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Supplementary material for this article is available [online](#)

Abstract

Hawking's black hole evaporation process suggests that we may need to choose between quantum unitarity and other basic physical principles such as no-signaling, entanglement monogamy, and the equivalence principle. We here show that the Hawking's quantum model for the black hole evaporation is consistent with the above fundamental principles. Our analysis does not involve exotic new physics, but rather uses standard quantum theory, general relativity, and the Einstein–Hilbert action including matter. We explicitly show that the whole state consisting of matter and radiation (in a joint superposition of different energy states) is pure at any stage of the evaporation process, including the particular case of 0 mass. Moreover, after full evaporation the state for radiation at infinity is pure and in one-to-one correspondence with the initial state forming the black hole. Thus there is no information loss upon full evaporation according to the quantum information theory. The original entanglement of the black hole matter (if any) gets transferred to the outgoing particles via a process similar to entanglement swapping, without violation of causality (as proved explicitly). On the other hand, if the initial state is a tensor product state, the entanglement of Hawking particles, present in the intermediate phase, is broken when the black hole evaporates completely. Therefore, the final state (entangled or tensor product depending on the nature of initial state) after the full black hole evaporation is pure without loss of information.

1. Introduction

Hawking's model for black hole evaporation implies a many-to-one mapping of initial pure states to a more random mixed state [1, 2]. This has been dubbed a 'loss of predictability'. In Hawking's model, pairs of particles are created from the vacuum near the event horizon: one of these (having negative energy⁷) falls into the black hole and the other flies away to future infinity (I^+). The particle of negative energy falling towards the black hole will eventually meet the black hole's matter and annihilate, causing the black hole mass to decrease [1–5]. As time passes, increasingly more particles are annihilated and the black hole eventually evaporates. The particle pairs created at the event horizon are, before the annihilation, in the following state [5],

⁷ Inside a Schwarzschild black hole, the time coordinate and the spatial radial coordinate interchange their roles. Therefore, the energy and momentum change their roles, too, allowing for a well-defined negative energy (actually, spatial momentum) inside the black hole. This fact is at the heart of the Hawking process for black holes' evaporation.

$$|\Psi\rangle = \bigotimes_{\omega>0} c_{\omega} \sum_{N_{\omega}=0} e^{-\frac{N_{\omega}\omega}{\kappa}} |N_{\omega}\rangle^{\text{out}} \otimes |N_{\omega}\rangle^{\text{int}}, \quad (1)$$

where $c_{\omega} \equiv \sqrt{1 - e^{-2\pi\omega/\kappa}}$ is a normalization factor, κ is the surface gravity at the event horizon (inversely proportional to the black-hole mass), N_{ω} is the number of particle pairs of energy ω , while ‘int’ and ‘out’ label the Hilbert spaces for the particles falling inside the black hole and those escaping to the future infinity, respectively [5]. The state $|\Psi\rangle$ of equation (1) is pure with the ‘int’ modes inside the black hole being correlated with the ‘out’ modes. After the annihilation, the ‘int’ modes and the black hole are in a vacuum state, and the ‘out’ modes are the non-trivial remnants. Thus according to this model of evaporation the black hole’s initial state is finally mapped to $\text{Tr}_{\text{int}}(|\Psi\rangle\langle\Psi|)$ [6]. As this state is mixed and even independent of the black hole’s initial state the evolution is non-unitary. Whilst non-unitary evolutions are allowed for subsystems, closed total systems are expected to evolve unitarily [7]. The black hole and its radiation are taken to be a closed total system. This contradiction is called the black-hole information paradox.

Multiple interesting approaches have been proposed in connection with resolving the paradox. Hawking’s semi-classical approximations were questioned [8]. Quantum gravitational corrections to general relativity were proposed that would leave a ‘remnant’ upon evaporation [9–13]. Modifying quantum theory through nonlinear effects, non-violent nonlocal effects, and generalized probabilistic theories was considered [14–18]. Of particular relevance here is that Page noted the possibility of quantum correlations between the early emitted radiation and the late radiation such that the total radiation state could be pure, but subsystems of the radiation mixed [19, 20]. There, Don Page derived his famous curve of the entanglement entropy in his particular model, it starts from zero (pure state) then it starts to maximally increase, until it reaches the Page time and starts to maximally decrease again until it reaches zero. However, that scenario was argued to be incompatible with the monogamy of entanglement [21, 22]. The ‘out’ modes and ‘int’ modes would be strongly entangled yet the early radiated particles would also be strongly entangled with the late radiated particles. New physical principles and phenomena like ‘complementarity’ and entanglement-breaking (and possibly equivalence-principle violating) ‘firewalls’ have also been considered [23–25]. Models with global unitary dynamics disentangling the radiation from the black hole, which may come with the price of non-causal signalling or a firewall, have also been proposed [26–28].

Here we propose an alternative approach. Our main result is a derivation, showing that upon evaporation, the entanglement and the information is transferred to the outside modes. We assume essentially Hawking’s model. The annihilation process inside the black hole induces a process similar to entanglement swapping which we call ‘conditional entanglement transfer’. After entanglement swapping, particles that never interacted become entangled conditional on a measurement outcome elsewhere. Entanglement swapping has been studied for discrete [29–37] as well as continuous-variable systems [38–44]. We show here that, similarly, there is entanglement transfer conditional on the annihilation of the black hole matter with in-falling Hawking particles. Conditional on the (full) annihilation of the black hole, the outside radiation is indeed entangled and in a pure state. There is no information loss and the final radiation state is one-to-one correspondence with the initial black hole state. No basic principles are violated because we use a standard black hole model and standard quantum theory, and in particular there is no signalling for the same reason that entanglement swapping is not signalling. Another attractive feature of the model is that the conditional transfer of entanglement appears naturally when modelling the annihilation explicitly as opposed to being postulated.

We proceed as follows. We carefully describe the evaporation process. Assuming the interaction between the ‘int-particles’ and the black hole matter, we show how the entanglement of the black hole matter gets transferred to the ‘out’ modes upon annihilation. Afterwards, we show that there is no information loss because the initial state of the black hole matter is in a one-to-one correspondence with the state of the radiation after full evaporation.

However, the problem of the interaction of int-particles with the black hole matter has been overlooked in most of the previous works despite being crucial in the complete evaporation of the black hole itself. Indeed, without a careful analysis of the interaction process we cannot believe neither the full evaporation of the black hole nor to the common information loss statement. Therefore, we here present an explicit model of singularity free black hole (in Einstein’s conformal gravity) where the above interactions are fully under control. Indeed, in conformal gravity the black hole spacetime is geodesically complete and nothing can reach $r = 0$ in a finite amount of time (or finite value of the affine parameter for mass-less particles). Hence, all the interactions take place in the fall towards the centre of the black hole. Unitarity is secured by the standard model \mathcal{S} -matrix, which is unitary at any perturbative order.

In the supplementary material (<https://stacks.iop.org/NJP/23/113011/mmedia>), we briefly review entanglement swapping and why it does not allow signalling.

2. Conditional entanglement transfer in black holes

In this section we look carefully at the evaporation process in all its phases until the complete disappearance of the black hole.

The Hawking radiation state (1) describes all the radiated particles, but for a better exposure and analysis of the problem we can focus on one pair being created near the event horizon. Therefore, the state (1) simplifies to⁸:

$$|\psi\rangle = \mathcal{N} \sum_{\omega} e^{-\frac{\pi\omega}{\kappa}} |\omega\rangle^{\text{out}} \otimes |-\omega\rangle^{\text{int}}, \quad (2)$$

up to a normalization factor. We now carefully look at the dynamics inside the black hole. We consider a black hole of mass M as a result of the gravitational collapse of a large number of entangled particles (we will also consider unentangled particles later). We take this state to be pure in order to address the black hole information paradox state⁹ (we will also consider the case of particles that are not entangled). Initially, in this paragraph, for the sake of simplicity we consider only one entangled pair inside the BH described at a time, and later a more general state will be treated. That means we focus on the following matter state inside the black hole:

$$|\phi\rangle = \sum_{\omega'} f(\omega') |\omega'\rangle_A \otimes |\omega'\rangle_B. \quad (3)$$

Therefore, the initial state is given by the tensor product of (2) and (3), namely¹⁰

$$|i\rangle = |\psi\rangle \otimes |\phi\rangle = \mathcal{N} \sum_{\omega'} \sum_{\omega} f(\omega') e^{-\frac{\pi\omega}{\kappa}} |\omega\rangle^{\text{out}} |-\omega\rangle^{\text{int}} |\omega'\rangle_A |\omega'\rangle_B. \quad (4)$$

We now assume that there is indeed an annihilation inside the black hole—see sections 5 and 6 for a justification. We assume this is mediated by a standard model (unitary) scattering \mathcal{S} -matrix, resulting in

$$|\omega' - \omega\rangle_A^{\text{int}} \equiv \mathcal{S} \left(|-\omega\rangle^{\text{int}} \otimes |\omega'\rangle_A \right), \quad (5)$$

$$= \sum_j c_j |\omega' - \omega, j\rangle_A^{\text{int}}, \quad (6)$$

where $|\omega' - \omega\rangle_A^{\text{int}}$ is some state with energy $\omega' - \omega$. Our analysis will not depend on the details of that state. The index j represents the number of (scalar field) particles resulting from the scattering. The amplitudes c_j are determined by the Feynman diagram of each process. Therefore, after the interaction has occurred, the state is:

$$|f\rangle = \mathcal{S}|i\rangle = \mathcal{N} \sum_{\omega', \omega} f(\omega') e^{-\frac{\pi\omega}{\kappa}} |\omega\rangle^{\text{out}} |\omega' - \omega\rangle_A^{\text{int}} |\omega'\rangle_B. \quad (7)$$

Now $|f\rangle$ is our new state and the mass of the black hole is reduced to $M - \omega$. Note that there is a sum over the ω 's, so the black hole is in a superposition of energy eigenstates. In other words, the above state is not an eigenstate of the energy or any other observable, and we are not considering any probability transition from $|i\rangle$ to $|f\rangle$. The latter state is simply the most general superposition of states after the interaction had taken place¹¹. In other words we are not making any measurement, but we are just assuming that the interaction happens. There is only another option, namely the interaction does not take place and the state stays (4). Moreover, for the sake of generality, the state $|\omega' - \omega\rangle_A^{\text{int}}$ in (7) does not mean it is necessarily one particle, the incident particles can scatter to make any allowed number of outgoing particles, even though in figure 2 we draw the case of a single particle state. Let us now consider a second Hawking pair created near the event horizon, namely $|\psi_2\rangle$. Using again (2) for $|\psi_2\rangle$ and assuming $|f\rangle$ as the initial state, the whole system is described by the tensor product $|i'\rangle = |f\rangle \otimes |\psi_2\rangle$ (see figures 2(a) and (b)), namely

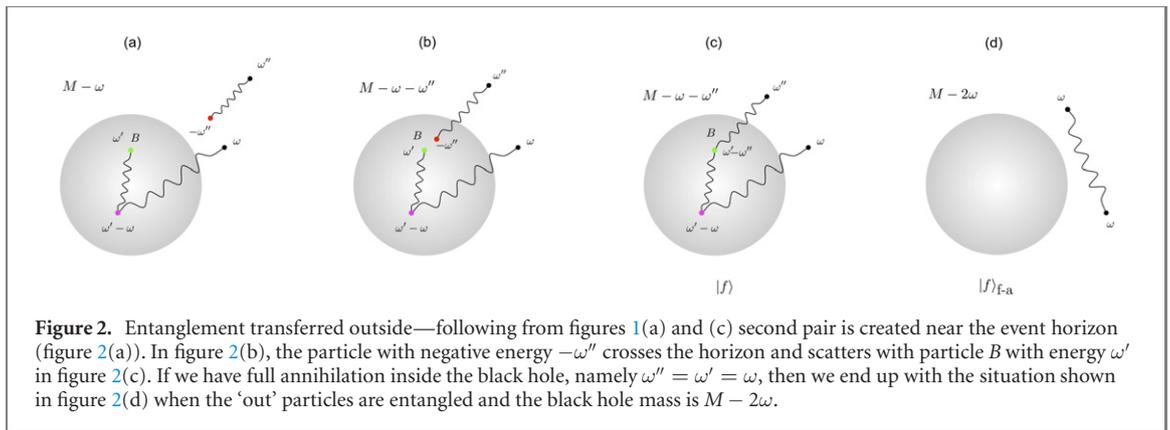
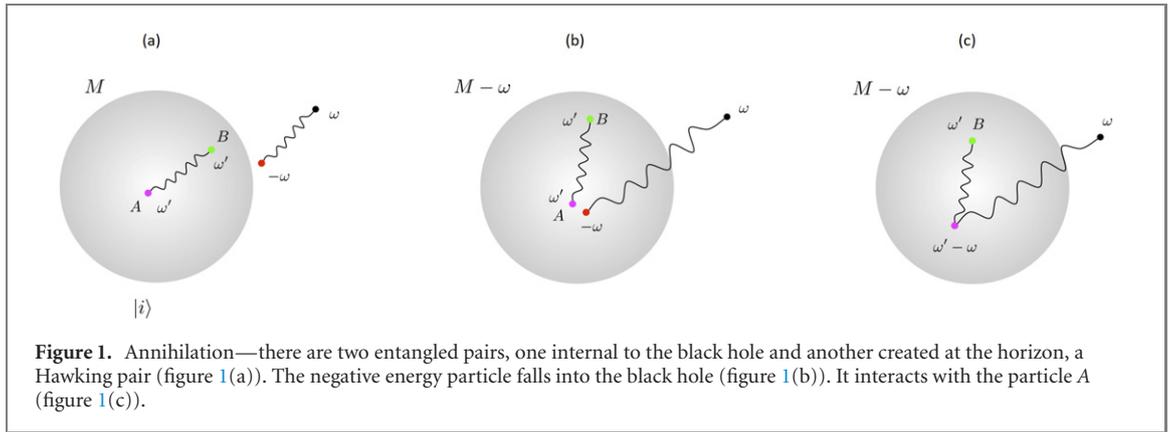
$$|i'\rangle = \mathcal{N}^2 \sum_{\omega'', \omega', \omega} f(\omega') e^{-\pi \left(\frac{\omega}{\kappa} + \frac{\omega''}{\kappa} \right)} |\omega\rangle^{\text{out}} |\omega' - \omega\rangle_A^{\text{int}} |\omega'\rangle_B |\omega''\rangle^{\text{out}} |-\omega''\rangle^{\text{int}}. \quad (8)$$

⁸ It is straightforward to see that the state (1) follows from this state and vice versa.

⁹ The connection between the entropy of the matter forming the black hole and the Bekenstein–Hawking entropy of the black hole is subtle. The Bekenstein–Hawking entropy does not fully depend on the entropy of the matter forming the black hole but only on the mass of the black hole [45].

¹⁰ In general the black hole's state consists of many particles and any number of Hawking pairs, but here for the sake of simplicity we only consider one Hawking pair and two entangled matter particles. We will later consider a significantly more general state.

¹¹ The interaction turns out to be unitary according to the standard model of particle physics, namely the sum over all final states gives the outcome '1'. However, we here just pick out one of these states, but all the arguments in these paper apply to any state consistent with the standard model of particle physics. Moreover, for the sake of simplicity we are here considering only massless scalar particles.



Now say the new created Hawking particle interacts with the particle B, we will get (see figure 2(c))

$$|f'\rangle = \mathcal{N}^2 \sum_{\omega'', \omega', \omega} f(\omega') e^{-\pi \left(\frac{\omega}{\kappa} + \frac{\omega''}{\kappa'} \right)} |\omega\rangle^{\text{out}} |\omega''\rangle^{\text{out}} |\omega' - \omega\rangle_A^{\text{int}} |\omega' - \omega''\rangle_B^{\text{int}}, \quad (9)$$

where we have introduced the notation $|\omega\rangle^{\text{int}} |0\rangle_{A,B} \equiv |\omega\rangle_{A,B}^{\text{int}}$ (see also the discussion after formula (7)). One of the possible states is drawn in figure 2(c) and it consists of two particles inside the black hole partially entangled between each other and with the two Hawking particles outside. Finally, assuming full annihilation of the two particles inside (or in this toy model: full evaporation of the black hole), it is easy to see that one gets

$$|f_{\text{Evap}}\rangle = \mathcal{N}^2 \sum_{\omega} f(\omega) e^{-\pi \omega \left(\frac{1}{\kappa} + \frac{1}{\kappa'(\omega)} \right)} |\omega\rangle^{\text{out}} |\omega\rangle^{\text{out}} |0\rangle_A^{\text{int}} |0\rangle_B^{\text{int}}, \quad (10)$$

which is clearly an entangled pair outside the black hole (see figure 2(d)). The pure state (3) has evolved in a similar pure state (10). Notice that we are not performing any measurement: the state (10) is only one of the possible final states all of which are pure.

If we now trace out the ‘int’ system the state (10) stays the same. In figure 3, the same scenario is represented in the Penrose diagram for the full black hole formation and evaporation process.

Notice that nothing changes if the second Hawking particle interacts with the particle A (instead of B). The whole process could eventually take longer but will be qualitatively the same. Moreover, it is possible that the incident Hawking particle scatters to produce more than one particle inside the black hole. In this case a multipartite entangled state is created (similar to the one we will study in section 3.5).

3. Interim summary and generalisations

We now summarise the arguments so-far before giving generalisations.

3.1. Interim summary

The black hole evaporation process is caused by a negative energy flux across the event horizon that balances the positive energy flux at infinity. Outside by near the horizon are produced virtual pairs from the

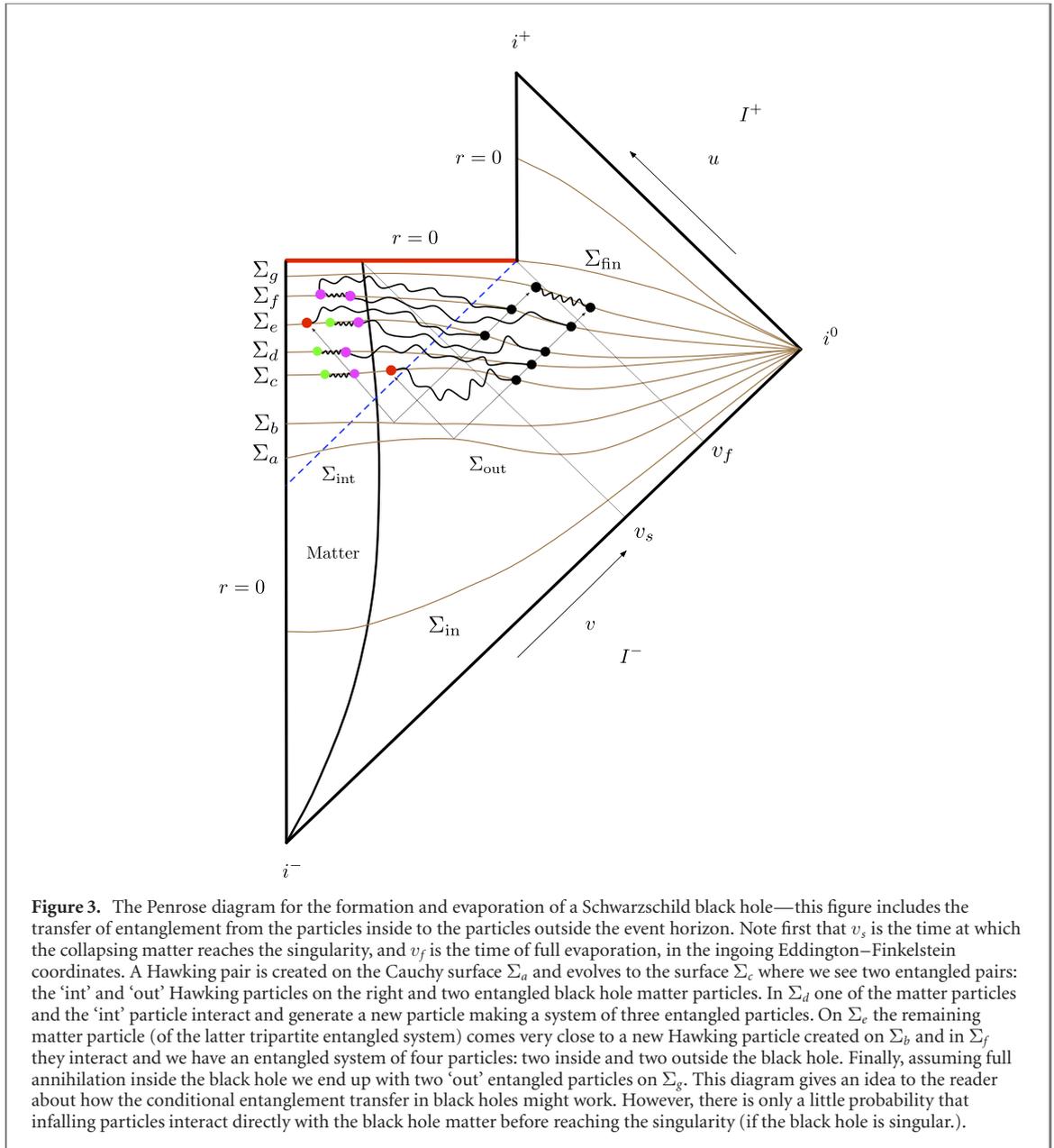


Figure 3. The Penrose diagram for the formation and evaporation of a Schwarzschild black hole—this figure includes the transfer of entanglement from the particles inside to the particles outside the event horizon. Note first that v_s is the time at which the collapsing matter reaches the singularity, and v_f is the time of full evaporation, in the ingoing Eddington–Finkelstein coordinates. A Hawking pair is created on the Cauchy surface Σ_a and evolves to the surface Σ_c where we see two entangled pairs: the ‘int’ and ‘out’ Hawking particles on the right and two entangled black hole matter particles. In Σ_d one of the matter particles and the ‘int’ particle interact and generate a new particle making a system of three entangled particles. On Σ_e the remaining matter particle (of the latter tripartite entangled system) comes very close to a new Hawking particle created on Σ_b and in Σ_f they interact and we have an entangled system of four particles: two inside and two outside the black hole. Finally, assuming full annihilation inside the black hole we end up with two ‘out’ entangled particles on Σ_g . This diagram gives an idea to the reader about how the conditional entanglement transfer in black holes might work. However, there is only a little probability that infalling particles interact directly with the black hole matter before reaching the singularity (if the black hole is singular.).

vacuum, one of negative energy and one with positive energy. The negative energy particle is in a region which is classically forbidden but it can tunnel through the event horizon to the region inside the black hole where the Killing vector, which represents the time translation invariance, is now spacelike. In this region the particle exists as a real particle with a timelike momentum vector even though its energy relative to an observer at infinity is negative. The other particle of the pair, having a positive energy, can escape to infinity where it will constitute part of the thermal radiation.

A quite general pure state that captures the above interpretation consists on N particles (the black hole’s matter) and n Hawking pairs,

$$|\psi_{\text{BH}}\rangle \otimes |\phi_{\text{H}}\rangle = \sum_{\omega} \sum_{\omega_1} \sum_{\omega_2} \dots \sum_{\omega_n} f(\omega) |\omega\rangle_{A_1} |\omega\rangle_{A_2} \dots |\omega\rangle_{A_N} \otimes \otimes e^{-\frac{\pi\omega_1}{\kappa}} |\omega_1\rangle^{\text{out}} |-\omega_1\rangle^{\text{int}} \otimes e^{-\frac{\pi\omega_2}{\kappa_2}} |\omega_2\rangle^{\text{out}} |-\omega_2\rangle^{\text{int}} \otimes \dots \otimes e^{-\frac{\pi\omega_n}{\kappa_n}} |\omega_n\rangle^{\text{out}} |-\omega_n\rangle^{\text{int}}. \quad (11)$$

After a long but finite amount of time the total energy of the black hole for an observer at infinity will be zero regardless of whether the annihilation processes took place inside the black hole. This is not due to any measurement of the observer, but a consequence of the Hawking’s pair production, which inexorably gradually carries away mass to the black hole. Indeed, if we wait enough time we will surely reach a configuration in which the total energy of the black hole for an observer at infinity is zero, and this is not due to any measurement, but it is a consequence of the unrelenting Hawking’s pair production. Therefore,

it is unavoidable that the int-particles annihilate the matter inside the black hole and the final state will be:

$$U(|\psi_{\text{BH}}\rangle \otimes |\phi_{\text{H}}\rangle) = \sum_{\omega} f(\omega) e^{-\pi\omega\left(\frac{1}{\kappa} + \frac{1}{\kappa_2} + \dots + \frac{1}{\kappa_n}\right)} |\omega\rangle^{\text{out}} |\omega\rangle^{\text{out}} \dots |\omega\rangle^{\text{out}} |0\rangle_{A_1}^{\text{int}} \dots |0\rangle_{A_n}^{\text{int}}, \quad (12)$$

which is a state of entangled particles at infinity. Hence, we can conclude that the initial state is evolved into a pure final state.

3.2. Product state inside the black hole: single-particle case

For completeness we also study the case in which the particle inside the black hole is not entangled with any other subsystem (we call this particle 'A'). Therefore, the state (3) is replaced with

$$|\phi_2\rangle = \sum_{\omega'} f(\omega') |\omega'\rangle_A. \quad (13)$$

An analysis similar to the one in (7), gives the following final state $|f''\rangle$,

$$\begin{aligned} |f''\rangle &= \mathcal{N} \sum_{\omega, \omega'} f(\omega') e^{-\frac{\pi\omega}{\kappa}} |\omega\rangle^{\text{out}} |\omega' - \omega\rangle^{\text{int}} |0\rangle_A = \mathcal{N} \sum_{\omega} f(\omega) e^{-\frac{\pi\omega}{\kappa}} |\omega\rangle^{\text{out}} |0\rangle^{\text{int}} |0\rangle_A \\ &+ \sum_{\omega' \neq \omega} f(\omega') e^{-\frac{\pi\omega}{\kappa}} |\omega\rangle^{\text{out}} |\omega' - \omega\rangle^{\text{int}} |0\rangle_A \equiv |f''\rangle_{\text{case (i)}} + |f''\rangle_{\text{case (ii)}}. \end{aligned} \quad (14)$$

Assuming full annihilation, we end up with the pure state $|f''\rangle_{\text{case (i)}}$. Indeed, the initial non entangled pure state has evolved to a non entangled pure state as well. Only in the intermediate stage the created Hawking pair is entangled.

3.3. Product state inside the black hole: two-particle case

Or we can assume having two particles inside the black hole, but in a product state. That is,

$$|p\rangle = |\omega_1\rangle_A \otimes |\omega_2\rangle_B. \quad (15)$$

Now, we consider two created Hawking pairs, each pair's incident particle interacts with one bh particle. We thus get,

$$|p_2\rangle = \mathcal{N}^2 \sum_{\omega', \omega''} e^{-\frac{\pi\omega'}{\kappa'} - \frac{\pi\omega''}{\kappa''}} |\omega_1 - \omega'\rangle_A \otimes |\omega_2 - \omega''\rangle_B \otimes |\omega'\rangle_{\text{out}} \otimes |\omega''\rangle_{\text{out}}. \quad (16)$$

Now as always, upon evaporation, the inside modes should fully annihilate,

$$|p_{\text{evap}}\rangle = \mathcal{N}^2 e^{-\frac{\pi\omega_1}{\kappa'} - \frac{\pi\omega_2}{\kappa''}} |0\rangle_A \otimes |0\rangle_B \otimes |\omega_1\rangle_{\text{out}} \otimes |\omega_2\rangle_{\text{out}}. \quad (17)$$

We get, therefore, a product state outside, in a one to one with the set of states we started with.

3.4. General state inside

For the analysis developed in the previous section we assumed all the particles inside the black hole to have the same energy, but it is straightforward to generalize to an arbitrary entangled state. Let us consider again a Hawking pair in the state (2), and a particle pair inside the black hole in the state $|\chi\rangle$ defined as

$$|\chi\rangle = \sum_{\omega'} f(\omega') |g(\omega')\rangle_A |\omega'\rangle_B, \quad (18)$$

where $g(\omega')$ is a general function of its argument. A pure bipartite entangled state can always be written in the form 18. The initial state (4) is replaced with $|i_g\rangle = |\psi\rangle \otimes |\chi\rangle$ and, if we assume the negative energy particle to interact with the particle B, the final state is:

$$|f_g\rangle = \mathcal{N} \sum_{\omega', \omega} f(\omega') e^{-\frac{\pi\omega}{\kappa}} |\omega\rangle^{\text{out}} |g(\omega')\rangle_A |\omega' - \omega\rangle^{\text{int}} |0\rangle_B \quad (19)$$

$$= \mathcal{N} \sum_{\omega} f(\omega) e^{-\frac{\pi\omega}{\kappa}} |\omega\rangle^{\text{out}} |g(\omega)\rangle_A |0\rangle^{\text{int}} |0\rangle_B + \mathcal{N} \sum_{\omega' \neq \omega} f(\omega') e^{-\frac{\pi\omega}{\kappa}} |\omega\rangle^{\text{out}} |g(\omega')\rangle_A |\omega' - \omega\rangle^{\text{int}} |0\rangle_B. \quad (20)$$

If we have annihilation, only the first term on the right-hand side of (20) survives (figure 1). However, the general case (19) is again elucidated in figure 2.

3.5. Multipartite entangled black hole matter

We now consider a general multipartite entangled pure state describing a black hole resulting from a gravitational collapse. For the sake of simplicity we do not here consider initial mixed states. However, our analysis applies in that case too. This will also help to understand the previously mentioned case where the incident Hawking particle scatters inside the black hole to produce more than one particle.

The multipartite matter state is a generalization of the simple bipartite state given in (18), namely

$$|\Psi\rangle = \sum_{\omega_1, \dots, \omega_k} f(\omega_1, \dots, \omega_k) |\omega_1\rangle_{A_0} |g_1(\omega_1, \dots, \omega_k)\rangle_{A_1} |g_2(\omega_1, \dots, \omega_k)\rangle_{A_2} \dots |g_k(\omega_1, \dots, \omega_k)\rangle_{A_k}, \quad (21)$$

where $f(\omega_1, \dots, \omega_k)$ is a general phase factor and A_0, \dots, A_k are $k + 1$ particles. Now consider an incident Hawking particle of energy ω that scatters with the particle A_0 to produce a particle of energy $\omega_1 - \omega$. The state of the whole system, before the interaction takes place, is the tensor product of (2) and (21), namely $|\Psi'\rangle \equiv |\Psi\rangle \otimes |\psi\rangle$,

$$|\Psi'\rangle = \sum_{\omega_1, \dots, \omega_k, \omega} f(\omega_1, \dots, \omega_k) e^{-\frac{\omega\omega_1}{\kappa}} |\omega_1\rangle_{A_0} |g_1(\omega_1, \dots, \omega_k)\rangle_{A_1} |g_2(\omega_1, \dots, \omega_k)\rangle_{A_2} \dots |g_k(\omega_1, \dots, \omega_k)\rangle_{A_k} \otimes |\omega\rangle^{\text{int}} |\omega\rangle^{\text{out}}. \quad (22)$$

When the ‘int’ particle interacts with the particle A_0 the state becomes:

$$|\Psi''\rangle = \sum_{\omega_1, \dots, \omega_k, \omega} f(\omega_1, \dots, \omega_k) e^{-\frac{\omega\omega_1}{\kappa}} |\omega_1\rangle_{A_0} |\omega_1 - \omega\rangle |g_1(\omega_1, \dots, \omega_k)\rangle_{A_1} |g_2(\omega_1, \dots, \omega_k)\rangle_{A_2} \dots |g_k(\omega_1, \dots, \omega_k)\rangle_{A_k} \otimes |\omega\rangle^{\text{out}}. \quad (23)$$

Therefore, the resulting particle of energy $\omega_1 - \omega$ is entangled with the black hole matter and the Hawking ‘out’ particle too. If more Hawking pairs are created, we have more ‘out’ particles entangled with the black hole matter and the state is:

$$\begin{aligned} |\Psi^{(k)}\rangle &= \sum_{\omega_1, \dots, \omega^{(k)}} f(\omega_1, \dots, \omega_k) e^{-\pi\left(\frac{\omega}{\kappa} + \frac{\omega'}{\kappa'} + \frac{\omega''}{\kappa''} + \dots\right)} |\omega_1\rangle_{A_0} |\omega_1 - \omega\rangle |g_1(\omega_1, \dots, \omega_k)_{A_1} - \omega'\rangle \dots \\ &\times |g_k(\omega_1, \dots, \omega_k)_{A_k} - \omega^{(k)}\rangle_{\text{BH}} \otimes |\omega, \omega', \dots, \omega^{(k)}\rangle^{\text{out}}, \end{aligned} \quad (24)$$

where the sum above is on all the frequencies $\omega_1, \dots, \omega_k, \omega, \omega', \omega'', \dots, \omega^{(k)}$. Now we have an entangled state involving all the particles inside and outside. If we assume full evaporation¹² of the black hole, the entanglement is swapped to the outside radiation and the state reads:

$$\begin{aligned} \sum_{\omega_1, \dots, \omega_k} f(\omega_1, \dots, \omega_k) e^{-\pi\left(\frac{\omega}{\kappa_1} + \frac{g_1}{\kappa_1 g_1} + \frac{g_2}{\kappa_2 g_2} + \dots\right)} |0\rangle_{\text{BH}} \otimes |\omega_1\rangle_{A_0}^{\text{out}} \\ \times |g_1(\omega_1, \dots, \omega_k)_{A_1}^{\text{out}} |g_2(\omega_1, \dots, \omega_k)_{A_2}^{\text{out}} \dots |g_k(\omega_1, \dots, \omega_k)_{A_k}^{\text{out}}, \end{aligned} \quad (25)$$

where we labelled the states also with the index A_i to keep track of the ‘int’ particles that have been annihilated with the particles A_1, \dots, A_k .

The state (25) is clearly an entangled pure state of Hawking’s ‘out’ particles after the black hole has fully evaporated. Notice that the state (24) is a superposition of all energy’s eigenstates. Therefore, the projection to the particular final state (25) is only due to the black hole full evaporation and not to an intrinsic unitarity violation.

The outcome of this section can be summarized as follows. The pure entangled state describing matter inside the black hole (21) evolves into the pure entangled state at I^+ (25). We here only assumed annihilation inside the black hole between negative and positive energy particles.

4. No loss of information

We consider again the black hole and radiation states in the same settings. We recall the black hole’s initial state

$$|\phi\rangle = \sum_{\omega'} f(\omega') |\omega'\rangle_A \otimes |\omega'\rangle_B, \quad (26)$$

¹² This is equivalent to saying that an observer at infinity makes a measurement of the black hole mass.

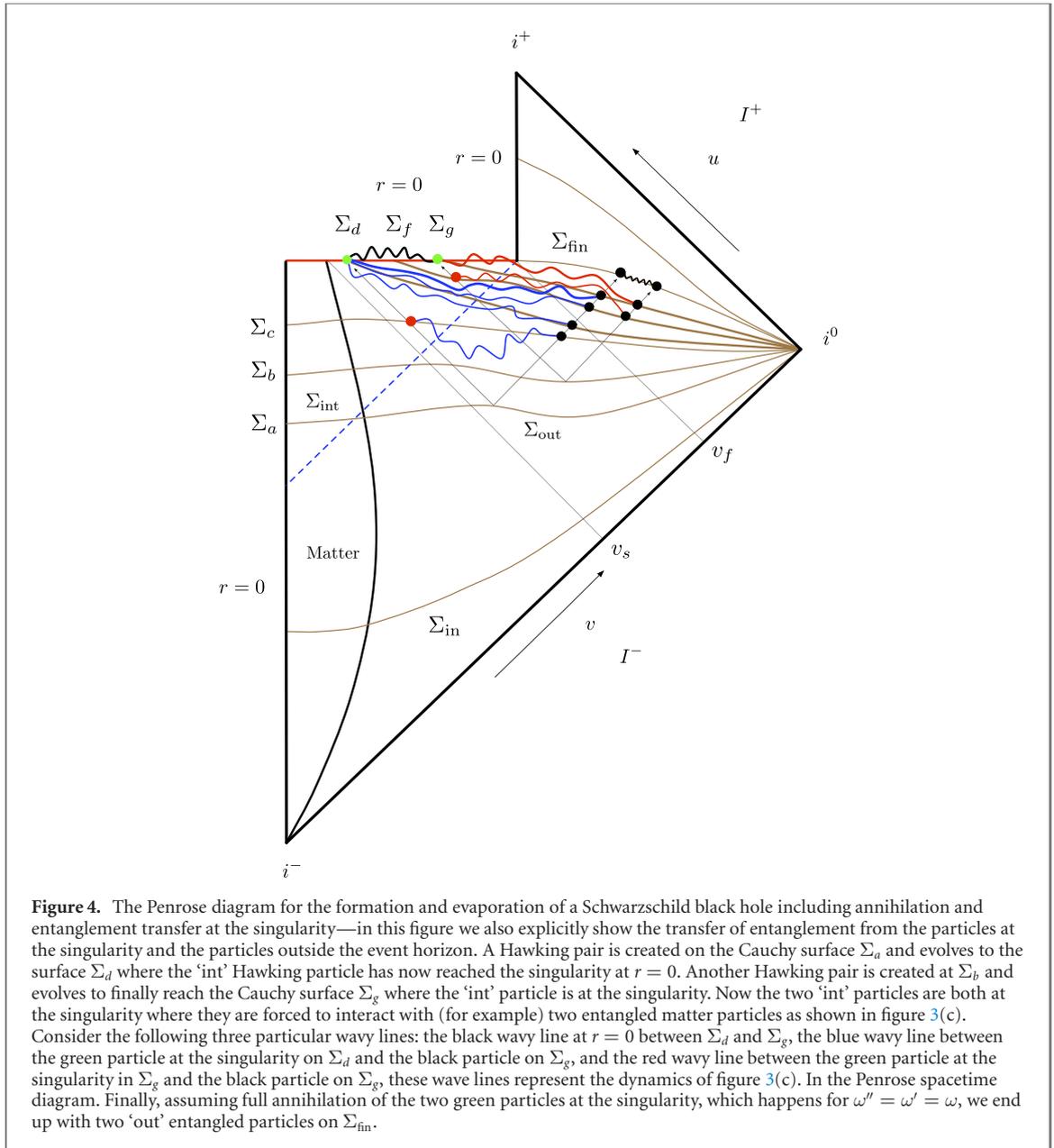


Figure 4. The Penrose diagram for the formation and evaporation of a Schwarzschild black hole including annihilation and entanglement transfer at the singularity—in this figure we also explicitly show the transfer of entanglement from the particles at the singularity and the particles outside the event horizon. A Hawking pair is created on the Cauchy surface Σ_d and evolves to the surface Σ_d where the ‘int’ Hawking particle has now reached the singularity at $r = 0$. Another Hawking pair is created at Σ_b and evolves to finally reach the Cauchy surface Σ_g where the ‘int’ particle is at the singularity. Now the two ‘int’ particles are both at the singularity where they are forced to interact with (for example) two entangled matter particles as shown in figure 3(c). Consider the following three particular wavy lines: the black wavy line at $r = 0$ between Σ_d and Σ_g , the blue wavy line between the green particle at the singularity on Σ_d and the black particle on Σ_g , and the red wavy line between the green particle at the singularity in Σ_g and the black particle on Σ_g , these wave lines represent the dynamics of figure 3(c). In the Penrose spacetime diagram. Finally, assuming full annihilation of the two green particles at the singularity, which happens for $\omega'' = \omega' = \omega$, we end up with two ‘out’ entangled particles on Σ_{fin} .

then, after emission of Hawking particles, we get

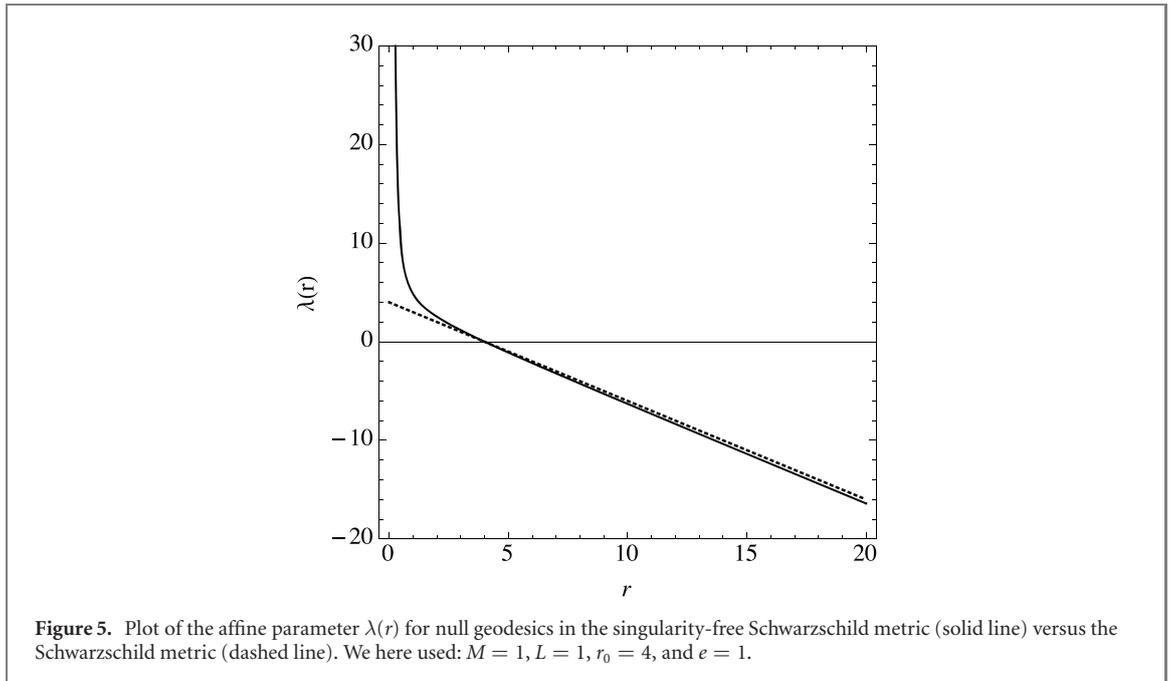
$$|f'\rangle = \mathcal{N}^2 \sum_{\omega'', \omega', \omega} f(\omega') e^{-\pi \left(\frac{\omega}{\kappa} + \frac{\omega''}{\kappa''} \right)} |\omega\rangle_{out} |\omega''\rangle_{out} \otimes |\omega' - \omega\rangle_A^{int} |\omega' - \omega''\rangle_B^{int}. \quad (27)$$

Here one can understand this annihilation state as a scattering outcome, it could include all possible numbers of scattering, for example,

$$|\omega' - \omega\rangle_A^{int} \equiv \mathcal{S} \left(|-\omega\rangle^{int} \otimes |\omega'\rangle_A \right) \quad (28)$$

$$= c_1 |\omega' - \omega, 1\rangle_A^{int} + c_2 |\omega' - \omega, 2\rangle_A^{int} + \dots + c_n |\omega' - \omega, n\rangle_A^{int}, \quad (29)$$

where the second index in each of the kets determines the number of particles outcome of the scattering, and the coefficients c_j determine the amplitude of each number state, determined by the Feynman diagram of each process. However, all the outgoing scattering particles have the same energy which is conserved after scattering. We label them thus with their energy, as in (29), coarse graining their number, since, in such a highly dynamic spacetime, the number of particles is not conserved, changing all the time.



Now in order to study the information loss in this process, we start from the final state:

$$|f_{\text{Evap}}\rangle = \mathcal{N}^2 \sum_{\omega} f(\omega) e^{-\pi\omega \left(\frac{1}{\kappa} + \frac{1}{\kappa'(\omega)} \right)} |\omega\rangle^{\text{out}} |\omega\rangle^{\text{out}} \otimes |0\rangle_A^{\text{int}} |0\rangle_B^{\text{int}}, \quad (30)$$

and the evolution operator that takes the initial black hole matter state to the final fully evaporated state

$$O = \mathcal{N}^2 \sum_{\omega} e^{-\pi\omega \left(\frac{1}{\kappa(\omega)} + \frac{1}{\kappa'(\omega)} \right)} |\omega_{\text{out}}, \omega_{\text{out}}, 0_{\text{int}}, 0_{\text{int}}\rangle \langle 0_{\text{out}}, 0_{\text{out}}, \omega_{\text{int}}, \omega_{\text{int}}|, \quad (31)$$

we would like to see whether we can reconstruct the initial state. Indeed it is quite simple to show that we can invert the evolution operator O and thus reconstruct the initial state. First of all, one can extract the relative factors in O from the measurement of ω , namely

$$\mathcal{N}^2 e^{-\pi\omega \left(\frac{1}{\kappa(\omega)} + \frac{1}{\kappa'(\omega)} \right)} = \langle 0, 0, \omega, \omega | O | 0, 0, \omega, \omega \rangle. \quad (32)$$

Then one can use the final state (30) to construct the initial one,

$$\begin{aligned} |\text{initial}\rangle &= \sum_{\omega} (\langle 0, 0, \omega, \omega | O | 0, 0, \omega, \omega \rangle)^{-1} \mathcal{N}^2 f(\omega) e^{-\pi\omega \left(\frac{1}{\kappa} + \frac{1}{\kappa'(\omega)} \right)} |\omega\rangle^{\text{out}} |\omega\rangle^{\text{out}} |0\rangle_A^{\text{int}} |0\rangle_B^{\text{int}} \\ &= \sum_{\omega'} f(\omega') |\omega'\rangle_A \otimes |\omega'\rangle_B. \end{aligned}$$

Thus, knowing the final state for radiation, and knowing the evolution operator equation (31), one can easily reconstruct the initial black hole matter state. Therefore, no information is lost after full evaporation.

5. The singularity issue

There are reasons to believe that our solution of the information loss problem seems to work regardless of whether the spacetime is singular or singularity-free [10, 11, 46–52, 54, 57]. Therefore, in this section we do not intend to address the singularity issue, but only provide some logical arguments on its role in the information loss problem. Indeed, our result seems to be valid for any black hole whose geometry allows interactions between the black hole's matter and the infalling Hawking particles. On the other hand, for black holes where such particles do not interact, there is no reason for the evaporation to happen, as we are going to explain. In the previous sections we never mention the spacetime singularity issue at $r = 0$. Indeed,

our analysis is based on the natural and historical commonly made assumption that particles inside the black hole are annihilated by the Hawking negative energy particles.

Now let us make some comments on the particular case of a singular black hole. As long as the ‘int’ particles interact with the matter inside the black hole that have not reached $r = 0$ yet, as in figure 4 for $v < v_s$, where v_s is the time at which the singularity is formed, the dynamics (the S -matrix of the standard model of particle physics) is well defined and the scattering takes place without violating unitarity. On the other hand, for $v > v_s$ the ‘int’ particles probably annihilate with matter particles that have already reached the singularity (see figure 5). In this paper as well as most others in the literature, it is assumed that the annihilation takes place regardless of the singularity¹³. Therefore, we are entitled to believe that if a singular black hole ever evaporates, then entanglement is transferred to (and/or from) the matter at the singularity. On the other hand, if there is no annihilation at the singularity we probably¹⁴ do not have evaporation and thus any information loss problem because there are correlations between the matter inside and the particles outside the black hole, that keep the state of the whole system pure. It is worth being stressed that the absence of local (or non-local) interactions between Hawking ‘int’ particles and the matter at the singularity implies that there is no black hole evaporation at all, contrary to what is commonly stated¹⁵. Furthermore, we do not have any information loss problem because the matter would still be there, and the Hawking particles would still be there too. Indeed, the whole information loss business is based on the assumption that the black hole completely evaporates (or nearly) and most of the mass evaporates after the creation of the singularity (the time v_s as depicted in figure 3). If we question the interaction of the ‘int’ particles with the singularity then we cannot trust the black hole evaporation after the instant v_s . However, at this stage of the evaporation process the black hole retains most of its mass, which is enormously bigger than the Planck mass. Why in such semiclassical regime should we not believe in the black hole evaporation? We here do not want to address this question, but we want only to point out that our resolution of the information loss problem is based on very reasonable and common assumptions.

Finally, in any singularity-free black hole our proof is *a priori* expected to apply and there is no information loss problem because in this case the spacetime is geodesically complete and the needed interactions for $v > v_s$ can happen smoothly. In the next section, we will provide an explicit example of singularity-free black hole in Einstein’s conformal gravity in which all the interactions for $v > v_s$ take place far from $r = 0$.

¹³ As proved in the paper [53], titled ‘The energy-momentum tensor of a black hole, or what curves the Schwarzschild geometry?’, the source of the Ricci flat solutions (in vacuum) has a well defined meaning in the space of distributions and the energy-momentum tensor is proportional to the Dirac’s delta, namely $T \propto M\delta(\mathbf{r})$ (this is also proved in many other textbooks like Landau–Lifshitz, etc). After the black hole formation, the matter is localized at $r = 0$ and can be reached in finite time (or finite value of the affine parameter in the massless case) by the Hawking ‘int’ particles. Therefore, all the ‘int’ particles annihilate for $r > 0$ in the first stage of the evaporation process or in $r = 0$ afterwards to finally end up with zero Bondi–Sachs mass. Notice that if there was no source at $r = 0$ then the spacetime would be Minkowski and not Schwarzschild.

¹⁴ Particles are likely created also inside the event horizon where the metric is actually Kantowski–Sachs. The latter cosmological metric, which is homogeneous but not isotropic, allows for the creation of particles at any time. However, such process can only make our analysis more complicated without any conceptual gain. Indeed, negative and positive energy particles inside the horizon must annihilate each other and the particle with positive energy cannot escape to infinity if we want to preserve causality. However, this technical complication can turn in our favour. It could be that negative energy particles created inside annihilate the matter, which is collapsing towards the singularity, while the partners with positive energy travel from left to right along or near the horizon annihilating the negative energy particles coming from outside. If so, we never need to consider the singularity and most of the annihilation happens near (inside) the horizon.

¹⁵ Assuming that no annihilation takes place at the singularity, we end up with a state consisting of an equal number of positive and negative energy particles in the black hole interior. Therefore, the mass of the black hole is zero (at least for a distant observer) and the final state is very similar to the one represented in figure 2(c). Although this possibility seems very unlikely from the physical point of view, we do not have any information loss problem. Indeed, after the ‘full’ evaporation we have a pure entangled state consisting on the out-particles in the future and a blob of matter with zero total energy in the past. In general, during the evaporation process we have positive energy particles that travel towards infinity and negative energy particles that reach and eventually cross the horizon. If the ‘int’ particles do not cross the horizon they must annihilate with other matter outside and there is no black hole evaporation. It could be that the black hole geometry is such that the particles seem never to cross the horizon. This is also what one observes sees from infinity in the Schwarzschild geometry. However, once the total amount of negative energy near the horizon is identical (or nearly equal) to M , then the total mass of the black hole for the observer at infinity is zero, there is no event horizon anymore, and the negative energy Hawking particles are forced to annihilate the whole mass inside the black hole. (Notice that the negative energy particles cannot annihilate the ‘out’ particles anymore because those are too far away.) Similarly, once an amount of particles of total mass equal, but opposite in sign, to the black hole mass is inside the black hole, the black hole is not black anymore because there is no more event horizon. Therefore, the two clouds of particles with positive energy (black hole’s matter) and with negative energy (‘int’ Hawking particles) are forced to annihilate. Notice that we cannot have an excess of negative particles with respect to the total amount of black hole mass because the evaporation process stops after the event horizon disappears.

6. Solving the singularity issue

In this section we provide an explicit example of regular and geodesically complete black hole spacetime based on Einstein's conformal gravity. The latter theory is defined by the following action,

$$S = \int d^4x \sqrt{-\hat{g}} (\phi^2 \hat{R} + 6\hat{g}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi), \quad (33)$$

where $\hat{g}_{\mu\nu}$ is the spacetime metric and ϕ is a scalar field (the dilaton). The presence of the dilaton field in the theory enlarges the symmetry from general coordinate invariance to also including conformal Weyl invariance, namely

$$\hat{g}'_{\mu\nu} = \Omega^2 \hat{g}_{\mu\nu}, \quad \phi' = \Omega^{-1} \phi. \quad (34)$$

The Einstein–Hilbert action for gravity,

$$S_{\text{EH}} = \frac{1}{16\pi G} \int d^4x \sqrt{-\hat{g}} \hat{R}, \quad (35)$$

is recovered when the Weyl conformal symmetry is broken spontaneously in exact analogy with the Higgs mechanism in the standard model of particle physics (for more details we refer the reader to [51, 55]). In the conformal invariant phase we have an entire gauge orbit of equivalent vacua (exact solutions of the EoM), but if we replace in the action the vacuum consistent with the solution $\phi = \text{const.} = 1/\sqrt{16\pi G}$, the theory (33) turns into (35). Hence, Einstein's gravity is simply the theory (33) in the spontaneously broken phase of conformal invariance [51].

Besides the constant vacuum, if a metric $\hat{g}_{\mu\nu}$ is an exact solution of the EoM, thus it is also the rescaled spacetime with a non trivial profile for the dilaton field, namely

$$\hat{g}_{\mu\nu}^* = S(r) \hat{g}_{\mu\nu} \quad \phi^* = S(r)^{-1/2} \phi. \quad (36)$$

Therefore, we can use the above rescaling to construct other exact and singularity-free solutions of the theory. In particular, in [51] it was proposed and extensively studied the following black hole (of mass M) metric,

$$ds^{*2} \equiv \hat{g}_{\mu\nu}^* dx^\mu dx^\nu = S(r) \hat{g}_{\mu\nu} dx^\mu dx^\nu = S(r) \left[\left(1 - \frac{2M}{r}\right) dt^2 + \frac{dr^2}{1 - \frac{2M}{r}} + r^2 d\Omega^2 \right], \quad (37)$$

$$\phi^* = S(r)^{-1/2} \kappa_4^{-1}, \quad (38)$$

where the following conformal factor $\Omega^2 = S$ depending only on the radial Schwarzschild coordinate r is

$$S(r) = 1 + \frac{L^4}{r^4}, \quad (39)$$

where L is a scale of length dimension that we will later identify to be proportional to the Schwarzschild radius of the black hole. In [51], it was proved that the Kretschmann invariant $\hat{K} = \hat{\mathbf{Riem}}^2$ and the Ricci scalar are both regular in $r = 0$, but most importantly it was shown that the spacetime is geodesically complete. Indeed, massive, conformally coupled particles, and photons can never reach $r = 0$. For the sake of simplicity, in this paper we focus on mass-less particles and we remind the geodesic radial equation as well as its solution [51], namely

$$-\frac{e^2}{S(r)^2} + \dot{r}^2 = 0 \implies S(r)|\dot{r}| = e, \quad (40)$$

where e is the conserved quantity due to the time translation invariance of the metric. The above differential equation (40) can be easily integrated for a photon trajectory approaching $r = 0$, namely for $\dot{r} < 0$,

$$\lambda(r) = \frac{1}{e} \left[\frac{L^4}{3r^3} - \frac{L^4}{3r_0^3} + \frac{2L^2}{r} - \frac{2L^2}{r_0} - r + r_0 \right], \quad (41)$$

where r_0 is the initial radial position. Hence, photons cannot reach $r = 0$ for any finite value of the affine parameter λ , as it is evident from figure 5. In order to get the same geodesic as in the empty spacetime when the mass goes to zero, we must select the scale $L = \alpha M$, where α is a dimensionless constant.

Therefore, (41) turns in:

$$\lambda(r) = \frac{1}{e} \left[\frac{(\alpha M)^4}{3r^3} - \frac{(\alpha M)^4}{3r_0^3} + \frac{2(\alpha M)^2}{r} - \frac{2(\alpha M)^2}{r_0} - r + r_0 \right]. \quad (42)$$

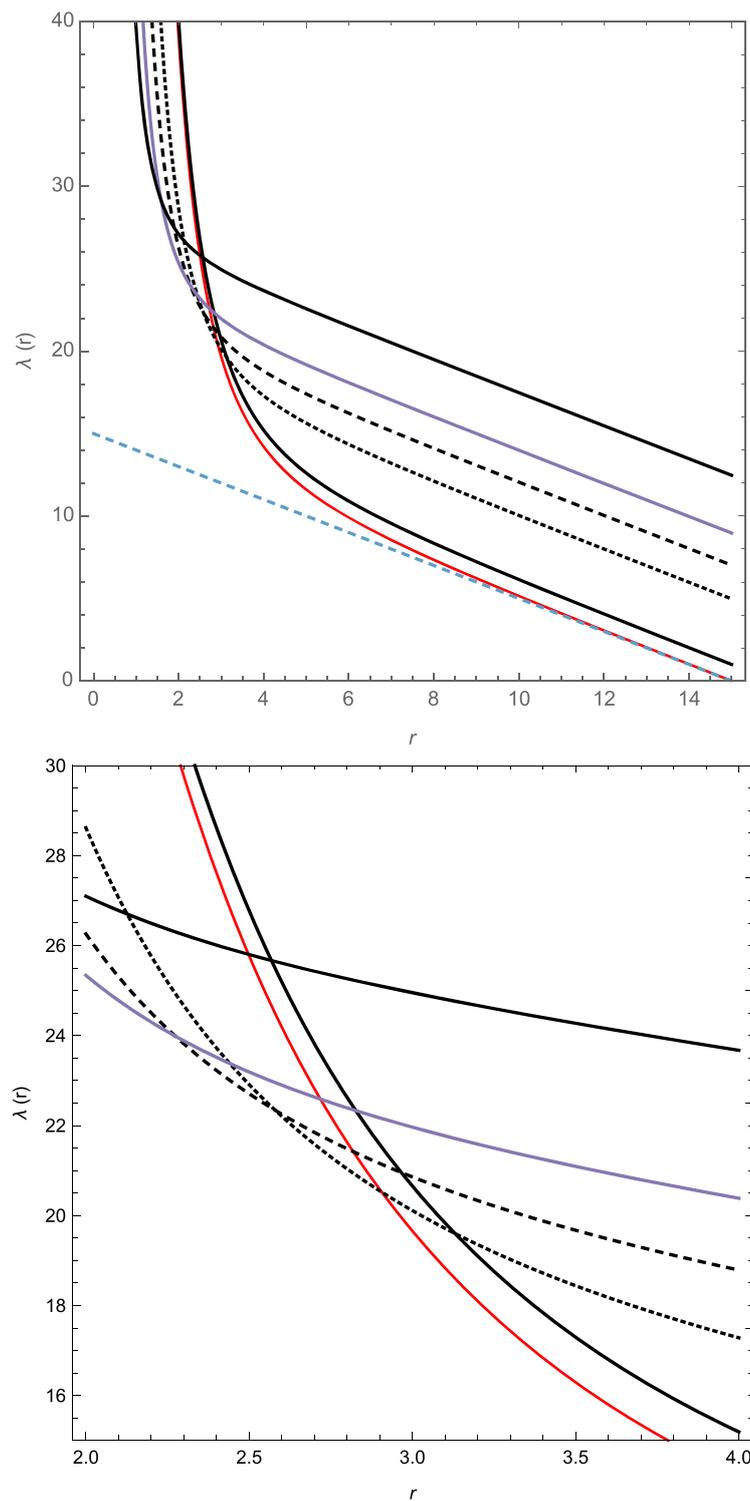


Figure 6. We here show the affine parameter $\lambda(r)$. The plot at the bottom is a zoom of the interacting region shown in the plot at the top. The initial mass of the black hole is $M = 5$ and the event horizon is located in $2M = 10$ while the initial value for the affine parameter is set to be zero in $r_0 = 3M$ for the collapsing shell, while for the other particles $\lambda(r_0) = 3.2M, 5M, 6.3M, 8M, 11M$ respectively. We here assumed natural units. The solid black line represents a particle with negative energy that can never hit the collapsing shell. However, other int-particles created later can hit such particle creating a new particle having even more negative energy, but now able to hit the collapsing matter. Notice that the value of e in (42) is harmless for the crossing of the trajectories. Indeed, a rescaling of λ cannot change the number of the interactions. Hence, we just took $e = 1$ in the plots.

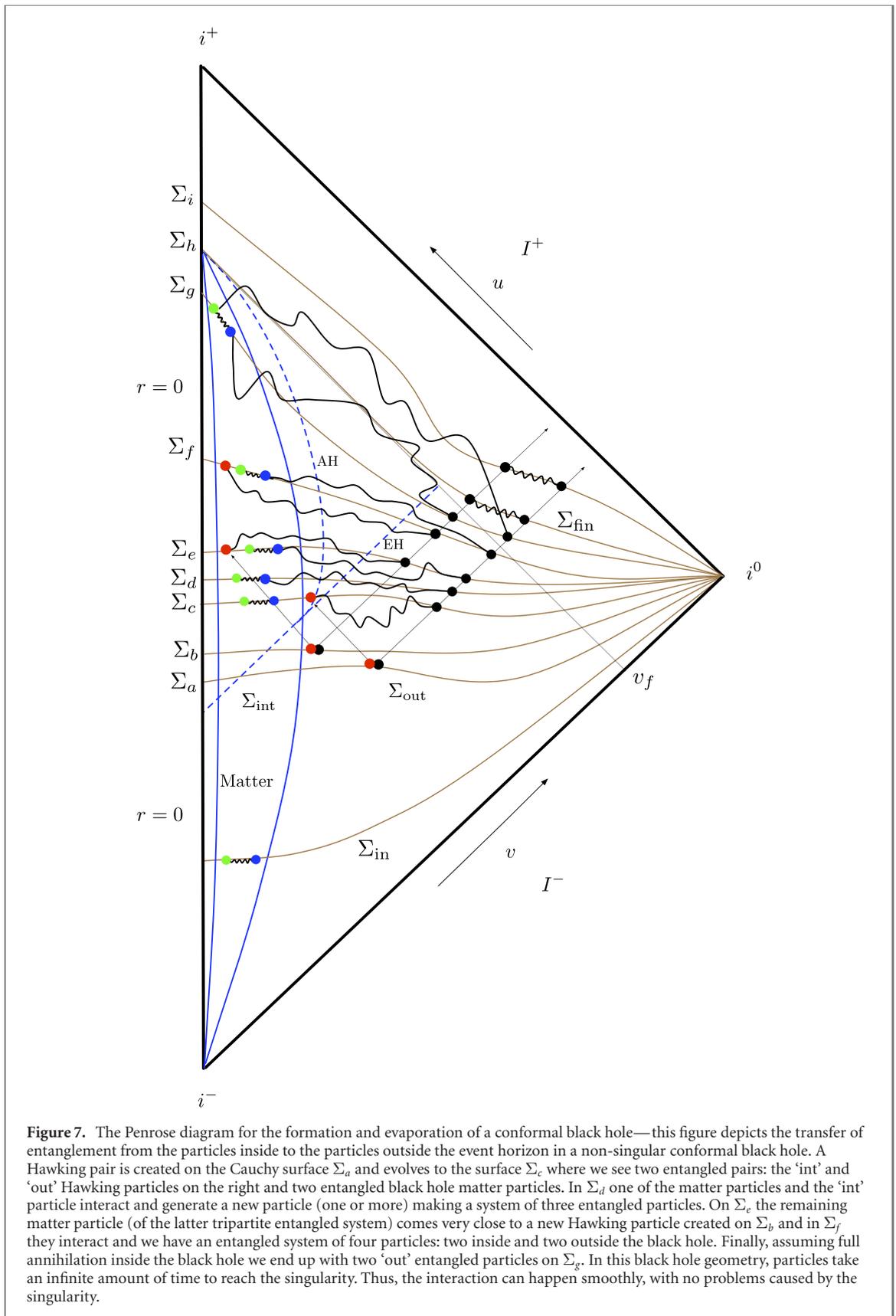


Figure 7. The Penrose diagram for the formation and evaporation of a conformal black hole—this figure depicts the transfer of entanglement from the particles inside to the particles outside the event horizon in a non-singular conformal black hole. A Hawking pair is created on the Cauchy surface Σ_a and evolves to the surface Σ_c where we see two entangled pairs: the ‘int’ and ‘out’ Hawking particles on the right and two entangled black hole matter particles. In Σ_d one of the matter particles and the ‘int’ particle interact and generate a new particle (one or more) making a system of three entangled particles. On Σ_e the remaining matter particle (of the latter tripartite entangled system) comes very close to a new Hawking particle created on Σ_b and in Σ_f they interact and we have an entangled system of four particles: two inside and two outside the black hole. Finally, assuming full annihilation inside the black hole we end up with two ‘out’ entangled particles on Σ_g . In this black hole geometry, particles take an infinite amount of time to reach the singularity. Thus, the interaction can happen smoothly, with no problems caused by the singularity.

6.1. Interactions inside the black-hole

In force of the results of the previous section and making use of toy model for the collapse, we are now able to show that the int-particles cannot avoid to hit the collapsing matter before to reach $r = 0$. We hereby assume both the int-particles and the collapsing matter to be in the form of massless particles. In particular, the collapsing matter is simply described by a null-shell of radiation that follows the geodesic of light, but

whose equation of state is defined by zero pressure. Therefore, assuming a geodesic motion in the semiclassical approximation the trajectory in spacetime of any particle is given by (42). Let us assume that the int-particles are created right after the collapse is started, and for all of them apart the first one the mass of the black hole is reduced of the correct amount consistently with the photons' energy. The process is described in figure 6. The lower dashed trajectory represents the classical geodesic for $L = 0$ in (41), the first one in red is the surface the collapsing null shell, the solid black trajectory is the first created particle (for it the mass of the black hole is still M), the other lines represent the trajectories for the later created int-particles. Notice, that the first created particle sees mass M and does not hit the collapsing shell, but the mass of the black hole shrinks anyway because when the particle cross the event horizon the surface gravity seen from outside is evaluated for a smaller mass. Given the hyperbolic solution (42) trajectories delayed in time surely meet somewhere, as evident from the plots (6). Therefore, the interaction between int-particles and matter take place until complete evaporation of the black hole occurs.

Finally, we remind the main results in the paper [50] about the black hole evaporation time computed making use of the Boltzmann law [5]. The Hawking temperature is a conformal invariant observable, hence, it turns out to be the same computed by Hawking for the Schwarzschild black hole (figure 7), i.e. $T_H = 1/8\pi M$. Also the event horizon area changes slightly, namely

$$A_H = 4\pi r_H^2 \left[1 + \frac{(\alpha M)^4}{r_H^4} \right] \approx 4\pi r_H^2, \quad (43)$$

whether we assume $\alpha \ll 1$ [56]. Therefore, the evaporation time is finite as for the Schwarzschild black hole. Last but not least, the black hole entropy is also conformal invariant, regardless of the corrections in α , when expressed in terms of the black hole mass [50], namely $S = 4\pi M^2$.

7. Comments and conclusions

Let us here summarize our result and make some comments on the usual information loss problem. Assuming no annihilation inside the black hole, the pure state (1) describes 'int' and 'out' radiation. Once we trace out the 'int' subsystem, we find the 'out' radiation in a mixed state. However, this does not imply any unitarity violation because the 'int' particles still exist in the black hole interior. If we now assume that some 'int' particles annihilate, then we must take into account that the entanglement is transferred to other particles inside and/or outside the event horizon through the process described in this paper. Commonly, people do not consider such swap of entanglement and information appears to be lost. On the base of figure 2, the mistake is to trace out the interior of figure 2(c) to end up with two non-entangled particles in figure 2(d), and of course the 'out' radiation is then in a mixed state. Similarly, at the end of the black hole's evaporation process (full annihilation of 'int' particles with the black hole matter), one has to trace out the 'int' states to end up (using the usual treatment) with 'out' particles in a mixed state. In contrast, throughout our analysis we keep track of the entanglement transfer at every step of the evaporation process and we finally get a pure entangled state outside (see (25)) which is in a one to one correspondence with initial states.

Let us summarize step by step the path taken in our paper. The summary consists of the following 5 + 1 items.

- (a) We start with the entangled pure state (3), which describes the black hole matter (in this toy model we consider the black hole made only of two particles with the same energy, but in sections 3.2 and 3.3 we also considered the general case of many particles with different energies.) For completeness, we here remind the reader of the state (3):

$$|\phi\rangle = \sum_{\omega'} f(\omega') |\omega'\rangle_A \otimes |\omega'\rangle_B. \quad (44)$$

- (b) Whereupon, we have the creation of a Hawking pair from the vacuum (state (2))

$$|\psi\rangle = \mathcal{N} \sum_{\omega} e^{-\frac{\pi\omega}{\kappa}} |\omega\rangle^{\text{out}} \otimes |-\omega\rangle^{\text{int}} \quad (45)$$

and the whole state is the tensor product of two entangled states (4), namely

$$|i\rangle = |\psi\rangle \otimes |\phi\rangle = \mathcal{N} \sum_{\omega'} \sum_{\omega} f(\omega') e^{-\frac{\pi\omega}{\kappa}} |\omega\rangle^{\text{out}} |-\omega\rangle^{\text{int}} |\omega'\rangle_A |\omega'\rangle_B. \quad (46)$$

- (c) Now assuming the black hole matter to interact with the ‘int’ Hawking particle, we get the new pure entangled state (9), which is described in figure 2(c).

$$|f\rangle = \mathcal{N} \sum_{\omega', \omega} f(\omega, \omega') e^{-\frac{\pi\omega}{\kappa}} |\omega\rangle^{\text{out}} |\omega' - \omega\rangle^{\text{int}} |0\rangle_A |\omega'\rangle_B. \quad (47)$$

- (d) Since another Hawking pair is surely created we have the new state (8), which is described in figures 2(a) and (b) and represented by the state:

$$|i'\rangle = \mathcal{N}^2 \sum_{\omega'', \omega', \omega} f(\omega, \omega') e^{-\pi\left(\frac{\omega}{\kappa} + \frac{\omega''}{\kappa}\right)} |\omega\rangle^{\text{out}} |\omega' - \omega\rangle_A^{\text{int}} |\omega'\rangle_B |\omega''\rangle^{\text{out}} |-\omega''\rangle^{\text{int}}. \quad (48)$$

- (e) Assuming again to have interaction, we end up with the state (9) (figure 2(c)), namely

$$|f'\rangle = \mathcal{N}^2 \sum_{\omega'', \omega', \omega} f(\omega, \omega', \omega'') e^{-\pi\left(\frac{\omega}{\kappa} + \frac{\omega''}{\kappa}\right)} |\omega\rangle^{\text{out}} |\omega''\rangle^{\text{out}} |\omega' - \omega\rangle_A^{\text{int}} |\omega' - \omega''\rangle_B^{\text{int}}. \quad (49)$$

- (f) Let us now assume that the black hole fully evaporates, which in our toy-model means: $\omega = \omega'$ and $\omega' = \omega''$ (that is, the infalling negative energy particles have energies that sum to the black hole energy). Therefore, the state is (10) and it is an entangled state within ‘out’ particles solely (see figure 2(d)). Here we remind the reader of the state,

$$|f_{\text{Evap}}\rangle = \mathcal{N}^2 \sum_{\omega} f(\omega) e^{-\pi\omega\left(\frac{1}{\kappa} + \frac{1}{\kappa(\omega)}\right)} |\omega\rangle^{\text{out}} |\omega\rangle^{\text{out}} |0\rangle_A^{\text{int}} |0\rangle_B^{\text{int}}. \quad (50)$$

It turns out that after full evaporation all entanglement is transferred to the ‘out’ particles, there is no black hole anymore, and the particles at future infinity are in a pure entangled state without any violation of the monogamy theorem, conservation of information, or equivalence principles. The most straightforward way to check whether there is any loss of information is to look at the final state and notice that -given the evolution of the system- there is a 1 to 1 correspondence between the initial and the final state. In fact the final state is almost identical to the initial state except for a relative phase factor which comes from the Hawking pairs states. We emphasize that the conditional transfer of entanglement from two particles inside the black hole to particles outside the black hole is a result of the full black hole evaporation and not an assumption in our proof. In other words, we do not assume any ‘conditional entanglement transfer’, it is actually the outcome of our computation only assuming full evaporation, energy conservation, and interaction between Hawking infalling particles and the black hole matter.

Therefore, as the reader can see, the information is recovered in the entanglement within the black hole radiation all done in a very standard formalism.

Let us now remark that the observer at infinity does not take any active part in the outcome of our analysis. The system is always in a pure state independently of the observer. The observer only takes part if we want to know in what particular state the black hole is, but the state is pure and information is conserved regardless of the measurement issue. Indeed, each interaction is compatible with the local unitary S-matrix of the standard model of particle physics.

The mistake commonly done is that people do not take care of the interactions inside the black hole and that the black hole is in a superposition of energy eigenstates. Therefore, they do not take into account how entanglement is transferred at any stage of the evaporation process. In this paper we just looked carefully at every single step only assuming local energy conservation and we ended up with the result (10) or (50). Furthermore, pure final states are in a one to one correspondence with all the possible initial states. Hence, there is no information loss, neither violation of monogamy theorem nor of the equivalence principle, and under a minimal number of very natural and common assumptions.

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Data availability statement

No new data were created or analysed in this study.

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