this equation will be studied for four different types of nonlinearity.

3.2.1. Kerr law nonlinearity. In this case we have $F_s(\Delta)=b_s\Delta$ $(1\leqslant s\leqslant M)$ and inserting it into equation (53) yields

$$\begin{aligned}&i\Gamma_\tau^{(s)}+a_s\Gamma_{xx}^{(s)}+b_s|\Gamma^{(s)}|^2\Gamma^{(s)}\\&\quad=\xi_s(|\Gamma^{(s)}|^2\Gamma^{(s)})_{xx}+\eta_s|\Gamma^{(s)}|^2\Gamma_{xx}^{(s)}+\zeta_s\Gamma^{(s)2}\Gamma_{xx}^{(s)*}\\&\qquad+K[\Gamma^{(s-1)}-2\Gamma^{(s)}+\Gamma^{(s+1)}].\end{aligned}\tag{61}$$

Equation (60) takes the following form

$$\begin{aligned}&ak^2U_s''-(\omega+a_sp^2)U_s+(b_s+2(\zeta_s-\xi_s)p^2)U_s^3\\&\quad-6\xi_sk^2U_s(U_s')^2-2k^2\zeta_sU_s^2U_s''=0.\end{aligned}\tag{62}$$

To obtain an analytic solution, we apply the transformations $\zeta_s=\xi_s=0$ in equation (62) to find

$$ak^2U_s''-(\omega+a_sp^2)U_s+b_sU_s^3=0.\tag{63}$$

Balancing U_s'' and U_s^3 in equation (63) give

$$N+2=3N\Leftrightarrow N=1.$$

We seek solutions of the form

$$U_l(\zeta)=\sigma_0+\sigma_1\varphi(\zeta),\tag{64}$$

where σ_0 and σ_1 are constants to be determined.

Substituting the solution (64) into (63) gives the polynomial equation in $\varphi(\zeta)$. Thus, we find algebraic equations system by setting all the coefficients to zero. The solution of this system for σ_0, σ_1, and ω with $\sigma_1\neq0$ gives

$$\begin{aligned}&\sigma_0=\pm\frac{k}{2}\sqrt{-\frac{2a_s}{b_s}}\ln(A),\qquad\sigma_1=\pm k\sqrt{-\frac{2a_s}{b_s}}\ln(A),\\&\omega=-a_sp^2-\frac{1}{2}a_sk^2\ln^2(A).\end{aligned}\tag{65}$$

Substituting equations (65) and (54) into equation (64), we obtain the following exact solutions of equation (61)

$$\begin{aligned}&\Gamma_7^{(s)}(x,\tau)=\pm k\sqrt{-\frac{2a_s}{b_s}}\ln(A)\left[\frac{1}{2}+\frac{1}{1+dA^{k(x+2a_sp\tau)}}\right]\\&\times\exp\left(i\left(-px+\left(-a_sp^2-\frac{1}{2}a_sk^2\ln^2(A)\right)\tau+\theta_0\right)\right).\end{aligned}$$

These solutions are valid for $a_s\,b_s<0$, $1\leqslant s\leqslant M$.

3.2.2. Power law nonlinearity. In this case, $F_s(\Delta)=b_s\Delta^n$ $(1\leqslant s\leqslant M)$, ($n$ indicates the power law nonlinearity factor) and inserting it into equation (53) yields

$$\begin{aligned}&i\Gamma_\tau^{(s)}+a_s\Gamma_{xx}^{(s)}+b_s|\Gamma^{(s)}|^{2n}\Gamma^{(s)}\\&\quad=\xi_s(|\Gamma^{(s)}|^2\Gamma^{(s)})_{xx}+\eta_s|\Gamma^{(s)}|^2\Gamma_{xx}^{(s)}+\zeta_s\Gamma^{(s)2}\Gamma_{xx}^{(s)*}\\&\qquad+K[\Gamma^{(s-1)}-2\Gamma^{(s)}+\Gamma^{(s+1)}],\end{aligned}\tag{66}$$

Equation (60) can be written as

$$ak^2U_s'' - (\omega + a_sp^2)U_s + b_sU_s^{2n+1} + 2(\zeta_s - \xi_s)p^2U_s^3 - 6\xi_sk^2U_s(U_s')^2 - 2k^2\zeta_sU_s^2U_s'' = 0. \tag{67}$$

To obtain an analytic solution, we apply the transformations $\zeta_s = \xi_s = 0$, equation (67) to find

$$ak^2U_s'' - (\omega + a_sp^2)U_s + b_sU_s^{2n+1} = 0. \tag{68}$$

In order to obtain the closed form solutions, the following transformation

$$U_s = V_s^{\frac{1}{2n}}, \tag{69}$$

is applied. The above equation therefore becomes

$$ak^2(2nV_sV_s'' + (1 - 2n)(V_s')^2) - 4n^2(\omega + a_sp^2)V_s^2 + 4n^2b_sV_s^3 = 0. \tag{70}$$

Balancing V_s^3 with $(V_s')^2$ gives $N = 2$. We seek solutions of the form

$$U_l(\zeta) = \sigma_0 + \sigma_1\varphi(\zeta) + \sigma_2\varphi^2(\zeta), \tag{71}$$

where σ_0, σ_1, and σ_2 are constants to be determined.

Substituting the solution (71) into (68) gives the polynomial equation in $\varphi(\zeta)$. Thus, we find an algebraic equation system by equating all the coefficients to zero. The solution of this system for σ_0, σ_1, σ_2, and ω with $\sigma_2 \neq 0$ gives

$$\sigma_0 = 0, \quad \sigma_1 = \frac{a_sk^2(n + 1)\ln^2(A)}{n^2b_s}, \quad \sigma_2 = -\frac{a_sk^2(n + 1)\ln^2(A)}{n^2b_s}, \quad \omega = \frac{a_sk^2\ln^2(A) - 4n^2a_sp^2}{4n^2}. \tag{72}$$

After substituting equations (72) and (54) into equation (71), we obtain the following exact solutions of equation (66)

$$\Gamma_8^{(s)}(x, \tau) = \left[\frac{a_sk^2(n + 1)\ln^2(A)dA^{k(x+2a_sp\tau)}}{n^2b_s(1 + dA^{k(x+2a_sp\tau)})^2}\right]^{\frac{1}{2n}} \times \exp\left(i\left(-px + \left(\frac{a_sk^2\ln^2(A) - 4n^2a_sp^2}{4n^2}\right)\tau + \theta_0\right)\right).$$

3.2.3. Parabolic law nonlinearity. In this case, $F_s(\Delta) = b_s\Delta + c_s\Delta^2$ $(1 \leqslant s \leqslant M)$ and inserting it into equation (53) yields

$$i\Gamma_\tau^{(s)} + a_s\Gamma_{xx}^{(s)} + (b_s|\Gamma^{(s)}|^2 + c_s|\Gamma^{(s)}|^4)\Gamma^{(s)} = \xi_s(|\Gamma^{(s)}|^2\Gamma^{(s)})_{xx} + \eta_s|\Gamma^{(s)}|^2\Gamma_{xx}^{(s)} + \zeta_s\Gamma^{(s)2}\Gamma_{xx}^{(s)*} + K[\Gamma^{(s-1)} - 2\Gamma^{(s)} + \Gamma^{(s+1)}], \tag{73}$$

Equation (60) turns into

$$ak^2U_s'' - (\omega + a_sp^2)U_s + (b_s + 2(\zeta_s - \xi_s)p^2)U_s^3 + c_sU_s^5 - 6\xi_sk^2U_s(U_s')^2 - 2k^2\zeta_sU_s^2U_s'' = 0. \tag{74}$$

To obtain an analytic solution, we apply the transformations $\zeta_s = \xi_s = 0$ in equation (74) to find

$$ak^2U_s'' - (\omega + a_sp^2)U_s + b_sU_s^3 + c_sU_s^5 = 0. \tag{75}$$

In order to obtain the closed form solutions, the following transformation

$$U_s = V_s^{\frac{1}{2}}, \tag{76}$$

is applied. The above equation therefore develops into

$$ak^2(2V_sV_s'' - (V_s')^2) - 4(\omega + a_sp^2)V_s^2 + 4b_sV_s^3 + 4c_sV_s^4 = 0. \tag{77}$$

Balancing V_s^4 with V_sV_s'' gives $N = 1$. We seek solutions of the form

$$U_1(\zeta) = \sigma_0 + \sigma_1\varphi(\zeta), \tag{78}$$

where σ_0 and σ_1 are constants to be determined.

Substituting the solution (78) into (75) gives the polynomial equation in $\varphi(\zeta)$. Thus, we find an algebraic equation system by equating all the coefficients to zero. The solution of this system for σ_0, σ_1, ω, and k with $\sigma_1 \neq 0$ gives

Case 1.

$$\sigma_0 = -\frac{3}{4}\frac{b_s}{c_s}, \quad \sigma_1 = \frac{3}{4}\frac{b_s}{c_s}, \quad \omega = -\frac{1}{16}\frac{16c_sa_sp^2 + 3b_s^2}{c_s}, \quad k = \pm\frac{b_s}{\ln(A)}\sqrt{-\frac{3}{4c_sa_s}}. \tag{79}$$

Case 2.

$$\sigma_0 = 0, \quad \sigma_1 = -\frac{3}{4}\frac{b_s}{c_s}, \quad \omega = -\frac{1}{16}\frac{16c_sa_sp^2 + 3b_s^2}{c_s}, \quad k = \pm\frac{b_s}{\ln(A)}\sqrt{-\frac{3}{4c_sa_s}}. \tag{80}$$

Substituting equations (79) and (54) into equation (78), we obtain the following exact solutions of equation (73)

$$\Gamma_9^{(s)}(x, \tau) = \left(\frac{3}{4}\frac{b_s}{c_s}\left[-1 + \frac{1}{1 + dA^{\pm\frac{b_s}{\ln(A)}\sqrt{-\frac{3}{4c_sa_s}}(x+2a_sp\tau)}}\right]\right)^{\frac{1}{2}} \times \exp\left(i\left(-px - \frac{1}{16}\frac{16c_sa_sp^2 + 3b_s^2}{c_s}\tau + \theta_0\right)\right).$$

Substituting equations (80) and (54) into equation (78), we obtain the following exact solutions of equation (73)

$$\Gamma_{10}^{(s)}(x,\tau)=\left(-\frac{3b_s}{4c_s\left(1+dA^{\pm\frac{b_s}{\ln(A)}\sqrt{-\frac{3}{4c_sa_s}}(x+2a_sp\tau)}\right)}\right)^{\frac{1}{2}}\times\exp\left(i\left(-px-\frac{1}{16}\frac{16c_sa_sp^2+3b_s^2}{c_s}\tau+\theta_0\right)\right).$$

These solutions are valid when $a_s\,c_s<0$, $1\leqslant s\leqslant M$.

3.2.4. Dual-power law nonlinearity. In this case, $F_s(\Delta)=b_s\Delta^n+c_s\Delta^{2n}$ $(1\leqslant s\leqslant M)$ and inserting it into equation (53) yields

$$\begin{aligned}&i\Gamma_\tau^{(s)}+a_s\Gamma_{xx}^{(s)}+(b_s\,|\Gamma^{(s)}|^{2n}+c_s\,|\Gamma^{(s)}|^{4n})\Gamma^{(s)}\\&=\xi_s(|\Gamma^{(s)}|^2\Gamma^{(s)})_{xx}+\eta_s\,|\Gamma^{(s)}|^2\Gamma_{xx}^{(s)}\\&+\zeta_s\Gamma^{(s)2}\Gamma_{xx}^{(s)*}+K[\Gamma^{(s-1)}-2\Gamma^{(s)}+\Gamma^{(s+1)}],\end{aligned}\tag{81}$$

Equation (60) takes the following structure

$$\begin{aligned}&ak^2U_s''-(\omega+a_sp^2)U_s\\&+b_sU_s^{2n+1}+2(\zeta_s-\xi_s)p^2U_s^3+c_sU_s^{4n+1}\\&-6\xi_sk^2U_s(U_s')^2-2k^2\zeta_sU_s^2U_s''=0.\end{aligned}\tag{82}$$

To obtain an analytic solution, we apply the transformations $\zeta_s=\xi_s=0$ in equation (82) to find

$$\begin{aligned}&ak^2U_s''-(\omega+a_sp^2)U_s\\&+b_sU_s^{2n+1}+c_sU_s^{4n+1}=0.\end{aligned}\tag{83}$$

In order to obtain the closed form solutions, the following transformation

$$U_s=V_s^{\frac{1}{2n}},\tag{84}$$

is applied. The above equation therefore is written as

$$\begin{aligned}&ak^2(2nV_sV_s''+(1-2n)(V_s')^2)\\&-4n^2(\omega+a_sp^2)V_s^2+4n^2b_sV_s^3+4n^2c_sV_s^4=0.\end{aligned}\tag{85}$$

Balancing V_s^4 with V_sV_s'' gives $N=1$. We seek solutions of the form

$$U_l(\zeta)=\sigma_0+\sigma_1\varphi(\zeta),\tag{86}$$

where σ_0 and σ_1 are constants to be determined.

Substituting the solution (86) into (83) gives the polynomial equation in $\varphi(\zeta)$. Thus, we find an algebraic equation system by equating all the coefficients to zero. The solution of this system for σ_0, σ_1, ω, and k with $\sigma_1\neq 0$ gives

Case 1.

$$\begin{aligned}&\sigma_0=-\frac{(2n+1)b_s}{2(n+1)c_s},\qquad\sigma_1=\frac{(2n+1)b_s}{2(n+1)c_s},\\&\omega=-\frac{1}{4}\frac{4c_s(n+1)^2a_sp^2+(2n+1)b_s^2}{(n+1)^2c_s},\\&k=\pm\frac{nb_s}{(n+1)\ln(A)}\sqrt{-\frac{2n+1}{4c_sa_s}}.\end{aligned}\tag{87}$$

Case 2.

$$\begin{aligned}&\sigma_0=0,\qquad\sigma_1=\frac{(2n+1)b_s}{2(n+1)c_s},\\&\omega=-\frac{1}{4}\frac{4c_s(n+1)^2a_sp^2+(2n+1)b_s^2}{(n+1)^2c_s},\\&k=\pm\frac{nb_s}{(n+1)\ln(A)}\sqrt{-\frac{2n+1}{4c_sa_s}}.\end{aligned}\tag{88}$$

Substituting equations (87) and (54) into equation (86), we obtain the exact solutions of equation (81)

$$\Gamma_{11}^{(s)}(x,\tau)=\left(\frac{(2n+1)b_s}{2(n+1)c_s}\left[-1+\frac{1}{1+dA^{\pm\frac{nb_s}{(n+1)\ln(A)}\sqrt{-\frac{2n+1}{4c_sa_s}}(x+2a_sp\tau)}}\right]\right)^{\frac{1}{2n}}\times\exp\left(i\left(-px-\frac{1}{4}\frac{4c_s(n+1)^2a_sp^2+(2n+1)b_s^2}{(n+1)^2c_s}\tau+\theta_0\right)\right).$$

Substituting equations (88) and (54) into equation (86), we obtain the exact solutions of equation (81)

$$\Gamma_{12}^{(s)}(x,\tau)=\left(-\frac{3b_s}{4c_s\left(1+dA^{\pm\frac{nb_s}{(n+1)\ln(A)}\sqrt{-\frac{2n+1}{4c_sa_s}}(x+2a_sp\tau)}\right)}\right)^{\frac{1}{2n}}\times\exp\left(i\left(-px-\frac{1}{4}\frac{4c_s(n+1)^2a_sp^2+(2n+1)b_s^2}{(n+1)^2c_s}\tau+\theta_0\right)\right).$$

These solutions are valid for $a_s\,c_s>0$, $1\leqslant s\leqslant M$.

3.3. Multiple-core couplers (coupling with all neighbors)

The governing system of equations for multiple-core couplers, where the coupling action is with all the existing neighbors, is given by [2, 12]

$$\begin{aligned}&i\Gamma_\tau^{(s)}+a_s\Gamma_{xx}^{(s)}+F_s(|\Gamma^{(s)}|^2)\Gamma^{(s)}=\xi_s(|\Gamma^{(s)}|^2\Gamma^{(s)})_{xx}\\&+\eta_s\,|\Gamma^{(s)}|^2\Gamma_{xx}^{(s)}+\zeta_s\Gamma^{(s)2}\Gamma_{xx}^{(s)*}+\sum_{m=1}^{M}\lambda_{sm}\Gamma^{(m)},\end{aligned}\tag{89}$$

where $1\leqslant s\leqslant M$. Here, λ_{sm} represents the coupling coefficient with all neighbors. In order to handle the governing model, the following are our starting hypothesis

$$\Gamma^{(s)}(x,\tau)=U_s(\xi)e^{i\Phi(x,\tau)},\tag{90}$$

where $U_s(\xi)$ represents the shape of the pulse. The phase $\Phi(x,\tau)$ is

$$\Phi(x,\tau)=-px+\omega\tau+\theta_0.$$

The traveling coordinate ξ is given by

$$\xi=k(x-v\tau),$$

where v, p, ω, and θ_0 represent the velocity, frequency, wave number, and phase constant of the soliton, respectively.

Substituting (90) into (89), the imaginary component offers

$$-(vk + 2a_s pk)U_s' + 2pk^2(3\xi_s + \eta_s - \zeta_s)U_s^2 U_s'' = 0, \quad 1 \leqslant s \leqslant M. \tag{91}$$

Setting the coefficients of the linearly independent functions in (91) to zero is possible to find the speed of the soliton

$$v = -2a_s p \tag{92}$$

and the constraint conditions

$$3\xi_s + \eta_s - \zeta_s = 0. \tag{93}$$

The real parts imply

$$\begin{aligned} &ak^2U_s'' - (\omega + a_s p^2)U_s + F_s(U_s^2)U_s \\ &+ 2(\zeta_s - \xi_s)p^2U_s^3 - 6\xi_s k^2 U_s(U_s')^2 - 2k^2\zeta_s U_s^2 U_s'' \\ &- \sum_{m=1}^{M} \lambda_{sm} U_m = 0. \end{aligned} \tag{94}$$

Next, balancing principle leads to

$$U_s = U_m. \tag{95}$$

Consequently, equation (94) is

$$\begin{aligned} &ak^2U_s'' - \left(\omega + \sum_{m=1}^{M} \lambda_{sm} + a_s p^2\right)U_s + F_s(U_s^2)U_s \\ &+ 2(\zeta_s - \xi_s)p^2U_s^3 - 6\xi_s k^2 U_s(U_s')^2 \\ &- 2k^2\zeta_s U_s^2 U_s'' = 0. \end{aligned} \tag{96}$$

In the following subsections, this equation will be studied for four different types of nonlinearity.

3.3.1. Kerr law nonlinearity. In this case, $F_s(\Delta) = b_s\Delta$ $(1 \leqslant s \leqslant M)$ and inserting it into equation (89) yields

$$\begin{aligned} &i\Gamma_\tau^{(s)} + a_s\Gamma_{xx}^{(s)} + b_s\,|\Gamma^{(s)}|^2\Gamma^{(s)} = \xi_s(|\Gamma^{(s)}|^2\Gamma^{(s)})_{xx} \\ &+ \eta_s\,|\Gamma^{(s)}|^2\Gamma_{xx}^{(s)} + \zeta_s\Gamma^{(s)2}\Gamma_{xx}^{(s)*} + \textstyle\sum_{m=1}^{M}\lambda_{sm}\Gamma^{(m)}, \end{aligned} \tag{97}$$

Equation (96) gives

$$\begin{aligned} &ak^2U_s'' - \left(\omega + \sum_{m=1}^{M} \lambda_{sm} + a_s p^2\right)U_s + (b_s + 2(\zeta_s - \xi_s)p^2) \\ &\times U_s^3 - 6\xi_s k^2 U_s(U_s')^2 - 2k^2\zeta_s U_s^2 U_s'' = 0. \end{aligned} \tag{98}$$

To obtain an analytic solution, we apply the transformations $\zeta_s = \xi_s = 0$ in equation (98) to find

$$ak^2U_s'' - \left(\omega + \sum_{m=1}^{M} \lambda_{sm} + a_s p^2\right)U_s + b_s U_s^3 = 0. \tag{99}$$

Balancing U_s'' with U_s^3 in equation (104) gives

$$N + 2 = 3N \Leftrightarrow N = 1.$$

We seek solutions of the form

$$U_l(\zeta) = \sigma_0 + \sigma_1\varphi(\zeta), \tag{100}$$

where σ_0 and σ_1 are constants to be determined.

Substituting the solution (100) into (99) gives the polynomial equation in $\varphi(\zeta)$. Thus, we find an algebraic equation system by equating all the coefficients to zero. The solution of this system for σ_0, σ_1, and ω with $\sigma_1 \neq 0$ gives

$$\begin{aligned} &\sigma_0 = \pm\frac{k}{2}\sqrt{-\frac{2a_s}{b_s}}\ln(A), \qquad \sigma_1 = \pm k\sqrt{-\frac{2a_s}{b_s}}\ln(A), \\ &\omega = -a_s p^2 - \sum_{m=1}^{M} \lambda_{sm} - \frac{1}{2}a_s k^2 \ln^2(A). \end{aligned} \tag{101}$$

Substituting equations (101) and (90) into equation (100), leads to the exact solutions of equation (97)

$$\begin{aligned} &\Gamma_{13}^{(s)}(x, \tau) = \pm k\sqrt{-\frac{2a_s}{b_s}}\ln(A)\left[\frac{1}{2} + \frac{1}{1 + dA^{k(x+2a_s p\tau)}}\right] \\ &\times \exp\left(i\left(-px + \left(-a_s p^2 - \sum_{m=1}^{M} \lambda_{sm} - \frac{1}{2}a_s k^2 \ln^2(A)\right)\tau + \theta_0\right)\right). \end{aligned}$$

These solutions are valid when $a_s\, b_s < 0$, $1 \leqslant s \leqslant M$.

3.3.2. Power law nonlinearity. In this case, $F_s(\Delta) = b_s\Delta^n$ $(1 \leqslant s \leqslant M)$, ($n$ indicates the power law nonlinearity factor) and inserting it into equation (89) yields

$$\begin{aligned} &i\Gamma_\tau^{(s)} + a_s\Gamma_{xx}^{(s)} + b_s\,|\Gamma^{(s)}|^{2n}\Gamma^{(s)} = \xi_s(|\Gamma^{(s)}|^2\Gamma^{(s)})_{xx} \\ &+ \eta_s\,|\Gamma^{(s)}|^2\Gamma_{xx}^{(s)} + \zeta_s\Gamma^{(s)2}\Gamma_{xx}^{(s)*} + \sum_{m=1}^{M}\lambda_{sm}\Gamma^{(m)}. \end{aligned} \tag{102}$$

Equation (96) results in the expression:

$$\begin{aligned} &ak^2U_s'' - \left(\omega + \sum_{m=1}^{M} \lambda_{sm} + a_s p^2\right)U_s + b_s U_s^{2n+1} \\ &+ 2(\zeta_s - \xi_s)p^2U_s^3 - 6\xi_s k^2 U_s(U_s')^2 \\ &- 2k^2\zeta_s U_s^2 U_s'' = 0. \end{aligned} \tag{103}$$

To obtain an analytic solution, we apply the transformations $\zeta_s = \xi_s = 0$ in equation (103) to find

$$ak^2U_s'' - \left(\omega + \sum_{m=1}^{M} \lambda_{sm} + a_s p^2\right)U_s + b_s U_s^{2n+1} = 0. \tag{104}$$

In order to obtain the closed form solutions, the following transformation

$$U_s = V_s^{\frac{1}{2n}}, \tag{105}$$

is applied. The above equation therefore shapes up as

$$\begin{aligned} &ak^2(2nV_sV_s'' + (1 - 2n)(V_s')^2) \\ &- 4n^2\left(\omega + \sum_{m=1}^{M} \lambda_{sm} + a_s p^2\right)V_s^2 + 4n^2 b_s V_s^3 = 0. \end{aligned} \tag{106}$$

Balancing V_s^3 with $(V_s{}')^2$ gives $N = 2$. We seek solutions of the form

$$U_l(\zeta) = \sigma_0 + \sigma_1\varphi(\zeta) + \sigma_2\varphi^2(\zeta), \tag{107}$$

where σ_0, σ_1, and σ_2 are constants to be determined.

Substituting the solution (107) into (104) gives the polynomial equation in $\varphi(\zeta)$. Thus, we find an algebraic equation system by equating all the coefficients to zero. The solution of this system for σ_0, σ_1, σ_2, and ω with $\sigma_2 \neq 0$ gives

$$\sigma_0 = 0, \qquad \sigma_1 = \frac{a_s k^2 (n+1)\ln^2(A)}{n^2 b_s},$$
$$\sigma_2 = -\frac{a_s k^2 (n+1)\ln^2(A)}{n^2 b_s},$$
$$\omega = \frac{a_s k^2 \ln^2(A) - 4n^2\left(a_s p^2 + \sum_{m=1}^{M}\lambda_{sm}\right)}{4n^2}. \tag{108}$$

Substituting equations (108) and (90) into equation (107), we obtain the exact solutions of equation (102)

$$\Gamma_{14}^{(s)}(x,\tau) = \left[\frac{a_s k^2 (n+1)\ln^2(A) dA^{k(x+2a_s p\tau)}}{n^2 b_s (1 + dA^{k(x+2a_s p\tau)})^2}\right]^{\frac{1}{2n}}$$
$$\times \exp\left(i\left(-px + \left(\frac{a_s k^2 \ln^2(A) - 4n^2\left(a_s p^2 + \sum_{m=1}^{M}\lambda_{sm}\right)}{4n^2}\right)\tau + \theta_0\right)\right).$$

3.3.3. Parabolic law nonlinearity. In this case, $F_s(\Delta) = b_s\Delta + c_s\Delta^2$ $(1 \leqslant s \leqslant M)$ and inserting it into equation (89) yields

$$i\Gamma_\tau^{(s)} + a_s\Gamma_{xx}^{(s)} + (b_s|\Gamma^{(s)}|^2 + c_s|\Gamma^{(s)}|^4)\Gamma^{(s)}$$
$$= \xi_s(|\Gamma^{(s)}|^2\Gamma^{(s)})_{xx} + \eta_s|\Gamma^{(s)}|^2\Gamma_{xx}^{(s)} + \zeta_s\Gamma^{(s)2}\Gamma_{xx}^{(s)*}$$
$$+ \sum_{m=1}^{M}\lambda_{sm}\Gamma^{(m)}. \tag{109}$$

Equation (96) takes the following form

$$ak^2U_s'' - \left(\omega + \sum_{m=1}^{M}\lambda_{sm} + a_s p^2\right)U_s$$
$$+ (b_s + 2(\zeta_s - \xi_s)p^2)U_s^3 + c_sU_s^5 - 6\xi_s k^2 U_s(U_s')^2$$
$$- 2k^2\zeta_s U_s^2 U_s'' = 0. \tag{110}$$

To obtain an analytic solution, we apply the transformations $\zeta_s = \xi_s = 0$ in equation (110) to find

$$ak^2U_s'' - \left(\omega + \sum_{m=1}^{M}\lambda_{sm} + a_s p^2\right)U_s + b_sU_s^3 + c_sU_s^5 = 0. \tag{111}$$

In order to obtain the closed form solutions, the following transformation

$$U_s = V_s^{\frac{1}{2}} \tag{112}$$

is applied. The above equation therefore can be written as

$$ak^2(2V_sV_s'' - (V_s')^2) - 4\left(\omega + \sum_{m=1}^{M}\lambda_{sm} + a_s p^2\right)V_s^2$$
$$+ 4b_sV_s^3 + 4c_sV_s^4 = 0. \tag{113}$$

Balancing V_s^4 with V_sV_s'' gives $N = 1$. We seek solutions of the form

$$U_1(\zeta) = \sigma_0 + \sigma_1\varphi(\zeta), \tag{114}$$

where σ_0 and σ_1 are constants to be determined.

Substituting the solution (114) into (111) gives the polynomial equation in $\varphi(\zeta)$. Thus, we find an algebraic equation system by equating all the coefficients to zero. The solution of this system for σ_0, σ_1, ω and k with $\sigma_1 \neq 0$ gives

Case 1.

$$\sigma_0 = -\frac{3}{4}\frac{b_s}{c_s}, \qquad \sigma_1 = \frac{3}{4}\frac{b_s}{c_s},$$
$$\omega = -\frac{1}{16}\frac{16c_s\left(a_s p^2 + \sum_{m=1}^{M}\lambda_{sm}\right) + 3b_s^2}{c_s},$$
$$k = \pm\frac{b_s}{\ln(A)}\sqrt{-\frac{3}{4c_s a_s}}. \tag{115}$$

Case 2.

$$\sigma_0 = 0, \qquad \sigma_1 = -\frac{3}{4}\frac{b_s}{c_s},$$
$$\omega = -\frac{1}{16}\frac{16c_s\left(a_s p^2 + \sum_{m=1}^{M}\lambda_{sm}\right) + 3b_s^2}{c_s},$$
$$k = \pm\frac{b_s}{\ln(A)}\sqrt{-\frac{3}{4c_s a_s}}. \tag{116}$$

Substituting equations (115) and (90) into equation (114), we obtain the exact solutions of equation (109)

$$\Gamma_{15}^{(s)}(x,\tau) = \left(\frac{3}{4}\frac{b_s}{c_s}\left[-1 + \frac{1}{1 + dA^{\pm\frac{b_s}{\ln(A)}\sqrt{-\frac{3}{4c_s a_s}}(x+2a_s p\tau)}}\right]\right)^{\frac{1}{2}}$$
$$\times \exp\left(i\left(-px - \frac{1}{16}\frac{16c_s\left(a_s p^2 + \sum_{m=1}^{M}\lambda_{sm}\right) + 3b_s^2}{c_s}\tau + \theta_0\right)\right).$$

Substituting equations (116) and (90) into equation (114), we obtain the exact solutions of equation (109)

$$\Gamma_{16}^{(s)}(x,\tau) = \left(-\frac{3b_s}{4c_s\left(1 + dA^{\pm\frac{b_s}{\ln(A)}\sqrt{-\frac{3}{4c_s a_s}}(x+2a_s p\tau)}\right)}\right)^{\frac{1}{2}}$$
$$\times \exp\left(i\left(-px - \frac{1}{16}\frac{16c_s\left(a_s p^2 + \sum_{m=1}^{M}\lambda_{sm}\right) + 3b_s^2}{c_s}\tau + \theta_0\right)\right).$$

These solutions are valid when $a_s\, c_s < 0$, $1 \leqslant s \leqslant M$.

3.3.4. Dual-power law nonlinearity. In this case, $F_s(\Delta) = b_s\Delta^n + c_s\Delta^{2n}$ $(1 \leqslant s \leqslant M)$ and inserting it into equation (89) yields

$$i\Gamma_\tau^{(s)} + a_s\Gamma_{xx}^{(s)} + (b_s\,|\Gamma^{(s)}|^{2n} + c_s\,|\Gamma^{(s)}|^{4n})\Gamma^{(s)} = \xi_s(|\Gamma^{(s)}|^2\Gamma^{(s)})_{xx} + \eta_s\,|\Gamma^{(s)}|^2\Gamma_{xx}^{(s)} + \zeta_s\Gamma^{(s)2}\Gamma_{xx}^{(s)*} + \sum_{m=1}^{M}\lambda_{sm}\Gamma^{(m)}, \tag{117}$$

Equation (96) results in

$$ak^2U_s'' - \left(\omega + \sum_{m=1}^{M}\lambda_{sm} + a_sp^2\right)U_s + b_sU_s^{2n+1} + 2(\zeta_s - \xi_s)p^2U_s^3 + c_sU_s^{4n+1} - 6\xi_sk^2U_s(U_s')^2 - 2k^2\zeta_sU_s^2U_s'' = 0. \tag{118}$$

To obtain an analytic solution, we apply the transformations $\zeta_s = \xi_s = 0$ in equation (118) to find

$$ak^2U_s'' - \left(\omega + \sum_{m=1}^{M}\lambda_{sm} + a_sp^2\right)U_s + b_sU_s^{2n+1} + c_sU_s^{4n+1} = 0. \tag{119}$$

In order to obtain the closed form solutions, the following transformation

$$U_s = V_s^{\frac{1}{2n}} \tag{120}$$

is applied. The above equation gives

$$ak^2(2nV_sV_s'' + (1 - 2n)(V_s')^2) - 4n^2\left(\omega + \sum_{m=1}^{M}\lambda_{sm} + a_sp^2\right) \times V_s^2 + 4n^2b_sV_s^3 + 4n^2c_sV_s^4 = 0. \tag{121}$$

Balancing V_s^4 with V_sV_s'' gives $N = 1$. We seek solutions of the form

$$U_l(\zeta) = \sigma_0 + \sigma_1\varphi(\zeta), \tag{122}$$

where σ_0 and σ_1 are constants to be determined.

Substituting the solution (122) into (119) gives the polynomial equation in $\varphi(\zeta)$. Thus, we find an algebraic equation system by equating all the coefficients to zero. The solution of this system for σ_0, σ_1, ω, and k with $\sigma_1 \neq 0$ gives

Case 1.

$$\sigma_0 = -\frac{(2n+1)b_s}{2(n+1)c_s}, \quad \sigma_1 = \frac{(2n+1)b_s}{2(n+1)c_s}, \quad \omega = -\frac{1}{4}\frac{4c_s(n+1)^2\left(a_sp^2 + \sum_{m=1}^{M}\lambda_{sm}\right) + (2n+1)b_s^2}{(n+1)^2c_s}, \quad k = \pm\frac{nb_s}{(n+1)\ln(A)}\sqrt{-\frac{2n+1}{4c_sa_s}}. \tag{123}$$

Case 2.

$$\sigma_0 = 0, \quad \sigma_1 = \frac{(2n+1)b_s}{2(n+1)c_s}, \quad \omega = -\frac{1}{4}\frac{4c_s(n+1)^2\left(a_sp^2 + \sum_{m=1}^{M}\lambda_{sm}\right) + (2n+1)b_s^2}{(n+1)^2c_s}, \quad k = \pm\frac{nb_s}{(n+1)\ln(A)}\sqrt{-\frac{2n+1}{4c_sa_s}}. \tag{124}$$

Substituting equations (123) and (90) into equation (122), we obtain the exact solutions of equation (117)

$$\Gamma_{16}^{(s)}(x,\tau) = \left(\frac{(2n+1)b_s}{2(n+1)c_s}\left[-1 + \frac{1}{1 + dA^{\pm\frac{nb_s}{(n+1)\ln(A)}\sqrt{-\frac{2n+1}{4c_sa_s}}(x+2a_sp\tau)}}\right]\right)^{\frac{1}{2n}} \times \exp\left(i\left(-px - \frac{1}{4}\frac{4c_s(n+1)^2\left(a_sp^2 + \sum_{m=1}^{M}\lambda_{sm}\right) + (2n+1)b_s^2}{(n+1)^2c_s}\tau + \theta_0\right)\right).$$

Substituting equations (124) and (90) into equation (122), we obtain the exact solutions of equation (117)

$$\Gamma_{17}^{(s)}(x,\tau) = \left(-\frac{3b_s}{4c_s\left(1 + dA^{\pm\frac{nb_s}{(n+1)\ln(A)}\sqrt{-\frac{2n+1}{4c_sa_s}}(x+2a_sp\tau)}\right)}\right)^{\frac{1}{2n}} \times \exp\left(i\left(-px - \frac{1}{4}\frac{4c_s(n+1)^2\left(a_sp^2 + \sum_{m=1}^{M}\lambda_{sm}\right) + (2n+1)b_s^2}{(n+1)^2c_s}\tau + \theta_0\right)\right).$$

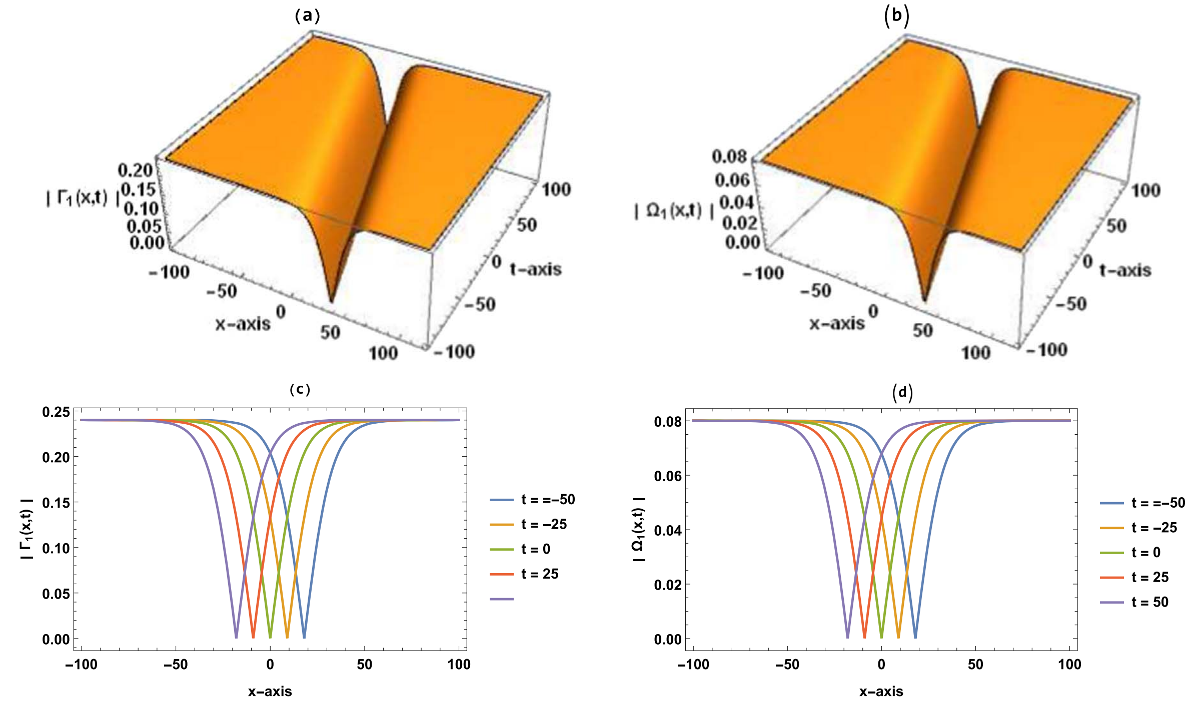

Figure 1. (a)–(d) The absolute values for the wave solutions $\Gamma_1(x, t)$ and $\Omega_1(x, t)$ in 3D- and 2D-plots when $A = 0.5$, $p = 0.3$, $b_1 = 0.1$, $b_2 = 0.9$, $k = 0.2$, $a = 0.6$ and $d = 1$.

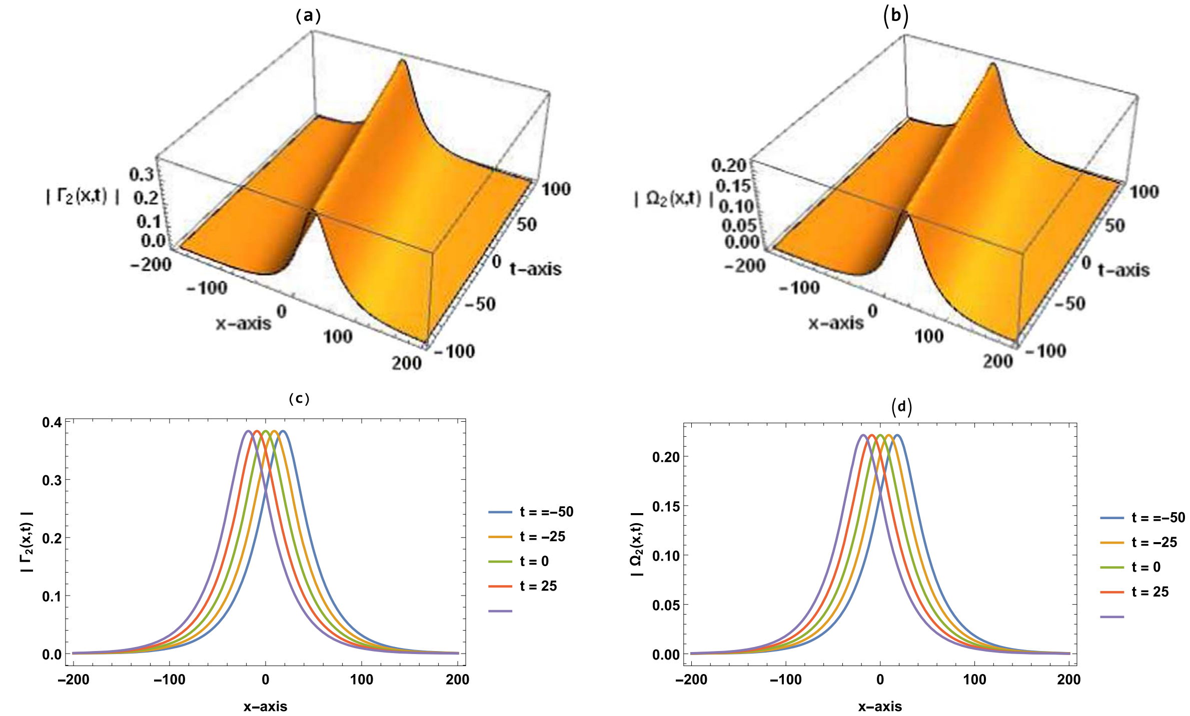

Figure 2. (a)–(d) The absolute values for the wave solutions $\Gamma_2(x, t)$ and $\Omega_2(x, t)$ in 3D- and 2D-plots when $A = 0.5$, $p = 0.3$, $b_1 = 0.1$, $b_2 = 0.9$, $k = 0.2$, $a = 0.6$, $d = 1$ and $n = 2$.

These solutions are valid when $a_s c_s > 0, 1 \leqslant s \leqslant M$.

4. Graphical description of the obtained solutions

In this section, the physical descriptions for some of the acquired solutions are explained graphically. The obtained solutions of the equations represent bright, dark and kink shaped solitons. However, a few numbers of the representative solutions are explained for the sake of straightforwardness. The 3D and 2D graphical illustrations of the explored solutions to three types of nonlinear directional couplers equation are presented to illustrate the solution's behavior by

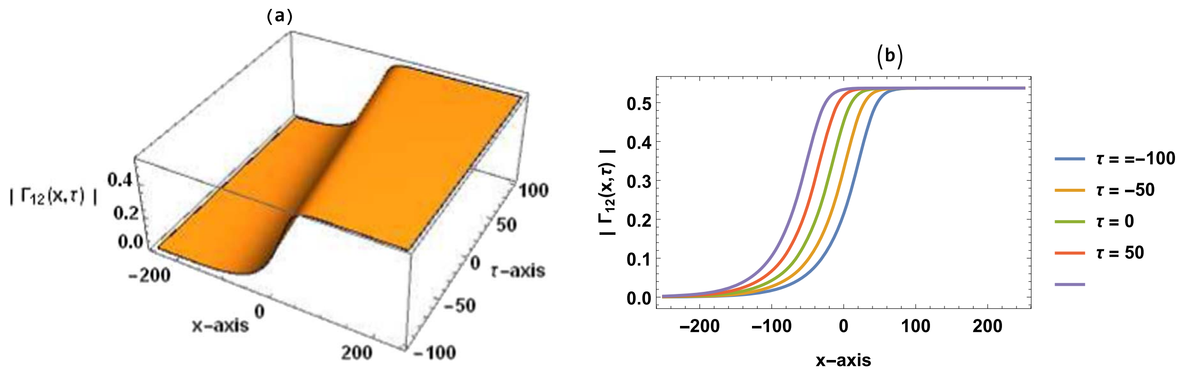

Figure 3. (a)–(b) The absolute value for the wave solution $\Gamma_{12}(x, t)$ in 3D- and 2D-plots when $A = 0.5$, $p = 0.3$, $a_s = -0.6$, $b_s = -0.1$, $c_s = 0.9$, $d = 1$ and $n = 2$.

considering particular values of the free parameters in these solutions. These graphics are visualized to display the spatiotemporal distribution of the obtained solutions.

Figures 1(a)–(d) represents the absolute values of the wave solutions $\Gamma_1(x, t)$ and $\Omega_1(x, t)$ in 3D- and 2D-plots which are bright solitons (the anti-bell shape soliton which rises from both sides) and they are stable. These waves are symmetric about the vertical axis that passing through the origin and propagated along the x-axis.

Figures 2(a)–(d) represents the absolute values of the wave solutions $\Gamma_2(x, t)$ and $\Omega_2(x, t)$ in 3D- and 2D-plots which are dark solitons (the bell shape soliton which is characterized by infinite tails or infinite wings) and they are also stable and symmetric about the vertical axis that passing through the origin and propagated along the x-axis.

Figures 3(a)–(b) represents the absolute value of the wave solution $\Gamma_{12}(x, t)$ in 3D- and 2D-plots which is stable kink wave that rises from left to right. The same discussion can be presented here as in figures 1 and 2, but the wave is not symmetric.

5. Conclusion

This paper investigates the traveling wave solution to nonlinear directional couplers in optical metamaterials. The modified Kudryashov method is applied to retrieve dark, singular and periodic soliton solutions. In future, the results will be extended with DWDM topology, also the phenomenon of birefringence in birefringent fibers that is described with this model will be explored. The 3D and 2D graphics for different values of the free parameter are represented to understand the physical meaning over the solutions. In comparison with previously attained solutions, the generated dark, bright and kink wave solutions are new in applied method senses and were not reported in previously published articles. The results are useful in telecommunication industry to enhance the performance capacity of transmission systems.

Conflict of interest.

This work does not have any conflicts of interest.

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